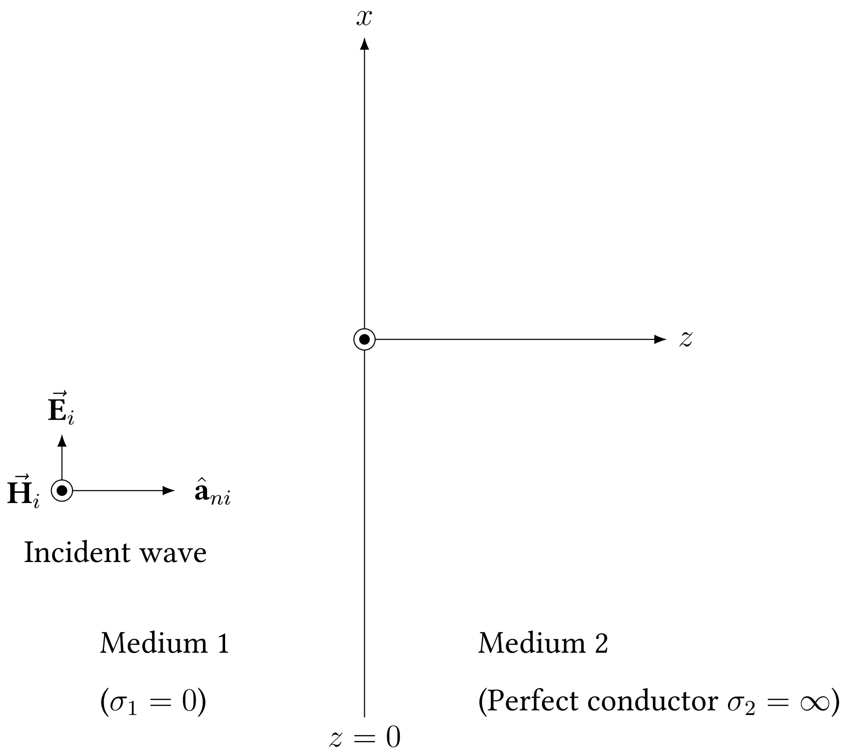
3. Normal Incidence at a Plane Conducting Boundary



$$\vec{\mathbf{E}}_i(z) = \hat{\mathbf{a}}_x \, E_{i0} \, e^{-j\beta_1 z} \tag{1}$$

$$\vec{\mathbf{H}}_i(z) = \hat{\mathbf{a}}_y \, \frac{E_{i0}}{\eta_1} \, e^{-j\beta_1 z} \tag{2}$$

E_{i0} : the magnitude of $\vec{\mathbf{E}}_i$ at $z = 0$

β_1 : phase constant of medium 1

η_1 : intrinsic impedance of medium 1

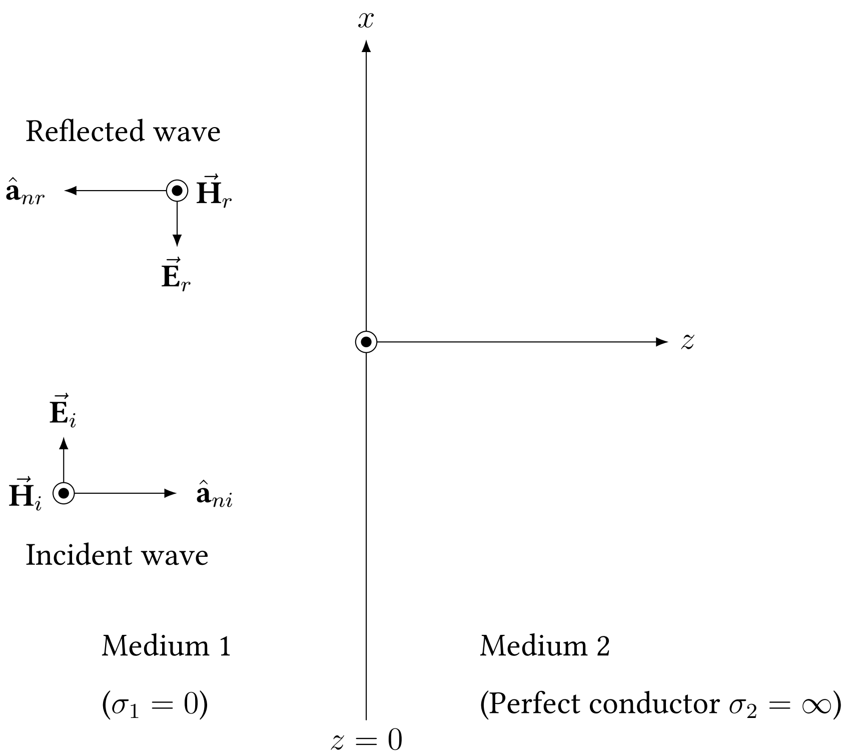
$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} \tag{3}$$

In a perfect conductor electric and magnetic fields are zero.

$$\vec{\mathbf{E}}_2 = 0 \tag{4}$$

$$\vec{\mathbf{H}}_2 = 0 \tag{5}$$

The incident wave is reflected giving rise to a reflected wave $(\vec{\mathbf{E}}_r, \vec{\mathbf{H}}_r)$.



$$\vec{\mathbf{E}}_r(z) = -\hat{\mathbf{a}}_x \, E_{r0} \, e^{j\beta_1 z} \tag{6}$$

Reflected wave travels in $-z$ direction.

Total field in medium 1

$$\vec{\mathbf{E}}_1(z) = \vec{\mathbf{E}}_i(z) + \vec{\mathbf{E}}_r(z) \tag{7}$$

$$\vec{\mathbf{E}}_1(z) = \hat{\mathbf{a}}_x \, E_{i0} \, e^{-j\beta_1 z} - \hat{\mathbf{a}}_x \, E_{r0} \, e^{j\beta_1 z} \tag{8}$$

$$\vec{\mathbf{E}}_1(z) = \hat{\mathbf{a}}_x \, \left(E_{i0} \, e^{-j\beta_1 z} - E_{r0} \, e^{j\beta_1 z}\right) \tag{9}$$

$$\vec{\mathbf{E}}_2 = 0 \tag{10}$$

$$E_{1t} = E_{2t} = 0 \tag{11}$$

$$\vec{\mathbf{E}}_1(0) = 0 = \hat{\mathbf{a}}_x \, (E_{i0} - E_{r0}) \tag{12}$$

$$\Rightarrow \boxed{E_{r0} = E_{i0}} \tag{13}$$

$$\vec{\mathbf{E}}_r(z) = -\hat{\mathbf{a}}_x \, E_{i0} \, e^{j\beta_1 z} \tag{14}$$

$$\vec{\mathbf{E}}_1(z) = \hat{\mathbf{a}}_x \, E_{i0} \, \left(e^{-j\beta_1 z} - e^{j\beta_1 z}\right) \tag{15}$$

$$\vec{\mathbf{E}}_1(z) = \hat{\mathbf{a}}_x \, E_{i0} \, (-2j) \sin \beta_1 z \tag{16}$$

$$\vec{\mathbf{E}}_1(z) = -\hat{\mathbf{a}}_x \, 2j \, E_{i0} \, \sin \beta_1 z \tag{17}$$

$$\vec{\mathbf{H}}_r(z) = \frac{1}{\eta_1} \, \hat{\mathbf{a}}_{nr} \times \vec{\mathbf{E}}_r(z) \tag{18}$$

$$\hat{\mathbf{a}}_{nr} = -\hat{\mathbf{a}}_z \tag{19}$$

$$\vec{\mathbf{H}}_r(z) = \frac{1}{\eta_1} (-\hat{\mathbf{a}}_z) \times (-\hat{\mathbf{a}}_x)\, E_{i0}\, e^{j\beta_1 z} \tag{20}$$

$$\vec{\mathbf{H}}_r(z) = \frac{1}{\eta_1}\, \hat{\mathbf{a}}_y\, E_{i0}\, e^{j\beta_1 z} \tag{21}$$

$$\vec{\mathbf{H}}_1(z) = \vec{\mathbf{H}}_i(z) + \vec{\mathbf{H}}_r(z) \tag{22}$$

$$\vec{\mathbf{H}}_1(z) = \hat{\mathbf{a}}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z} + \hat{\mathbf{a}}_y \frac{E_{i0}}{\eta_1} e^{j\beta_1 z} \tag{23}$$

$$\vec{\mathbf{H}}_1(z) = \hat{\mathbf{a}}_y \frac{E_{i0}}{\eta_1} \left(e^{-j\beta_1 z} + e^{j\beta_1 z}\right) \tag{24}$$

$$\vec{\mathbf{H}}_1(z) = \hat{\mathbf{a}}_y\, 2\, \frac{E_{i0}}{\eta_1}\, \cos\beta_1 z \tag{25}$$

$$\vec{\mathbf{E}}_1(z,t) = \text{Re}\left[\vec{\mathbf{E}}_1(z)\, e^{j\omega t}\right] \tag{26}$$

$$\vec{\mathbf{E}}_1(z,t) = \text{Re}\left[-\hat{\mathbf{a}}_x\, 2j\, E_{i0}\, \sin\beta_1 z\, e^{j\omega t}\right] \tag{27}$$

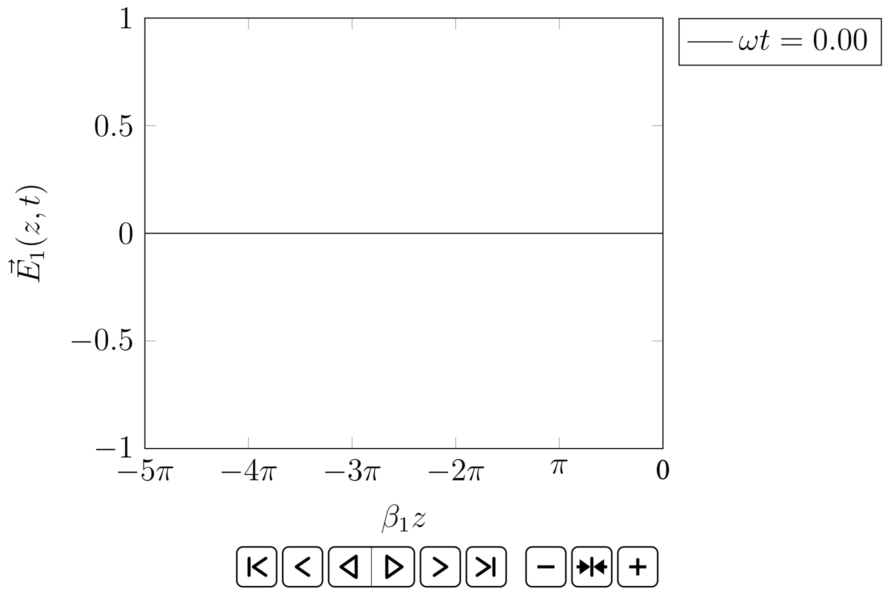
$$\vec{\mathbf{E}}_1(z,t) = \text{Re}\left[-\hat{\mathbf{a}}_x\, 2j\, E_{i0}\, \sin\beta_1 z\, (\cos\omega t + j\sin\omega t)\right] \tag{28}$$

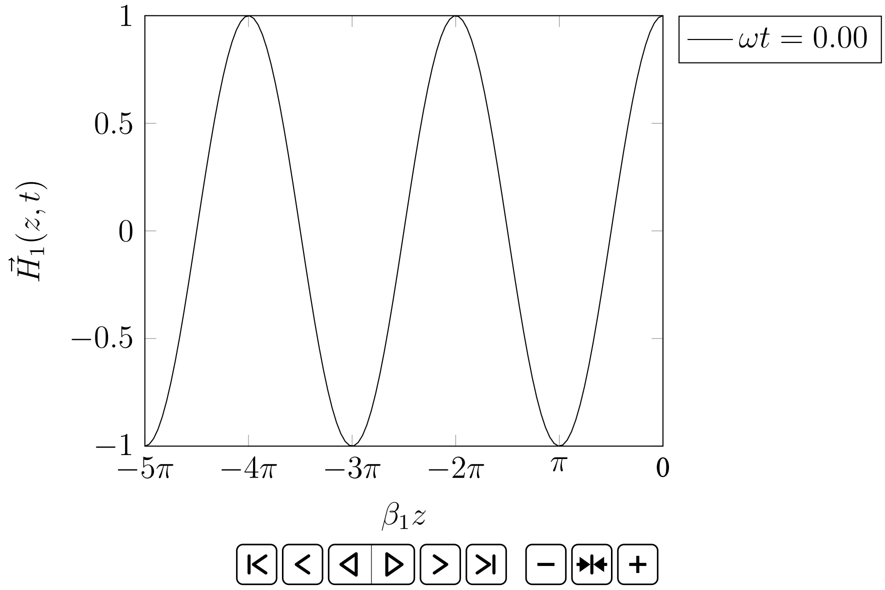
$$\vec{\mathbf{E}}_1(z,t) = \hat{\mathbf{a}}_x\, 2\, E_{i0}\, \sin\beta_1 z\, \sin\omega t \tag{29}$$

$$\vec{\mathbf{H}}_1(z,t) = \text{Re}\left[\vec{\mathbf{H}}_1(z)\, e^{j\omega t}\right] \tag{30}$$

$$\vec{\mathbf{H}}_1(z,t) = \text{Re}\left[\hat{\mathbf{a}}_y\, 2\, \frac{E_{i0}}{\eta_1}\, \cos\beta_1 z\, e^{j\omega t}\right] \tag{31}$$

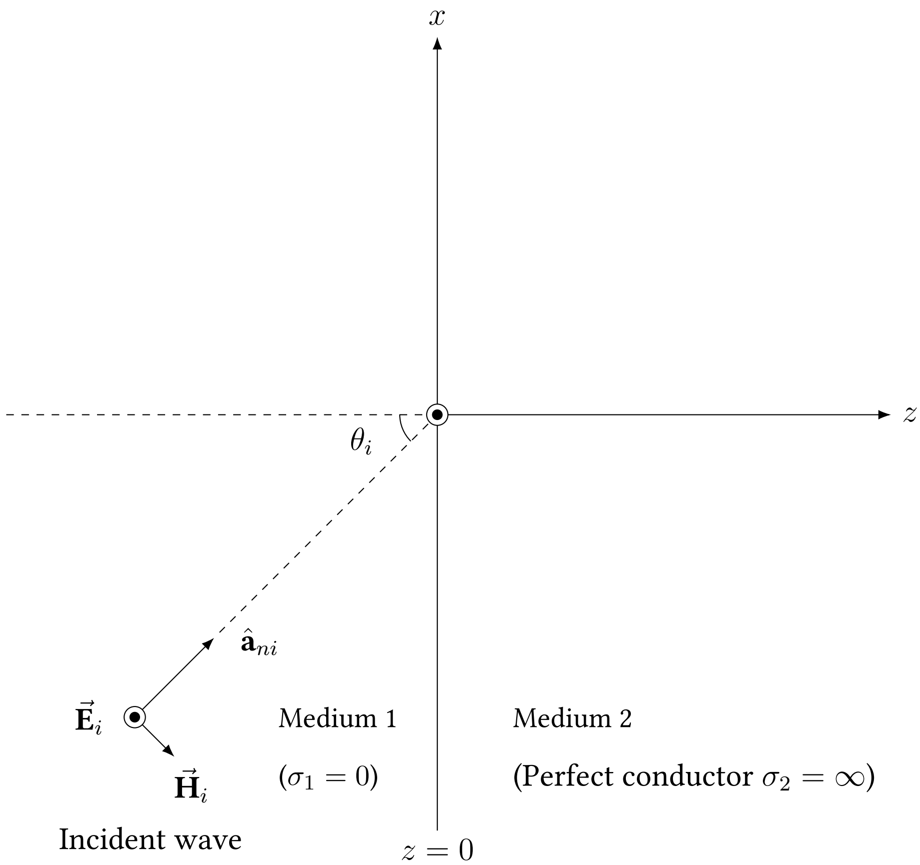
$$\vec{\mathbf{H}}_1(z,t) = \hat{\mathbf{a}}_y\, 2\, \frac{E_{i0}}{\eta_1}\, \cos\beta_1 z\, \cos\omega t \tag{32}$$



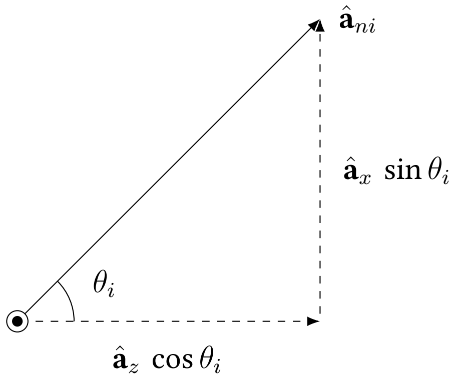

1
0.5
0
−0.5
−1
$\vec{H}_1(z,t)$
-5π
-4π
-3π
-2π
π
0
$\beta_1 z$
$\omega t = 0.00$

4. Oblique Incidence at a Plane Conducting Boundary

4.1 Horizontal Polarization (Yatay Kutuplanma)



θ_i: angle of incidence



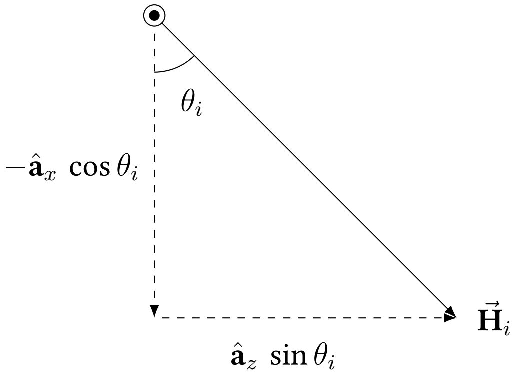
$$\hat{\mathbf{a}}_{ni} = \hat{\mathbf{a}}_x \ \sin\theta_i + \hat{\mathbf{a}}_z \ \cos\theta_i \tag{33}$$

$$\vec{\mathbf{E}}_i(x,z) = \hat{\mathbf{a}}_y \, E_{i0} \, e^{-j\beta_1 \, \hat{\mathbf{a}}_{ni}\cdot\vec{\mathbf{R}}} \tag{34}$$

$$\hat{\mathbf{a}}_{ni}\cdot\vec{\mathbf{R}} = (\hat{\mathbf{a}}_x \ \sin\theta_i + \hat{\mathbf{a}}_z \ \cos\theta_i)\cdot(\hat{\mathbf{a}}_x \, x + \hat{\mathbf{a}}_y \, y + \hat{\mathbf{a}}_z \, z) \tag{35}$$

$$\hat{\mathbf{a}}_{ni}\cdot\vec{\mathbf{R}} = (x\sin\theta_i + z\cos\theta_i) \tag{36}$$

$$\vec{\mathbf{E}}_i(x,z) = \hat{\mathbf{a}}_y \, E_{i0} \, e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)} \tag{37}$$

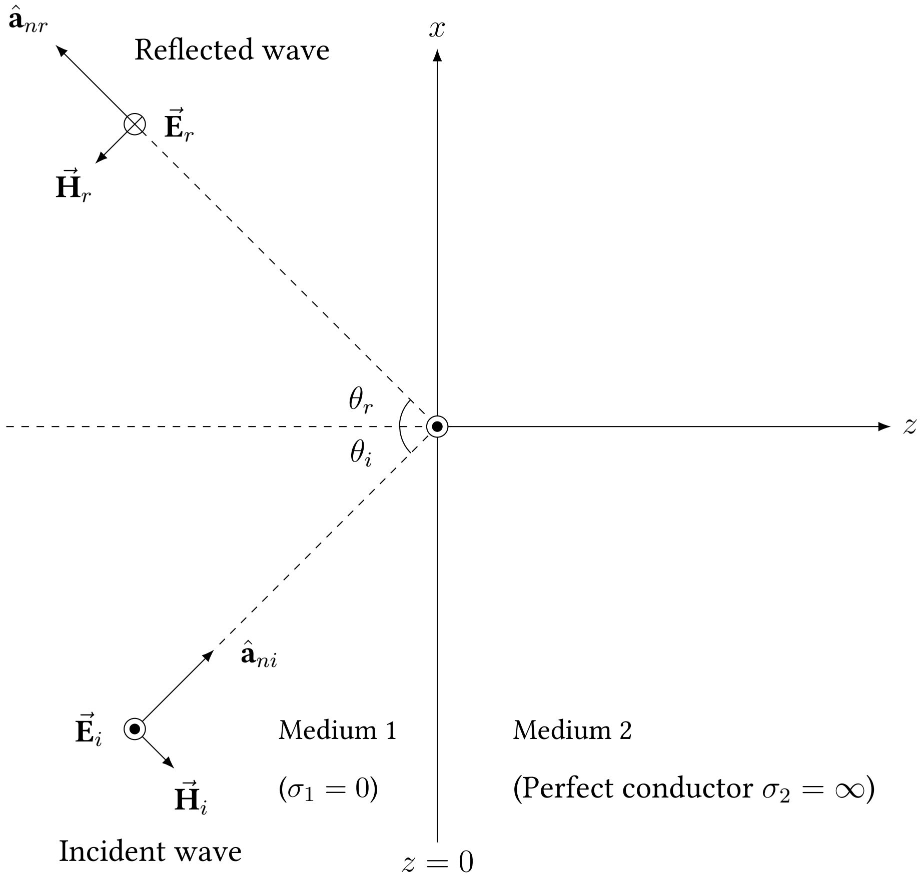


$$\vec{\mathbf{H}}_i(x,z) = \frac{1}{\eta_1}\left[\hat{\mathbf{a}}_{ni} \times \vec{\mathbf{E}}_i(x,z)\right] \tag{38}$$

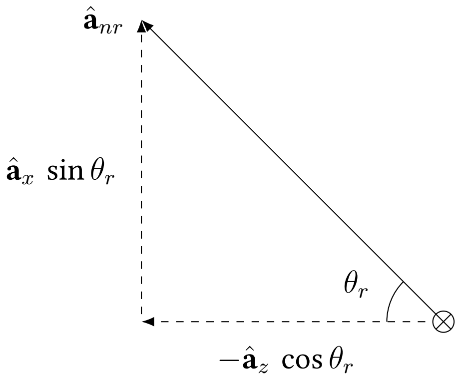
$$\vec{\mathbf{H}}_i(x,z) = \frac{1}{\eta_1}\left[(\hat{\mathbf{a}}_x \sin\theta_i + \hat{\mathbf{a}}_z \cos\theta_i) \times \hat{\mathbf{a}}_y E_{i0}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\right] \tag{39}$$

$$\vec{\mathbf{H}}_i(x,z) = (\hat{\mathbf{a}}_z \sin\theta_i - \hat{\mathbf{a}}_x \cos\theta_i)\,\frac{E_{i0}}{\eta_1}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{40}$$

$$\vec{\mathbf{H}}_i(x,z) = (-\hat{\mathbf{a}}_x \cos\theta_i + \hat{\mathbf{a}}_z \sin\theta_i)\,\frac{E_{i0}}{\eta_1}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{41}$$



θ_r: angle of reflection



$$\vec{\mathbf{E}}_r(x,z) = -\hat{\mathbf{a}}_y\, E_{r0}\, e^{-j\beta_1\, \hat{\mathbf{a}}_{nr}\cdot\vec{\mathbf{R}}} \tag{42}$$

$$\hat{\mathbf{a}}_{nr} = \hat{\mathbf{a}}_x \sin\theta_r - \hat{\mathbf{a}}_z \cos\theta_r \tag{43}$$

$$\hat{\mathbf{a}}_{nr}\cdot\vec{\mathbf{R}} = (\hat{\mathbf{a}}_x \sin\theta_r - \hat{\mathbf{a}}_z \cos\theta_r)\cdot(\hat{\mathbf{a}}_x\, x + \hat{\mathbf{a}}_y\, y + \hat{\mathbf{a}}_z\, z) \tag{44}$$

$$\hat{\mathbf{a}}_{nr}\cdot\vec{\mathbf{R}} = (x\sin\theta_r - z\cos\theta_r) \tag{45}$$

$$\vec{\mathbf{E}}_r(x,z) = -\hat{\mathbf{a}}_y\, E_{r0}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \tag{46}$$

At the boundary surface, $z=0$, the tangential components of $\vec{\mathbf{E}}_1$ is zero ($E_{1t} = E_{2t} = 0$).

$$\vec{\mathbf{E}}_1(x,z) = \vec{\mathbf{E}}_i(x,z) + \vec{\mathbf{E}}_r(x,z) \tag{47}$$

$$\vec{\mathbf{E}}_1(x,z=0) = \vec{\mathbf{E}}_i(x,z=0) + \vec{\mathbf{E}}_r(x,z=0) = 0 \tag{48}$$

$$\vec{\mathbf{E}}_1(x,z=0) = \hat{\mathbf{a}}_y\, E_{i0}\, e^{-j\beta_1 x\sin\theta_i} - \hat{\mathbf{a}}_y\, E_{r0}\, e^{-j\beta_1 x\sin\theta_r} = 0 \tag{49}$$

$$E_{i0}\, e^{-j\beta_1 x\sin\theta_i} = E_{r0}\, e^{-j\beta_1 x\sin\theta_r} \qquad \text{valid for all } x \tag{50}$$

For $x=0$,

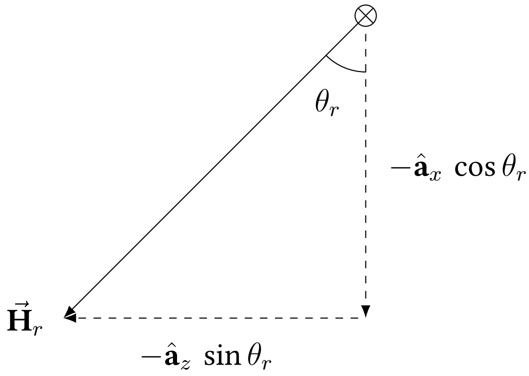
$$\boxed{E_{r0} = E_{i0}} \tag{51}$$

$$\Rightarrow \boxed{\theta_r = \theta_i} \qquad \text{(Snell's law of reflection)} \tag{52}$$

The angle of reflection is equal to the angle of incidence. So we have

$$\vec{\mathbf{E}}_r(x,z) = -\hat{\mathbf{a}}_y\, E_{i0}\, e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \tag{53}$$

$$\vec{\mathbf{H}}_r(x,z) = \frac{1}{\eta_1}\left[\hat{\mathbf{a}}_{nr}\times\vec{\mathbf{E}}_r(x,z)\right] \tag{54}$$



$$\vec{\mathbf{H}}_r(x,z) = \frac{1}{\eta_1}\ \left[(\hat{\mathbf{a}}_x\ \sin\theta_r - \hat{\mathbf{a}}_z\ \cos\theta_r) \times (-)\,\hat{\mathbf{a}}_y\, E_{i0}\, e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}\right] \tag{55}$$

$$\vec{\mathbf{H}}_r(x,z) = (-\hat{\mathbf{a}}_z\ \sin\theta_r - \hat{\mathbf{a}}_x\ \cos\theta_r)\ \frac{E_{i0}}{\eta_1}\ e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \tag{56}$$

$$\vec{\mathbf{H}}_r(x,z) = (-\hat{\mathbf{a}}_x\ \cos\theta_i - \hat{\mathbf{a}}_z\ \sin\theta_i)\ \frac{E_{i0}}{\eta_1}\ e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \tag{57}$$

The total electric field

$$\vec{\mathbf{E}}_1(x,z) = \vec{\mathbf{E}}_i(x,z) + \vec{\mathbf{E}}_r(x,z) \tag{58}$$

$$\vec{\mathbf{E}}_1(x,z) = \hat{\mathbf{a}}_y\, E_{i0}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} - \hat{\mathbf{a}}_y\, E_{i0}\, e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \tag{59}$$

$$\vec{\mathbf{E}}_1(x,z) = \hat{\mathbf{a}}_y\, E_{i0}\left(e^{-j\beta_1 z\cos\theta_i} - e^{j\beta_1 z\cos\theta_i}\right)\ e^{-j\beta_1 x\sin\theta_i} \tag{60}$$

$$e^{-j\beta_1 z\cos\theta_i} - e^{j\beta_1 z\cos\theta_i} = -2j\,\sin(\beta_1 z\cos\theta_i) \tag{61}$$

$$\vec{\mathbf{E}}_1(x,z) = -\hat{\mathbf{a}}_y\, j2\, E_{i0}\,\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} \tag{62}$$

The total magnetic field

$$\vec{\mathbf{H}}_1(x,z) = \vec{\mathbf{H}}_i(x,z) + \vec{\mathbf{H}}_r(x,z) \tag{63}$$

$$\begin{aligned}\vec{\mathbf{H}}_1(x,z) =& (-\hat{\mathbf{a}}_x\ \cos\theta_i + \hat{\mathbf{a}}_z\ \sin\theta_i)\ \frac{E_{i0}}{\eta_1}\ e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\\ &+ (-\hat{\mathbf{a}}_x\ \cos\theta_i - \hat{\mathbf{a}}_z\ \sin\theta_i)\ \frac{E_{i0}}{\eta_1}\ e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}\end{aligned} \tag{64}$$

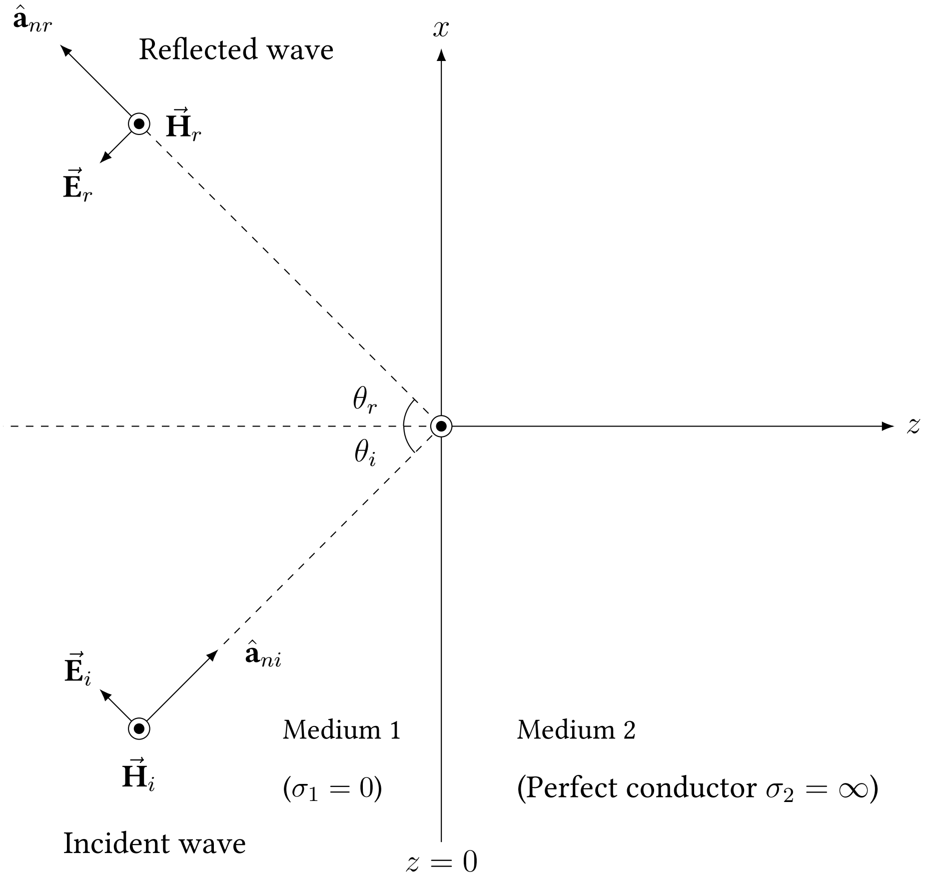
$$\begin{aligned}\vec{\mathbf{H}}_1(x,z) =& -\hat{\mathbf{a}}_x\ \cos\theta_i\ \frac{E_{i0}}{\eta_1}\ \left(e^{-j\beta_1 z\cos\theta_i} + e^{j\beta_1 z\cos\theta_i}\right) e^{-j\beta_1 x\sin\theta_i}\\ &+\hat{\mathbf{a}}_z\ \sin\theta_i\ \frac{E_{i0}}{\eta_1}\ \left(e^{-j\beta_1 z\cos\theta_i} - e^{j\beta_1 z\cos\theta_i}\right) e^{-j\beta_1 x\sin\theta_i}\end{aligned} \tag{65}$$

$$e^{-j\beta_1 z\cos\theta_i} + e^{j\beta_1 z\cos\theta_i} = 2\cos(\beta_1 z\cos\theta_i) \tag{66}$$

$$e^{-j\beta_1 z\cos\theta_i} - e^{j\beta_1 z\cos\theta_i} = -2j\,\sin(\beta_1 z\cos\theta_i) \tag{67}$$

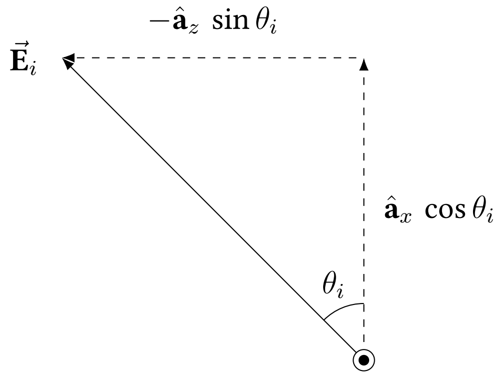
$$\begin{aligned}\vec{\mathbf{H}}_1(x,z) =& -\hat{\mathbf{a}}_x\ \frac{2E_{i0}}{\eta_1}\ \cos\theta_i\ \cos(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i}\\ &-\hat{\mathbf{a}}_z\ \frac{2jE_{i0}}{\eta_1}\ \sin\theta_i\ \sin(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i}\end{aligned} \tag{68}$$

4.2 Vertical Polarization (Dikey Kutuplanma)



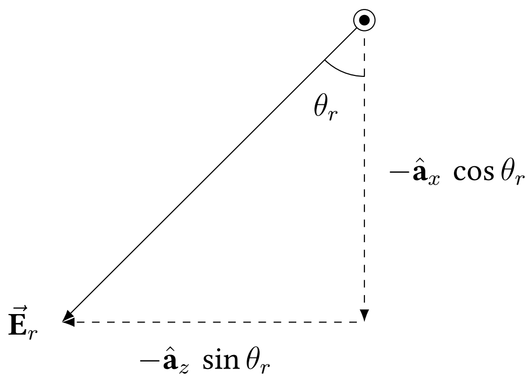
$$\hat{\mathbf{a}}_{ni} = \hat{\mathbf{a}}_x \, \sin\theta_i + \hat{\mathbf{a}}_z \, \cos\theta_i \tag{69}$$

$$\hat{\mathbf{a}}_{nr} = \hat{\mathbf{a}}_x \, \sin\theta_r - \hat{\mathbf{a}}_z \, \cos\theta_r \tag{70}$$



$$\vec{\mathbf{E}}_i(x,z) = (\hat{\mathbf{a}}_x \, \cos\theta_i - \hat{\mathbf{a}}_z \, \sin\theta_i) \; E_{i0} \, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{71}$$

$$\vec{\mathbf{H}}_i(x,z) = \hat{\mathbf{a}}_y \, \frac{E_{i0}}{\eta_1} \, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{72}$$



$$\vec{\mathbf{E}}_r(x,z) = (-\hat{\mathbf{a}}_x\ \cos\theta_r - \hat{\mathbf{a}}_z\ \sin\theta_r)\ E_{r0}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \tag{73}$$

$$\vec{\mathbf{H}}_r(x,z) = \hat{\mathbf{a}}_y\ \frac{E_{r0}}{\eta_1}\ e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \tag{74}$$

At $z = 0$, the tangential component of the total electric field intensity must vanish.

$$E_{ix}(x, z=0) + E_{rx}(x, z=0) = 0 \tag{75}$$

$$E_{i0}\ \cos\theta_i\, e^{-j\beta_1 x\sin\theta_i} - E_{r0}\ \cos\theta_r\, e^{-j\beta_1 x\sin\theta_r} = 0 \tag{76}$$

$$\Rightarrow \boxed{E_{r0} = E_{i0}} \tag{77}$$

$$\boxed{\theta_r = \theta_i} \qquad \text{(Snell's law of reflection)} \tag{78}$$

The total electric field intensity in medium 1

$$\vec{\mathbf{E}}_1(x,z) = \vec{\mathbf{E}}_i(x,z) + \vec{\mathbf{E}}_r(x,z) \tag{79}$$

$$\begin{aligned}\vec{\mathbf{E}}_1(x,z) = &(\hat{\mathbf{a}}_x\ \cos\theta_i - \hat{\mathbf{a}}_z\ \sin\theta_i)\ E_{i0}\ e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\\ &- (\hat{\mathbf{a}}_x\ \cos\theta_i + \hat{\mathbf{a}}_z\ \sin\theta_i)\ E_{i0}\ e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}\end{aligned} \tag{80}$$

$$\begin{aligned}\vec{\mathbf{E}}_1(x,z) = &\hat{\mathbf{a}}_x\ \cos\theta_i\, E_{i0}\ \left(e^{-j\beta_1 z\cos\theta_i} - e^{j\beta_1 z\cos\theta_i}\right) e^{-j\beta_1 x\sin\theta_i}\\ &-\hat{\mathbf{a}}_z\ \sin\theta_i\, E_{i0}\ \left(e^{-j\beta_1 z\cos\theta_i} + e^{j\beta_1 z\cos\theta_i}\right) e^{-j\beta_1 x\sin\theta_i}\end{aligned} \tag{81}$$

$$e^{-j\beta_1 z\cos\theta_i} - e^{j\beta_1 z\cos\theta_i} = -2j\ \sin(\beta_1 z\cos\theta_i) \tag{82}$$

$$e^{-j\beta_1 z\cos\theta_i} + e^{j\beta_1 z\cos\theta_i} = 2\cos(\beta_1 z\cos\theta_i) \tag{83}$$

$$\begin{aligned}\vec{\mathbf{E}}_1(x,z) = &-\hat{\mathbf{a}}_x\, 2jE_{i0}\ \cos\theta_i\ \sin(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i}\\ &-\hat{\mathbf{a}}_z\, 2E_{i0}\ \sin\theta_i\ \cos(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i}\end{aligned} \tag{84}$$

Total magnetic field in medium 1

$$\vec{\mathbf{H}}_1(x,z) = \vec{\mathbf{H}}_i(x,z) + \vec{\mathbf{H}}_r(x,z) \tag{85}$$

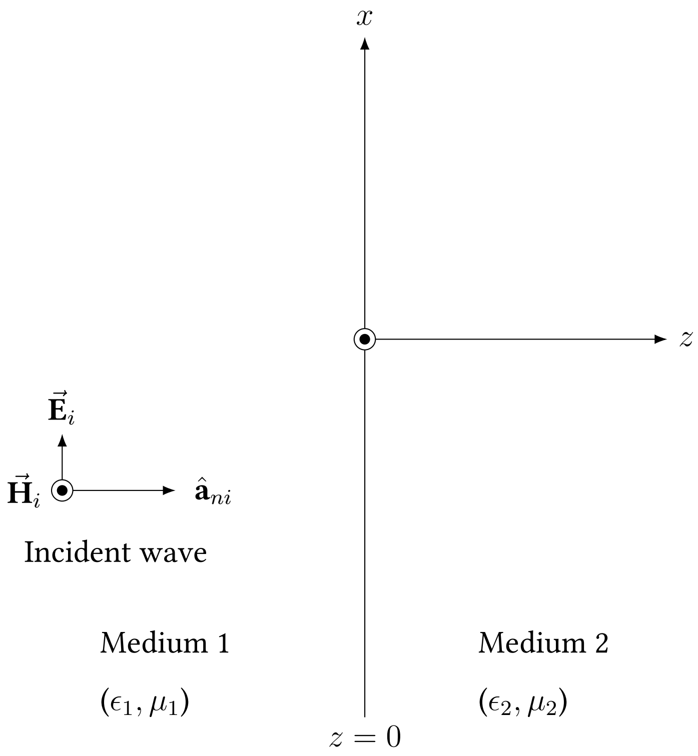
$$\vec{\mathbf{H}}_1(x,z) = \hat{\mathbf{a}}_y\ \frac{E_{i0}}{\eta_1}\ e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} + \hat{\mathbf{a}}_y\ \frac{E_{i0}}{\eta_1}\ e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \tag{86}$$

$$\vec{\mathbf{H}}_1(x,z) = \hat{\mathbf{a}}_y \frac{E_{i0}}{\eta_1} \left(e^{-j\beta_1 z\cos\theta_i} + e^{j\beta_1 z\cos\theta_i}\right) e^{-j\beta_1 x\sin\theta_i} \tag{87}$$

$$e^{-j\beta_1 z\cos\theta_i} + e^{j\beta_1 z\cos\theta_i} = 2\cos(\beta_1 z\cos\theta_i) \tag{88}$$

$$\vec{\mathbf{H}}_1(x,z) = \hat{\mathbf{a}}_y \frac{2E_{i0}}{\eta_1} \cos(\beta_1 z\cos\theta_i)\, e^{-j\beta_1 x\sin\theta_i} \tag{89}$$

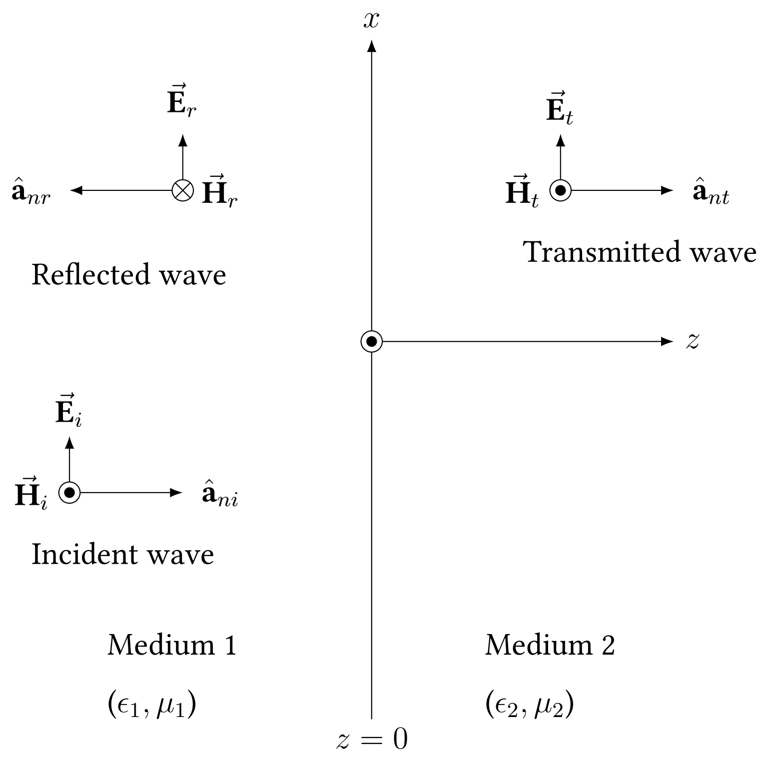
5. Normal Incidence at a Plane Dielectric Boundary



$$\vec{\mathbf{E}}_i(z) = \hat{\mathbf{a}}_x\, E_{i0}\, e^{-j\beta_1 z} \tag{90}$$

$$\vec{\mathbf{H}}_i(z) = \hat{\mathbf{a}}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z} \tag{91}$$

When an electromagnetic wave is incident on the surface of a different dielectric medium, part of the incident power is reflected and part is transmitted.



$$\vec{\mathbf{E}}_r(z) = \hat{\mathbf{a}}_x\, E_{r0}\, e^{j\beta_1 z} \tag{92}$$

$$\vec{\mathbf{H}}_r(z) = \frac{1}{\eta_1}\,(-\hat{\mathbf{a}}_z) \times \vec{\mathbf{E}}_r(z) = -\hat{\mathbf{a}}_y\, \frac{E_{r0}}{\eta_1}\, e^{j\beta_1 z} \tag{93}$$

$$\vec{\mathbf{E}}_t(z) = \hat{\mathbf{a}}_x\, E_{t0}\, e^{-j\beta_2 z} \tag{94}$$

$$\vec{\mathbf{H}}_t(z) = \frac{1}{\eta_2}\,\hat{\mathbf{a}}_z \times \vec{\mathbf{E}}_t(z) = \hat{\mathbf{a}}_y\, \frac{E_{t0}}{\eta_2}\, e^{-j\beta_2 z} \tag{95}$$

E_{t0} : the magnitude of $\vec{\mathbf{E}}_t$ at $z = 0$.

β_2 : phase constant of medium 2.

η_2 : intrinsic impedance of medium 2.

$$\beta_2 = \omega\,\sqrt{\mu_2\,\epsilon_2} \tag{96}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \tag{97}$$

E_{r0} and E_{t0} may be positive or negative, depending on $\epsilon_1, \mu_1, \epsilon_2, \mu_2$. E_{r0} and E_{t0} must be determined.

The tangential components (the $x-$ and $y-$ components) of the electric and magnetic field intensities must be continous at $z = 0$.

$$\vec{\mathbf{E}}_i(0) + \vec{\mathbf{E}}_r(0) = \vec{\mathbf{E}}_t(0) \tag{98}$$

$$\Rightarrow E_{i0} + E_{r0} = E_{t0} \tag{99}$$

$$\vec{\mathbf{H}}_i(0) + \vec{\mathbf{H}}_r(0) = \vec{\mathbf{H}}_t(0) \tag{100}$$

$$\Rightarrow \frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2} \tag{101}$$

$$\begin{aligned} E_{i0} + E_{r0} &= E_{t0} \\ \frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} &= \frac{E_{t0}}{\eta_2} \end{aligned} \tag{102}$$

$$\begin{aligned} E_{r0} - E_{t0} &= -E_{i0} \\ -\frac{E_{r0}}{\eta_1} - \frac{E_{t0}}{\eta_2} &= -\frac{E_{i0}}{\eta_1} \qquad (//\,\eta_1) \end{aligned} \tag{103}$$

$$\begin{aligned} E_{r0} - E_{t0} &= -E_{i0} \\ -E_{r0} - \frac{\eta_1}{\eta_2}\,E_{t0} &= -E_{i0} \end{aligned} \tag{104}$$

$$-E_{t0}\left(1 + \frac{\eta_1}{\eta_2}\right) = -2\,E_{i0} \tag{105}$$

$$E_{t0}\left(1 + \frac{\eta_1}{\eta_2}\right) = 2\,E_{i0} \tag{106}$$

$$E_{t0} = \frac{2}{1 + \frac{\eta_1}{\eta_2}}\,E_{i0} = \frac{2}{\frac{\eta_2+\eta_1}{\eta_2}}\,E_{i0} \tag{107}$$

$$\boxed{E_{t0} = \frac{2\,\eta_2}{\eta_1 + \eta_2}\,E_{i0}} \tag{108}$$

$$E_{r0} = E_{t0} - E_{i0} \tag{109}$$

$$E_{r0} = \frac{2\,\eta_2}{\eta_1 + \eta_2}\,E_{i0} - E_{i0} \tag{110}$$

$$E_{r0} = \left(\frac{2\,\eta_2}{\eta_1 + \eta_2} - 1\right)\,E_{i0} \tag{111}$$

$$E_{r0} = \frac{2\,\eta_2 - (\eta_1 + \eta_2)}{\eta_1 + \eta_2}\,E_{i0} \tag{112}$$

$$E_{r0} = \frac{2\,\eta_2 - \eta_1 - \eta_2}{\eta_1 + \eta_2}\,E_{i0} \tag{113}$$

$$\boxed{E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\,E_{i0}} \tag{114}$$

$$\boxed{\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \quad \text{(dimensionless)} \quad \text{(Reflection coefficient)} \tag{115}$$

$$\boxed{\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\,\eta_2}{\eta_2 + \eta_1}} \quad \text{(dimensionless)} \quad \text{(Transmission coefficient)} \tag{116}$$

The reflection coefficient Γ can be positive or negative. The tranmission coefficient τ is always positive.

$$E_{r0} = E_{t0} - E_{i0} \tag{117}$$

$$\frac{E_{r0}}{E_{i0}} = \frac{E_{t0}}{E_{i0}} - 1 \tag{118}$$

$$\Gamma = \tau - 1 \tag{119}$$

$$\boxed{\tau = \Gamma + 1} \tag{120}$$

The total field in medium 1

$$\vec{\mathbf{E}}_1(z) = \vec{\mathbf{E}}_i(z) + \vec{\mathbf{E}}_r(z) = \hat{\mathbf{a}}_x\, E_{i0}\, \left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right) \tag{121}$$