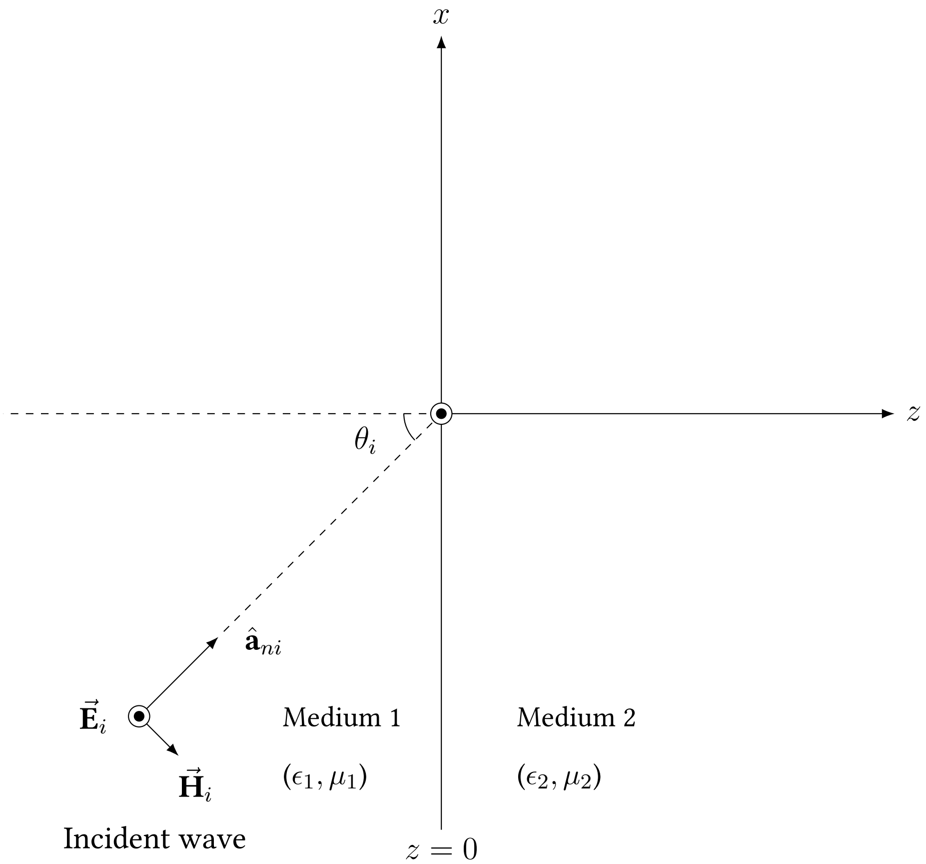
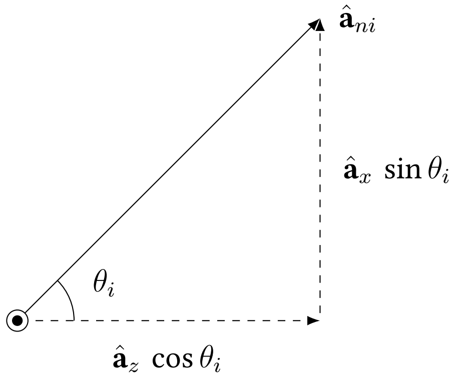
6. Oblique Incidence at a Plane Dielectric Boundary

6.1 Horizontal Polarization (Yatay Kutuplanma)





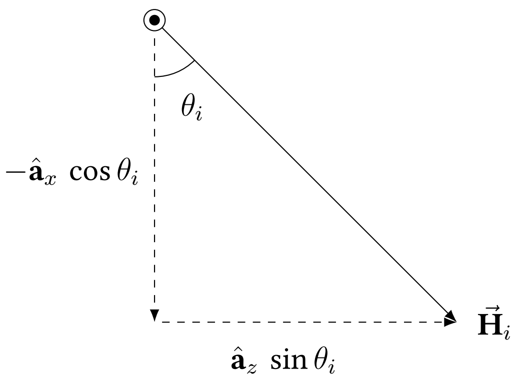
$$\vec{\mathbf{E}}_i(x,z) = \hat{\mathbf{a}}_y\, E_{i0}\, e^{-j\beta_1\, \hat{\mathbf{a}}_{ni}\cdot\vec{\mathbf{R}}} \tag{1}$$

$$\hat{\mathbf{a}}_{ni} = \hat{\mathbf{a}}_x\ \sin\theta_i + \hat{\mathbf{a}}_z\ \cos\theta_i \tag{2}$$

$$\hat{\mathbf{a}}_{ni}\cdot\vec{\mathbf{R}} = (\hat{\mathbf{a}}_x\ \sin\theta_i + \hat{\mathbf{a}}_z\ \cos\theta_i)\cdot(\hat{\mathbf{a}}_x\, x + \hat{\mathbf{a}}_y\, y + \hat{\mathbf{a}}_z\, z) \tag{3}$$

$$\hat{\mathbf{a}}_{ni}\cdot\vec{\mathbf{R}} = (x\sin\theta_i + z\cos\theta_i) \tag{4}$$

$$\vec{\mathbf{E}}_i(x,z) = \hat{\mathbf{a}}_y\, E_{i0}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{5}$$

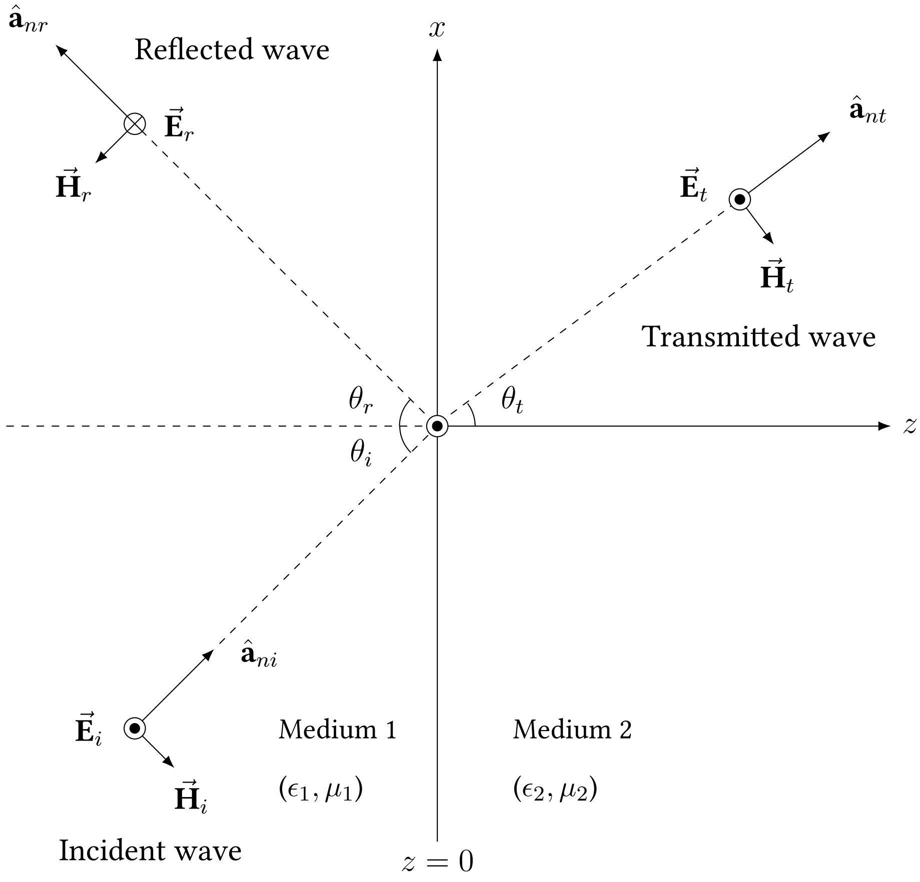


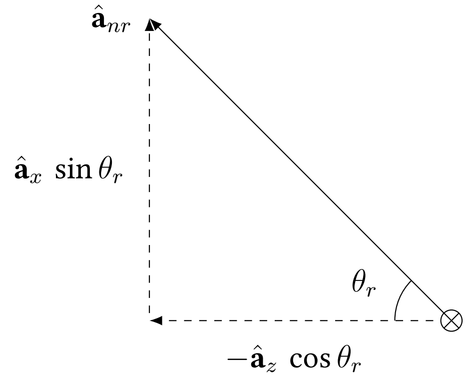
$$\vec{\mathbf{H}}_i(x,z) = \frac{1}{\eta_1}\left[\hat{\mathbf{a}}_{ni} \times \vec{\mathbf{E}}_i(x,z)\right] \tag{6}$$

$$\vec{\mathbf{H}}_i(x,z) = \frac{1}{\eta_1}\left[(\hat{\mathbf{a}}_x \sin\theta_i + \hat{\mathbf{a}}_z \cos\theta_i) \times \hat{\mathbf{a}}_y E_{i0}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\right] \tag{7}$$

$$\vec{\mathbf{H}}_i(x,z) = (\hat{\mathbf{a}}_z \sin\theta_i - \hat{\mathbf{a}}_x \cos\theta_i)\,\frac{E_{i0}}{\eta_1}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{8}$$

$$\vec{\mathbf{H}}_i(x,z) = (-\hat{\mathbf{a}}_x \cos\theta_i + \hat{\mathbf{a}}_z \sin\theta_i)\,\frac{E_{i0}}{\eta_1}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{9}$$





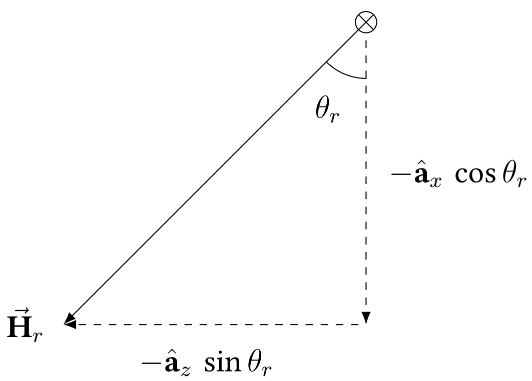
$$\vec{\mathbf{E}}_r(x,z) = -\hat{\mathbf{a}}_y \, E_{r0} \, e^{-j\beta_1 \, \hat{\mathbf{a}}_{nr} \cdot \vec{\mathbf{R}}} \tag{10}$$

$$\hat{\mathbf{a}}_{nr} = \hat{\mathbf{a}}_x \ \sin\theta_r - \hat{\mathbf{a}}_z \ \cos\theta_r \tag{11}$$

$$\hat{\mathbf{a}}_{nr} \cdot \vec{\mathbf{R}} = (\hat{\mathbf{a}}_x \ \sin\theta_r - \hat{\mathbf{a}}_z \ \cos\theta_r) \cdot (\hat{\mathbf{a}}_x \, x + \hat{\mathbf{a}}_y \, y + \hat{\mathbf{a}}_z \, z) \tag{12}$$

$$\hat{\mathbf{a}}_{nr} \cdot \vec{\mathbf{R}} = (x \sin\theta_r - z \cos\theta_r) \tag{13}$$

$$\vec{\mathbf{E}}_r(x,z) = -\hat{\mathbf{a}}_y \, E_{r0} \, e^{-j\beta_1 (x \sin\theta_r - z\cos\theta_r)} \tag{14}$$



$$\vec{\mathbf{H}}_r(x,z) = \frac{1}{\eta_1} \ \left[\hat{\mathbf{a}}_{nr} \times \vec{\mathbf{E}}_r(x,z)\right] \tag{15}$$

$$\vec{\mathbf{H}}_r(x,z) = \frac{1}{\eta_1} \ \left[(\hat{\mathbf{a}}_x \ \sin\theta_r - \hat{\mathbf{a}}_z \ \cos\theta_r) \times (-) \, \hat{\mathbf{a}}_y \, E_{r0} \, e^{-j\beta_1 (x \sin\theta_r - z\cos\theta_r)}\right] \tag{16}$$

$$\vec{\mathbf{H}}_r(x,z) = (-\hat{\mathbf{a}}_z \ \sin\theta_r - \hat{\mathbf{a}}_x \ \cos\theta_r) \ \frac{E_{r0}}{\eta_1} \, e^{-j\beta_1 (x \sin\theta_r - z\cos\theta_r)} \tag{17}$$

$$\vec{\mathbf{H}}_r(x,z) = (-\hat{\mathbf{a}}_x \ \cos\theta_r - \hat{\mathbf{a}}_z \ \sin\theta_r) \ \frac{E_{r0}}{\eta_1} \, e^{-j\beta_1 (x \sin\theta_r - z\cos\theta_r)} \tag{18}$$

$$\vec{\mathbf{E}}_t(x,z) = \hat{\mathbf{a}}_y\, E_{t0}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \tag{19}$$

$$\vec{\mathbf{H}}_t(x,z) = (-\hat{\mathbf{a}}_x\, \cos\theta_t + \hat{\mathbf{a}}_z\, \sin\theta_t)\, \frac{E_{t0}}{\eta_2}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \tag{20}$$

E_{r0}, E_{t0}, θ_r and θ_t are unknown quantities.

Tangential components of $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$ are continuous at $z = 0$.

$$E_{iy}(x, z=0) + E_{ry}(x, z=0) = E_{ty}(x, z=0) \tag{21}$$

$$H_{ix}(x, z=0) + H_{rx}(x, z=0) = H_{tx}(x, z=0) \tag{22}$$

$$E_{i0}\, e^{-j\beta_1\, x\sin\theta_i} - E_{r0}\, e^{-j\beta_1\, x\sin\theta_r} = E_{t0}\, e^{-j\beta_2\, x\sin\theta_t} \tag{23}$$

$$-\frac{E_{i0}}{\eta_1}\cos\theta_i\, e^{-j\beta_1\, x\sin\theta_i} - \frac{E_{r0}}{\eta_1}\cos\theta_r\, e^{-j\beta_1\, x\sin\theta_r} = -\frac{E_{t0}}{\eta_2}\cos\theta_t\, e^{-j\beta_2\, x\sin\theta_t} \tag{24}$$

These equations must be satisfied for all x. So

$$\beta_1\, x\sin\theta_i = \beta_1\, x\sin\theta_r = \beta_2\, x\sin\theta_t \qquad \text{(phase matching)} \tag{25}$$

$$\beta_1\, x\sin\theta_i = \beta_1\, x\sin\theta_r \tag{26}$$

$$\sin\theta_i = \sin\theta_r \tag{27}$$

$$\boxed{\theta_r = \theta_i} \qquad \text{(Snell's law of reflection)} \tag{28}$$

$$\beta_1\, \sin\theta_i = \beta_2\, \sin\theta_t \tag{29}$$

$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \frac{\beta_1}{\beta_2} = \frac{\omega\sqrt{\mu_1\,\epsilon_1}}{\omega\sqrt{\mu_2\,\epsilon_2}} = \frac{\sqrt{\mu_1\,\epsilon_1}}{\sqrt{\mu_2\,\epsilon_2}} = \frac{n_1}{n_2}} \qquad \text{(Snell's law of refraction)} \tag{30}$$

n_1 : the index of refraction for medium 1.

n_2 : the index of refraction for medium 2.

$$E_{i0} - E_{r0} = E_{t0} \tag{31}$$

$$-\frac{E_{i0}}{\eta_1}\cos\theta_i - \frac{E_{r0}}{\eta_1}\cos\theta_r = -\frac{E_{t0}}{\eta_2}\cos\theta_t \tag{32}$$

$$E_{i0} - E_{r0} = E_{t0} \tag{33}$$

$$-\frac{E_{i0}}{\eta_1}\cos\theta_i - \frac{E_{r0}}{\eta_1}\cos\theta_i = -\frac{E_{t0}}{\eta_2}\cos\theta_t \tag{34}$$

$$E_{r0} + E_{t0} = E_{i0} \qquad // \, (\frac{1}{\eta_1} \cos\theta_i) \tag{35}$$

$$-\frac{E_{r0}}{\eta_1} \cos\theta_i + \frac{E_{t0}}{\eta_2} \cos\theta_t = \frac{E_{i0}}{\eta_1} \cos\theta_i \tag{36}$$

$$\frac{E_{r0}}{\eta_1} \cos\theta_i + \frac{E_{t0}}{\eta_1} \cos\theta_i = \frac{E_{i0}}{\eta_1} \cos\theta_i \tag{37}$$

$$-\frac{E_{r0}}{\eta_1} \cos\theta_i + \frac{E_{t0}}{\eta_2} \cos\theta_t = \frac{E_{i0}}{\eta_1} \cos\theta_i \tag{38}$$

$$\frac{E_{t0}}{\eta_1} \cos\theta_i + \frac{E_{t0}}{\eta_2} \cos\theta_t = 2\,\frac{E_{i0}}{\eta_1} \cos\theta_i \tag{39}$$

$$E_{t0} \left(\frac{1}{\eta_1} \cos\theta_i + \frac{1}{\eta_2} \cos\theta_t \right) = 2\,\frac{E_{i0}}{\eta_1} \cos\theta_i \tag{40}$$

$$E_{t0} \left(\frac{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}{\eta_1\,\eta_2} \right) = 2\,\frac{E_{i0}}{\eta_1} \cos\theta_i \tag{41}$$

$$E_{t0}\,(\eta_2 \cos\theta_i + \eta_1 \cos\theta_t) = 2\,\eta_2\,E_{i0} \cos\theta_i \tag{42}$$

$$E_{t0} = \frac{2\,\eta_2\,E_{i0} \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \tag{43}$$

$$\boxed{\tau_\perp = \frac{E_{t0}}{E_{i0}} = \frac{2\,\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}} \tag{44}$$

$\tau_\perp$: transmission coefficient for horizontal polarization

$$E_{r0} = E_{i0} - E_{t0} \tag{45}$$

$$E_{r0} = E_{i0} \left(1 - \frac{E_{t0}}{E_{i0}} \right) \tag{46}$$

$$E_{r0} = E_{i0}\,(1 - \tau_\perp) \tag{47}$$

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = 1 - \tau_\perp \tag{48}$$

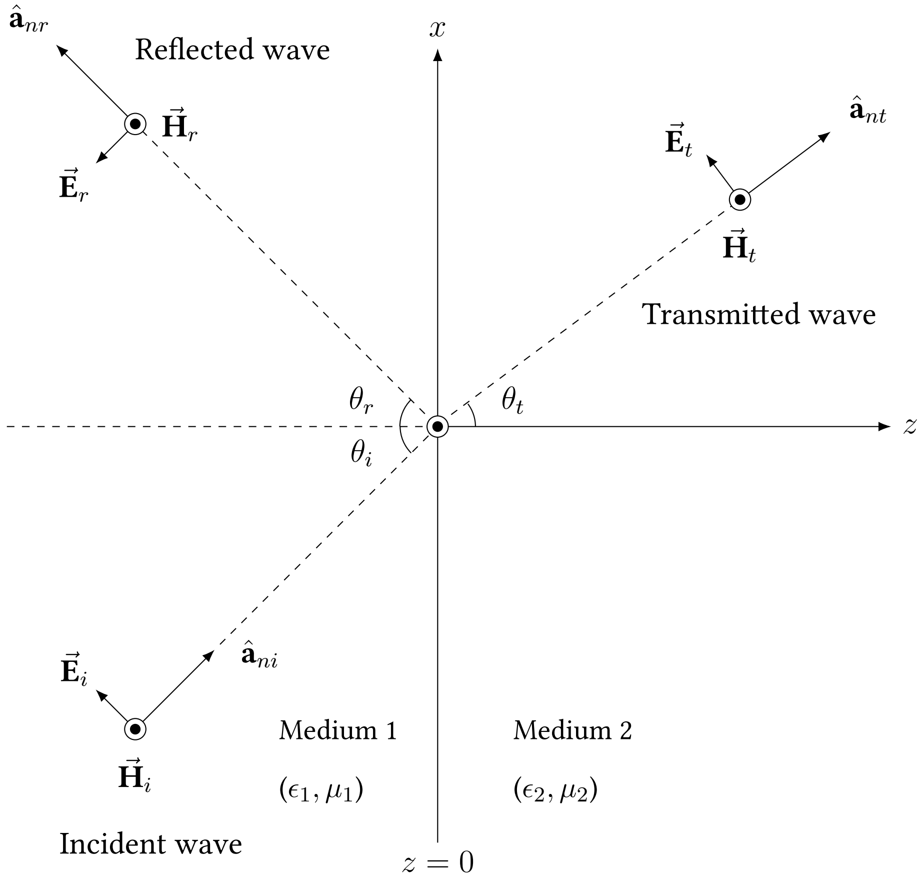
$$\Gamma_\perp = 1 - \frac{2\,\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \tag{49}$$

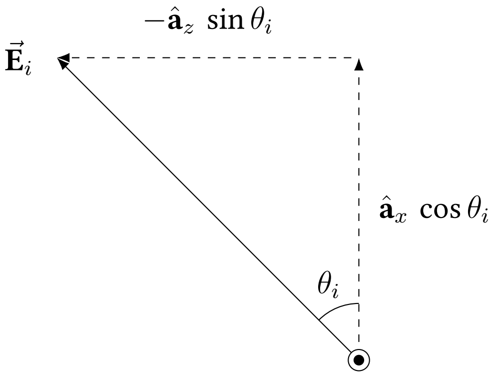
$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t - 2\,\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \tag{50}$$

$$\boxed{\Gamma_\perp = \frac{\eta_1 \cos\theta_t - \eta_2 \cos\theta_i}{\eta_1 \cos\theta_t + \eta_2 \cos\theta_i}} \tag{51}$$

$\Gamma_\perp$: reflection coefficient for horizontal polarization

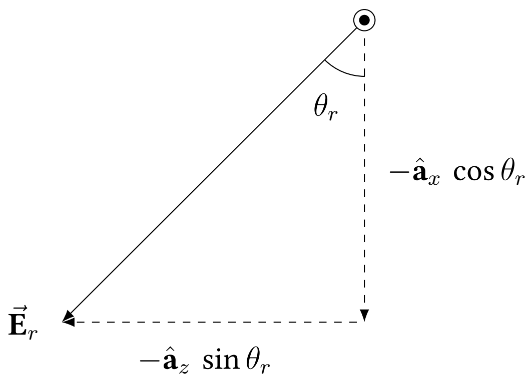
6.2 Vertical Polarization (Dikey Kutuplanma)





$$\vec{\mathbf{E}}_i(x,z) = (\hat{\mathbf{a}}_x \cos\theta_i - \hat{\mathbf{a}}_z \sin\theta_i)\ E_{i0}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{52}$$

$$\vec{\mathbf{H}}_i(x,z) = \hat{\mathbf{a}}_y \frac{E_{i0}}{\eta_1}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{53}$$



$$\vec{\mathbf{E}}_r(x,z) = (-\hat{\mathbf{a}}_x \cos\theta_r - \hat{\mathbf{a}}_z \sin\theta_r)\; E_{r0}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \tag{54}$$

$$\vec{\mathbf{H}}_r(x,z) = \hat{\mathbf{a}}_y \frac{E_{r0}}{\eta_1}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \tag{55}$$

$$\vec{\mathbf{E}}_t(x,z) = (\hat{\mathbf{a}}_x \cos\theta_t - \hat{\mathbf{a}}_z \sin\theta_t)\; E_{t0}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \tag{56}$$

$$\vec{\mathbf{H}}_t(x,z) = \hat{\mathbf{a}}_y \frac{E_{t0}}{\eta_2}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \tag{57}$$

E_{r0}, E_{t0}, θ_r and θ_t are unknown quantities.

Tangential components of $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$ are continuous at $z = 0$.

$$E_{ix}(x, z=0) + E_{rx}(x, z=0) = E_{tx}(x, z=0) \tag{58}$$

$$H_{iy}(x, z=0) + H_{ry}(x, z=0) = H_{ty}(x, z=0) \tag{59}$$

$$E_{i0}\, \cos\theta_i\, e^{-j\beta_1\, x\sin\theta_i} - E_{r0}\, \cos\theta_r\, e^{-j\beta_1\, x\sin\theta_r} = E_{t0}\, \cos\theta_t\, e^{-j\beta_2\, x\sin\theta_t} \tag{60}$$

$$\frac{E_{i0}}{\eta_1}\, e^{-j\beta_1\, x\sin\theta_i} + \frac{E_{r0}}{\eta_1}\, e^{-j\beta_1\, x\sin\theta_r} = \frac{E_{t0}}{\eta_2}\, e^{-j\beta_2\, x\sin\theta_t} \tag{61}$$

These equations must be satisfied for all x. So

$$\beta_1\, x\sin\theta_i = \beta_1\, x\sin\theta_r = \beta_2\, x\sin\theta_t \qquad \text{(phase matching)} \tag{62}$$

$$\beta_1\, x\sin\theta_i = \beta_1\, x\sin\theta_r \tag{63}$$

$$\sin\theta_i = \sin\theta_r \tag{64}$$

$$\boxed{\theta_r = \theta_i} \qquad \text{(Snell's law of reflection)} \tag{65}$$

$$\beta_1\, \sin\theta_i = \beta_2\, \sin\theta_t \tag{66}$$

$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \frac{\beta_1}{\beta_2} = \frac{\omega\sqrt{\mu_1\,\epsilon_1}}{\omega\sqrt{\mu_2\,\epsilon_2}} = \frac{\sqrt{\mu_1\,\epsilon_1}}{\sqrt{\mu_2\,\epsilon_2}} = \frac{n_1}{n_2}} \qquad \text{(Snell's law of refraction)} \tag{67}$$

n_1 : the index of refraction for medium 1.

n_2 : the index of refraction for medium 2.

For $x = 0$

$$E_{i0}\,\cos\theta_i - E_{r0}\,\cos\theta_r = E_{t0}\,\cos\theta_t \tag{68}$$

$$\frac{E_{i0}}{\eta_1} + \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2} \tag{69}$$

$$E_{i0}\,\cos\theta_i - E_{r0}\,\cos\theta_i = E_{t0}\,\cos\theta_t \tag{70}$$

$$\frac{E_{i0}}{\eta_1} + \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2} \tag{71}$$

$$E_{r0}\,\cos\theta_i + E_{t0}\,\cos\theta_t = E_{i0}\,\cos\theta_i \tag{72}$$

$$E_{i0} + E_{r0} = \frac{\eta_1}{\eta_2}\,E_{t0} \tag{73}$$

$$E_{r0}\,\cos\theta_i + E_{t0}\,\cos\theta_t = E_{i0}\,\cos\theta_i \tag{74}$$

$$E_{r0} - \frac{\eta_1}{\eta_2}\,E_{t0} = -E_{i0} \qquad //\,(-\cos\theta_i) \tag{75}$$

$$E_{r0}\,\cos\theta_i + E_{t0}\,\cos\theta_t = E_{i0}\,\cos\theta_i \tag{76}$$

$$-E_{r0}\,\cos\theta_i + \frac{\eta_1}{\eta_2}\,E_{t0}\,\cos\theta_i = E_{i0}\,\cos\theta_i \tag{77}$$

$$E_{t0}\,\cos\theta_t + \frac{\eta_1}{\eta_2}\,E_{t0}\,\cos\theta_i = 2\,E_{i0}\,\cos\theta_i \qquad //\,(\eta_2) \tag{78}$$

$$\eta_2\,E_{t0}\,\cos\theta_t + \eta_1\,E_{t0}\,\cos\theta_i = 2\,\eta_2\,E_{i0}\,\cos\theta_i \tag{79}$$

$$E_{t0}\,(\eta_2\,\cos\theta_t + \eta_1\,\cos\theta_i) = 2\,\eta_2\,E_{i0}\,\cos\theta_i \tag{80}$$

$$\frac{E_{t0}}{E_{i0}} = \frac{2\,\eta_2\,\cos\theta_i}{\eta_2\,\cos\theta_t + \eta_1\,\cos\theta_i} \tag{81}$$

$$\boxed{\tau_{\parallel} = \frac{E_{t0}}{E_{i0}} = \frac{2\,\eta_2\,\cos\theta_i}{\eta_2\,\cos\theta_t + \eta_1\,\cos\theta_i}} \tag{82}$$

$\tau_{\parallel}$: transmission coefficient for vertical polarization

$$E_{r0} = \frac{\eta_1}{\eta_2}\,E_{t0} - E_{i0} \tag{83}$$

$$\frac{E_{r0}}{E_{i0}} = \frac{\eta_1}{\eta_2}\,\frac{E_{t0}}{E_{i0}} - 1 \tag{84}$$

$$\frac{E_{r0}}{E_{i0}} = \frac{\eta_1}{\eta_2}\tau_\parallel - 1 \tag{85}$$

$$\frac{E_{r0}}{E_{i0}} = \frac{\eta_1}{\eta_2}\frac{2\,\eta_2\,\cos\theta_i}{\eta_2\,\cos\theta_t + \eta_1\,\cos\theta_i} - 1 \tag{86}$$

$$\frac{E_{r0}}{E_{i0}} = \frac{2\,\eta_1\,\cos\theta_i}{\eta_2\,\cos\theta_t + \eta_1\,\cos\theta_i} - 1 \tag{87}$$

$$\frac{E_{r0}}{E_{i0}} = \frac{2\,\eta_1\,\cos\theta_i - \eta_2\,\cos\theta_t - \eta_1\,\cos\theta_i}{\eta_2\,\cos\theta_t + \eta_1\,\cos\theta_i} \tag{88}$$

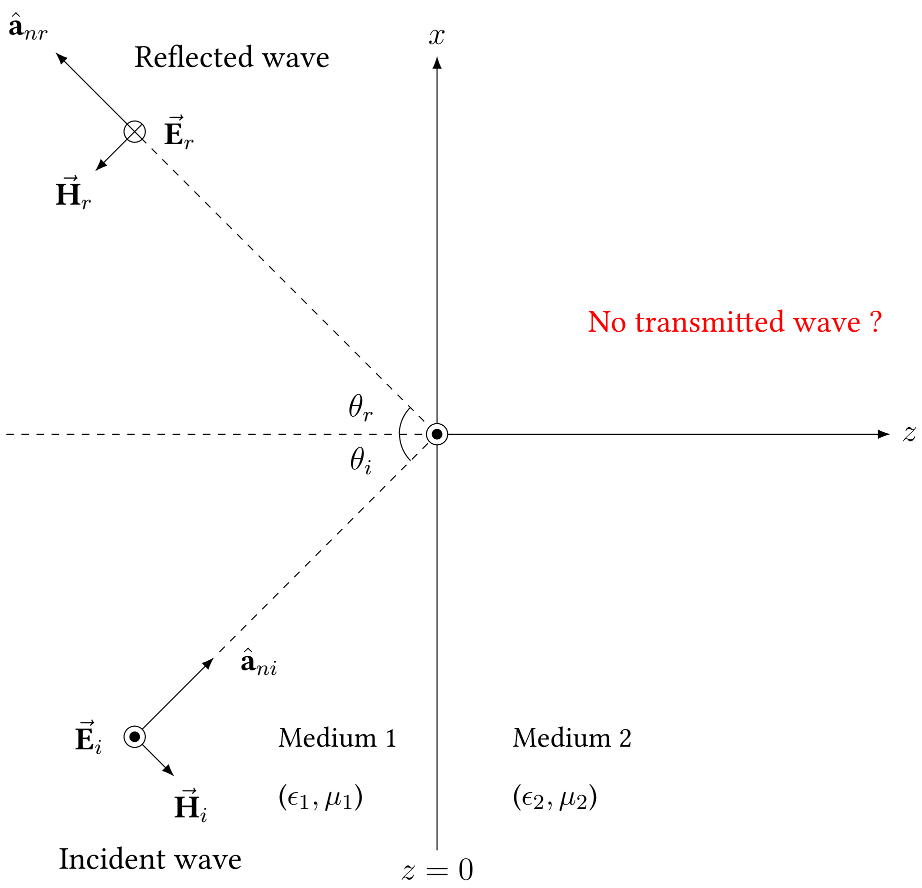
$$\frac{E_{r0}}{E_{i0}} = \frac{\eta_1\,\cos\theta_i - \eta_2\,\cos\theta_t}{\eta_2\,\cos\theta_t + \eta_1\,\cos\theta_i} \tag{89}$$

$$\boxed{\Gamma_\parallel = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1\,\cos\theta_i - \eta_2\,\cos\theta_t}{\eta_2\,\cos\theta_t + \eta_1\,\cos\theta_i}} \tag{90}$$

$\Gamma_\parallel$: reflection coefficient for vertical polarization

6.3 Total Reflection

a) Horizontal Polarization (Yatay Kutuplanma)



For total reflection we need $|\Gamma_\perp| = 1$ or $\Gamma_\perp = \mp 1$.

Let's first try $\Gamma_\perp = 1$.

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 \cos\theta_t - \eta_2 \cos\theta_i}{\eta_1 \cos\theta_t + \eta_2 \cos\theta_i} = 1 \tag{91}$$

$$\eta_1 \cos\theta_t - \eta_2 \cos\theta_i = \eta_1 \cos\theta_t + \eta_2 \cos\theta_i \tag{92}$$

$$\Rightarrow 2\,\eta_2 \cos\theta_i = 0 \tag{93}$$

$$\cos\theta_i = 0 \Rightarrow \theta_i = 90^\circ \tag{94}$$

This does not give us an interesting solution.

Now let's try $\Gamma_\perp = -1$.

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 \cos\theta_t - \eta_2 \cos\theta_i}{\eta_1 \cos\theta_t + \eta_2 \cos\theta_i} = -1 \tag{95}$$

$$\eta_1 \cos\theta_t - \eta_2 \cos\theta_i = -\eta_1 \cos\theta_t - \eta_2 \cos\theta_i \tag{96}$$

$$2\,\eta_1 \cos\theta_t = 0 \tag{97}$$

$$\cos\theta_t = 0 \Rightarrow \theta_t = 90^\circ \tag{98}$$

From Snell's law

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\sqrt{\mu_1\,\epsilon_1}}{\sqrt{\mu_2\,\epsilon_2}} \tag{99}$$

For nonmagnetic media $\mu_1 = \mu_2 = \mu_0$.

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \tag{100}$$

$$\theta_t = 90^\circ \Rightarrow \sin\theta_t = 1 \tag{101}$$

$$\frac{1}{\sin\theta_i} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \tag{102}$$

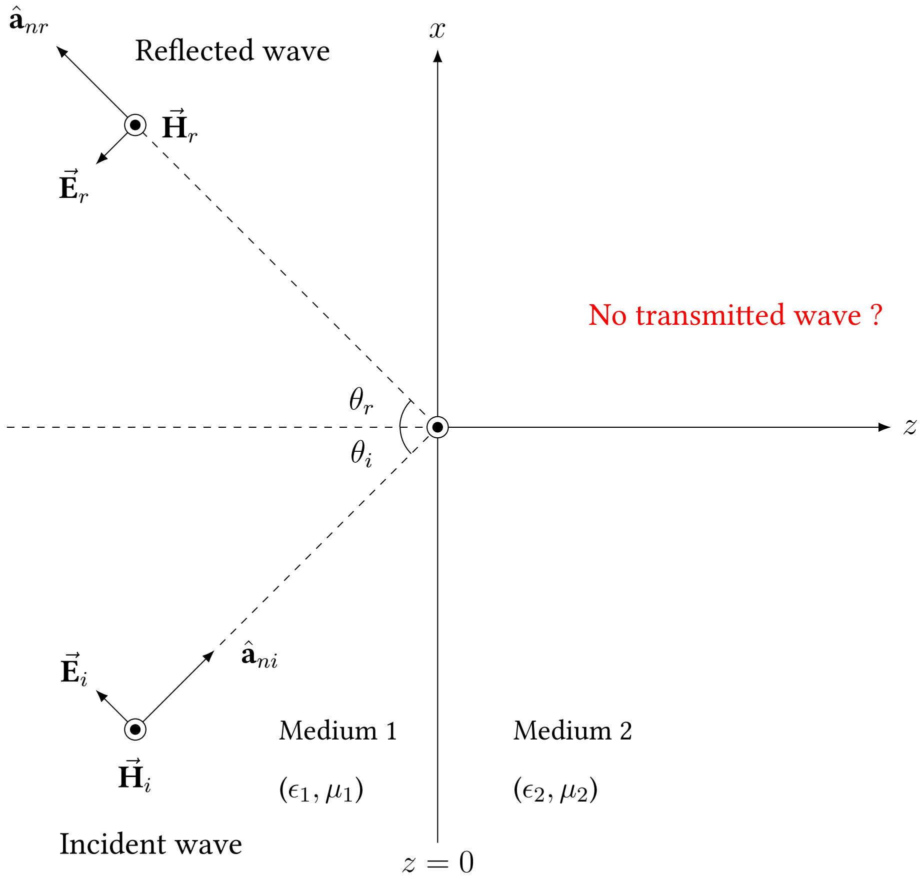
$$\sin\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \tag{103}$$

$$\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \tag{104}$$

$$\boxed{\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}} \tag{105}$$

The angle of incidence θ_c is called the **critical angle**.

b) Vertical Polarization (Dikey Kutuplanma)



For total reflection we need $|\Gamma_{\|}| = 1$ or $\Gamma_{\|} = \mp 1$.

Let's first try $\Gamma_{\|} = -1$.

$$\Gamma_{\|} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = -1 \tag{106}$$

$$\eta_1 \cos\theta_i - \eta_2 \cos\theta_t = -\eta_2 \cos\theta_t - \eta_1 \cos\theta_i \tag{107}$$

$$\Rightarrow 2\,\eta_1 \cos\theta_i = 0 \tag{108}$$

$$\cos\theta_i = 0 \Rightarrow \theta_i = 90^\circ \tag{109}$$

This does not give us an interesting solution.

Now let's try $\Gamma_{\|} = 1$.

$$\Gamma_{\|} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = 1 \tag{110}$$

$$\eta_1 \cos\theta_i - \eta_2 \cos\theta_t = \eta_2 \cos\theta_t + \eta_1 \cos\theta_i \tag{111}$$

$$2\,\eta_2 \cos\theta_t = 0 \tag{112}$$

$$\cos\theta_t = 0 \Rightarrow \theta_t = 90^\circ \tag{113}$$

From Snell's law

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\sqrt{\mu_1\,\epsilon_1}}{\sqrt{\mu_2\,\epsilon_2}} \tag{114}$$

For nonmagnetic media $\mu_1 = \mu_2 = \mu_0$.

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \tag{115}$$

$$\theta_t = 90^\circ \Rightarrow \sin\theta_t = 1 \tag{116}$$

$$\frac{1}{\sin\theta_i} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \tag{117}$$

$$\sin\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \tag{118}$$

$$\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \tag{119}$$

$$\boxed{\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}} \tag{120}$$

The angle of incidence θ_c is called the **critical angle**.

If $\theta_i = \theta_c$ then $\theta_t = 90^\circ$.

If $\theta_i > \theta_c$ then

$$\sin\theta_i > \sin\theta_c \tag{121}$$

$$\sin\theta_i > \sqrt{\frac{\epsilon_2}{\epsilon_1}} \tag{122}$$

$$\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i > 1 \tag{123}$$

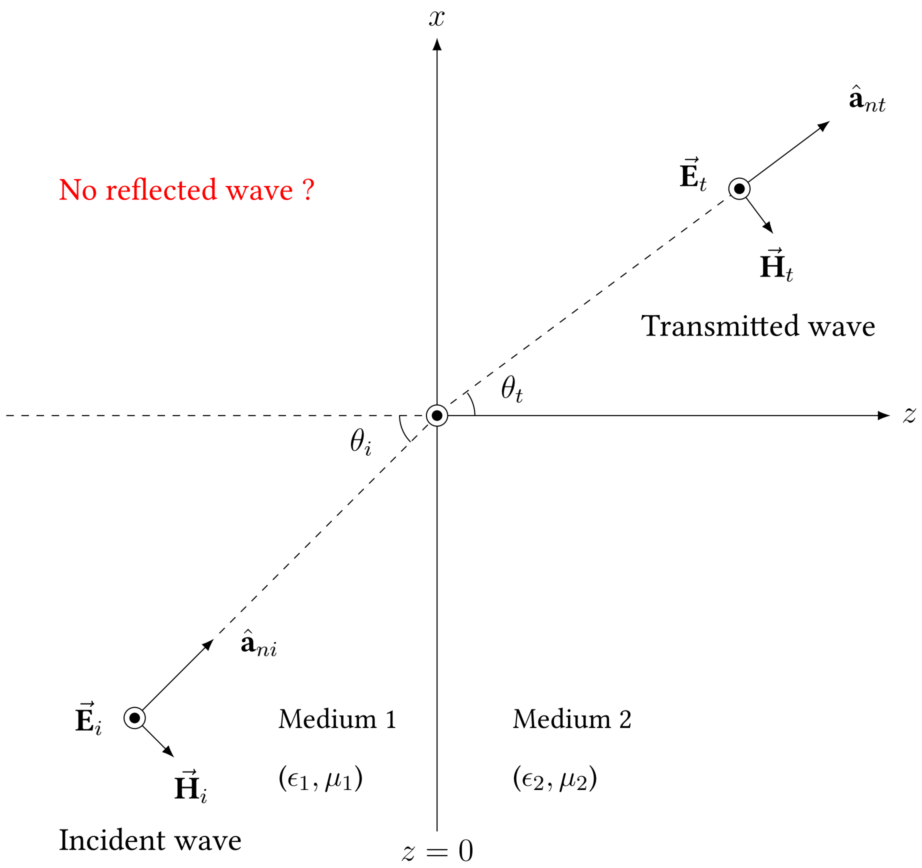
$$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i \tag{124}$$

$$\Rightarrow \sin\theta_t > 1 \tag{125}$$

There is no real solution for θ_t and total reflection occurs.

6.4 Total Transmission

a) Horizontal Polarization (Yatay Kutuplanma)



For total transmission we need $\Gamma_\perp = 0$.

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 \cos\theta_t - \eta_2 \cos\theta_i}{\eta_1 \cos\theta_t + \eta_2 \cos\theta_i} = 0 \tag{126}$$

$$\Rightarrow \eta_1 \cos\theta_t - \eta_2 \cos\theta_i = 0 \tag{127}$$

$$\Rightarrow \eta_1 \cos\theta_t = \eta_2 \cos\theta_i \tag{128}$$

From Snell's law

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} \tag{129}$$

$$\sin\theta_t = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} \sin\theta_i \tag{130}$$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2} \sin^2\theta_i} \tag{131}$$

$$\cos\theta_i = \sqrt{1 - \sin^2\theta_i} \tag{132}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} \tag{133}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \tag{134}$$

$$\eta_1 \cos\theta_t = \eta_2 \cos\theta_i \tag{135}$$

$$\sqrt{\frac{\mu_1}{\epsilon_1}} \sqrt{1 - \frac{\mu_1\,\epsilon_1}{\mu_2\,\epsilon_2} \sin^2\theta_i} = \sqrt{\frac{\mu_2}{\epsilon_2}} \sqrt{1 - \sin^2\theta_i} \tag{136}$$

$$\frac{\mu_1}{\epsilon_1} \left(1 - \frac{\mu_1\,\epsilon_1}{\mu_2\,\epsilon_2} \sin^2\theta_i\right) = \frac{\mu_2}{\epsilon_2} \left(1 - \sin^2\theta_i\right) \tag{137}$$

$$\frac{\mu_1}{\epsilon_1} - \frac{\mu_1^2}{\mu_2\,\epsilon_2} \sin^2\theta_i = \frac{\mu_2}{\epsilon_2} - \frac{\mu_2}{\epsilon_2} \sin^2\theta_i \tag{138}$$

$$\frac{\mu_2}{\epsilon_2} \sin^2\theta_i - \frac{\mu_1^2}{\mu_2\,\epsilon_2} \sin^2\theta_i = \frac{\mu_2}{\epsilon_2} - \frac{\mu_1}{\epsilon_1} \tag{139}$$

$$\left(\frac{\mu_2}{\epsilon_2} - \frac{\mu_1^2}{\mu_2\,\epsilon_2}\right) \sin^2\theta_i = \frac{\mu_2}{\epsilon_2} - \frac{\mu_1}{\epsilon_1} \tag{140}$$

$$\sin^2\theta_i = \frac{\frac{\mu_2}{\epsilon_2} - \frac{\mu_1}{\epsilon_1}}{\frac{\mu_2}{\epsilon_2} - \frac{\mu_1^2}{\mu_2\,\epsilon_2}} \tag{141}$$

$$\sin^2\theta_i = \frac{\epsilon_1\,\mu_2 - \epsilon_2\,\mu_1}{\epsilon_1\,\epsilon_2} \frac{\mu_2\,\epsilon_2}{\mu_2^2 - \mu_1^2} \tag{142}$$

$$\sin^2\theta_i = \frac{\epsilon_1\,\mu_2 - \epsilon_2\,\mu_1}{\epsilon_1} \frac{\mu_2}{\mu_2^2 - \mu_1^2} \tag{143}$$

$$\sin^2\theta_i = \frac{\epsilon_1\,\mu_2 - \epsilon_2\,\mu_1}{\epsilon_1} \frac{\mu_2}{\mu_2^2\,[1 - (\mu_1/\mu_2)^2]} \tag{144}$$

$$\sin^2\theta_i = \frac{\epsilon_1\,\mu_2 - \epsilon_2\,\mu_1}{\epsilon_1\,\mu_2} \frac{1}{[1 - (\mu_1/\mu_2)^2]} \tag{145}$$

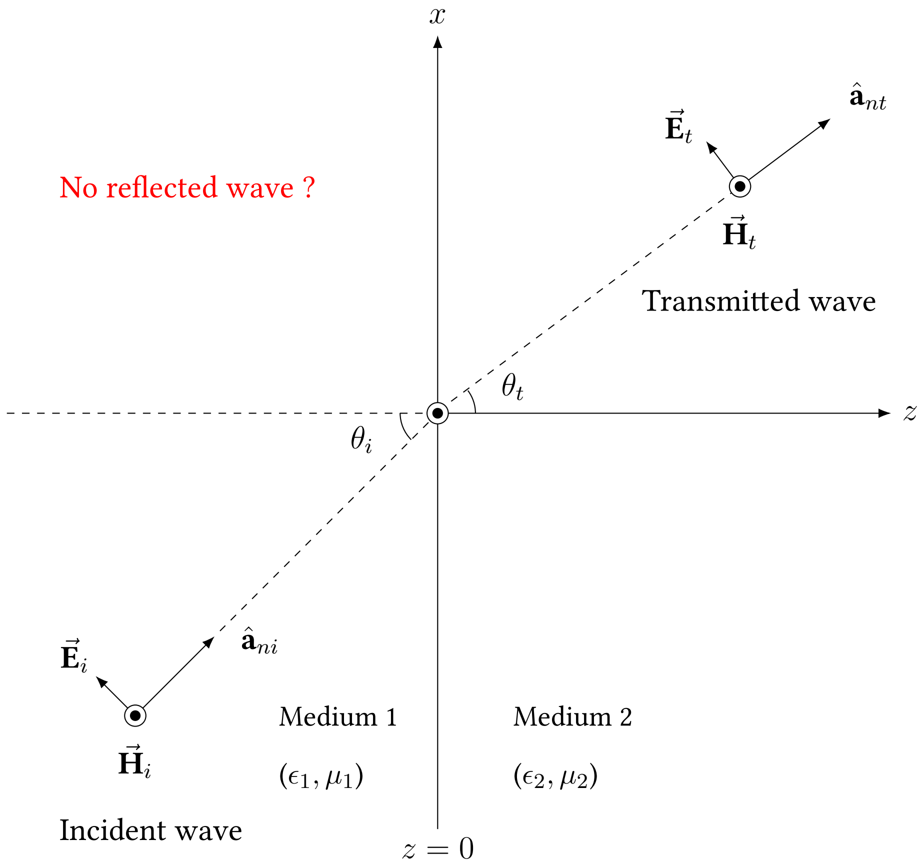
$$\sin^2\theta_i = \frac{1 - \frac{\epsilon_2\,\mu_1}{\epsilon_1\,\mu_2}}{1 - \frac{\mu_1^2}{\mu_2^2}} \tag{146}$$

For nonmagnetic media $\mu_1 = \mu_2 = \mu_0$ and we have

$$\sin^2\theta_i = \infty \tag{147}$$

So for horizontal polarization there is no incidence angle that makes the reflection coefficient $\Gamma_\perp = 0$.

b) Vertical Polarization (Dikey Kutuplanma)



For total transmission we need $\Gamma_{\parallel} = 0$.

$$\Gamma_{\parallel} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = 0 \tag{148}$$

$$\Rightarrow \eta_1 \cos\theta_i - \eta_2 \cos\theta_t = 0 \tag{149}$$

$$\Rightarrow \eta_1 \cos\theta_i = \eta_2 \cos\theta_t \tag{150}$$

From Snell's law

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} \tag{151}$$

$$\sin\theta_t = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} \sin\theta_i \tag{152}$$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2} \sin^2\theta_i} \tag{153}$$

$$\cos\theta_i = \sqrt{1 - \sin^2\theta_i} \tag{154}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} \tag{155}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \tag{156}$$

$$\eta_1 \cos\theta_i = \eta_2 \cos\theta_t \tag{157}$$

$$\sqrt{\frac{\mu_1}{\epsilon_1}}\sqrt{1-\sin^2\theta_i} = \sqrt{\frac{\mu_2}{\epsilon_2}}\sqrt{1-\frac{\mu_1\,\epsilon_1}{\mu_2\,\epsilon_2}\sin^2\theta_i} \tag{158}$$

$$\frac{\mu_1}{\epsilon_1}\left(1-\sin^2\theta_i\right) = \frac{\mu_2}{\epsilon_2}\left(1-\frac{\mu_1\,\epsilon_1}{\mu_2\,\epsilon_2}\sin^2\theta_i\right) \tag{159}$$

$$\frac{\mu_1}{\epsilon_1}-\frac{\mu_1}{\epsilon_1}\sin^2\theta_i = \frac{\mu_2}{\epsilon_2}-\frac{\mu_2}{\epsilon_2}\frac{\mu_1\,\epsilon_1}{\mu_2\,\epsilon_2}\sin^2\theta_i \tag{160}$$

$$\frac{\mu_1}{\epsilon_1}-\frac{\mu_1}{\epsilon_1}\sin^2\theta_i = \frac{\mu_2}{\epsilon_2}-\frac{\mu_1\,\epsilon_1}{\epsilon_2\,\epsilon_2}\sin^2\theta_i \tag{161}$$

$$-\frac{\mu_1}{\epsilon_1}\sin^2\theta_i + \frac{\mu_1\,\epsilon_1}{\epsilon_2\,\epsilon_2}\sin^2\theta_i = \frac{\mu_2}{\epsilon_2}-\frac{\mu_1}{\epsilon_1} \tag{162}$$

$$-\frac{\mu_1}{\epsilon_1}\sin^2\theta_i + \frac{\mu_1\,\epsilon_1}{\epsilon_2^2}\sin^2\theta_i = \frac{\mu_2}{\epsilon_2}-\frac{\mu_1}{\epsilon_1} \tag{163}$$

$$\left(-\frac{\mu_1}{\epsilon_1}+\frac{\mu_1\,\epsilon_1}{\epsilon_2^2}\right)\sin^2\theta_i = \frac{\mu_2}{\epsilon_2}-\frac{\mu_1}{\epsilon_1} \tag{164}$$

$$\left(\frac{\mu_1\,\epsilon_1}{\epsilon_2^2}-\frac{\mu_1}{\epsilon_1}\right)\sin^2\theta_i = \frac{\mu_2}{\epsilon_2}-\frac{\mu_1}{\epsilon_1} \tag{165}$$

$$\sin^2\theta_i = \frac{\frac{\mu_2}{\epsilon_2}-\frac{\mu_1}{\epsilon_1}}{\frac{\mu_1\,\epsilon_1}{\epsilon_2^2}-\frac{\mu_1}{\epsilon_1}} \tag{166}$$

$$\sin^2\theta_i = \frac{\frac{\epsilon_1\,\mu_2-\epsilon_2\,\mu_1}{\epsilon_1\,\epsilon_2}}{\frac{\epsilon_1^2\,\mu_1-\epsilon_2^2\,\mu_1}{\epsilon_1\,\epsilon_2^2}} \tag{167}$$

$$\sin^2\theta_i = \frac{\epsilon_1\,\mu_2-\epsilon_2\,\mu_1}{\epsilon_1\,\epsilon_2}\,\frac{\epsilon_1\,\epsilon_2^2}{\epsilon_1^2\,\mu_1-\epsilon_2^2\,\mu_1} \tag{168}$$

$$\sin^2\theta_i = [\epsilon_1\,\mu_2-\epsilon_2\,\mu_1]\,\frac{\epsilon_2}{\epsilon_1^2\,\mu_1-\epsilon_2^2\,\mu_1} \tag{169}$$

$$\sin^2\theta_i = [\epsilon_1\,\mu_2-\epsilon_2\,\mu_1]\,\frac{\epsilon_2}{[\epsilon_1^2-\epsilon_2^2]\,\mu_1} \tag{170}$$

$$\sin^2\theta_i = \epsilon_2\,\mu_1\left[\frac{\epsilon_1\,\mu_2}{\epsilon_2\,\mu_1}-1\right]\frac{\epsilon_2}{\epsilon_2^2\left[\frac{\epsilon_1^2}{\epsilon_2^2}-1\right]\mu_1} \tag{171}$$

$$\sin^2\theta_i = \frac{\left[\frac{\epsilon_1\,\mu_2}{\epsilon_2\,\mu_1}-1\right]}{\left[\frac{\epsilon_1^2}{\epsilon_2^2}-1\right]} \tag{172}$$

$$\sin^2\theta_i = \frac{1-\frac{\epsilon_1\,\mu_2}{\epsilon_2\,\mu_1}}{1-\frac{\epsilon_1^2}{\epsilon_2^2}} \tag{173}$$

For nonmagnetic media $\mu_1 = \mu_2 = \mu_0$ and we have

$$\sin^2\theta_i = \frac{1-\frac{\epsilon_1}{\epsilon_2}}{1-\frac{\epsilon_1^2}{\epsilon_2^2}} = \frac{1-\frac{\epsilon_1}{\epsilon_2}}{\left(1-\frac{\epsilon_1}{\epsilon_2}\right)\left(1+\frac{\epsilon_1}{\epsilon_2}\right)} \tag{174}$$

$$\sin^2\theta_i = \frac{1}{1+\frac{\epsilon_1}{\epsilon_2}} \tag{175}$$

$$\boxed{\sin\theta_i = \frac{1}{\sqrt{1+\frac{\epsilon_1}{\epsilon_2}}}} \tag{176}$$

The angle satisfying this equation is called as the Brewster angle θ_B. If $\theta_i = \theta_B$ then $\Gamma_\parallel = 0$ and there is no reflection from the interface, the wave is totally transmitted to the second medium.