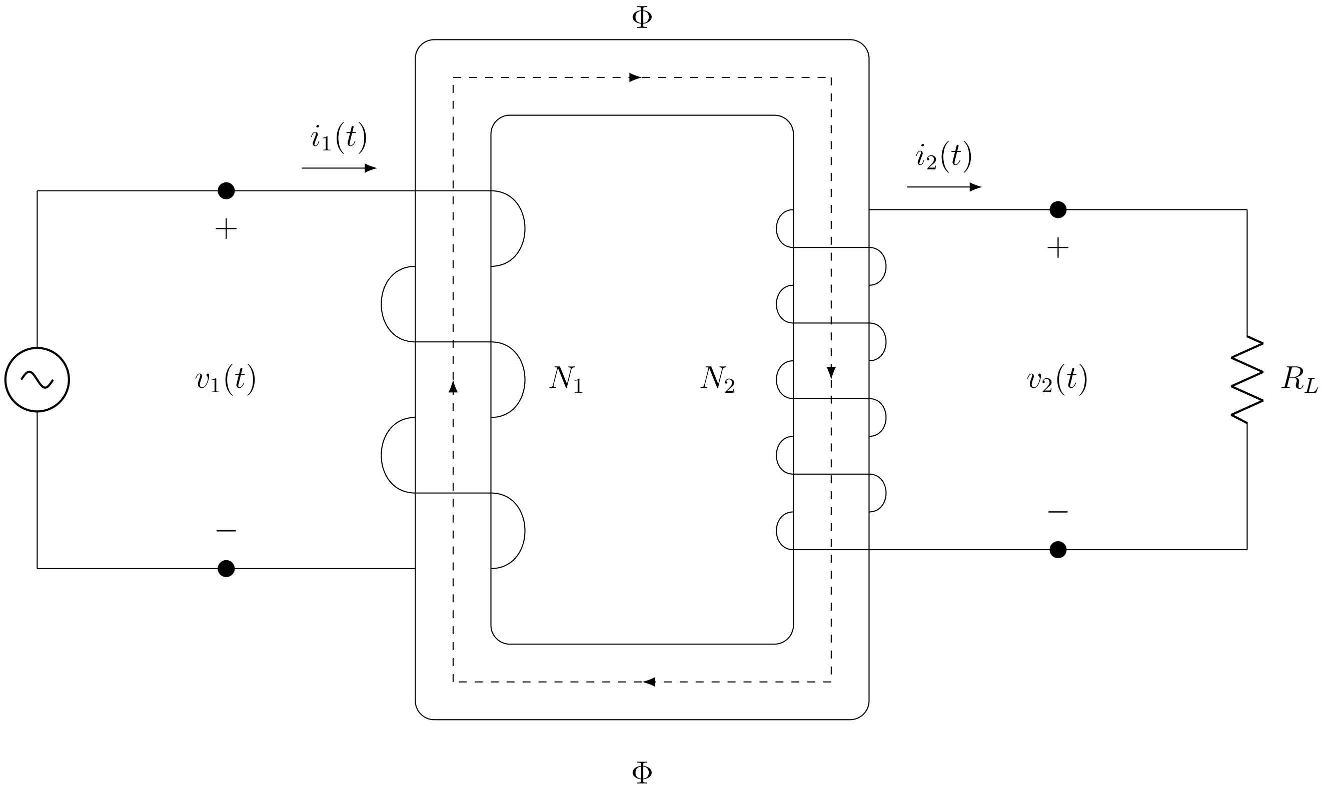
2.2 Transformers

A transformer is an alternating current (a-c) device that transforms voltages, currents and impedances. A transformer consists of two or more coils (sargı) coupled magnetically through a common ferromagnetic core (çekirdek).



A transformer

For any closed path in a magnetic circuit

$$\sum_j N_j\, I_j = \sum_k \mathcal{R}_k\, \Phi_k \tag{1}$$

The algebraic sum of ampere-turns (amper-sarım) around a closed path in a magnetic circuit is equal to the algebraic sum of the products of the reluctances and fluxes.

For the closed path in the magnetic circuit traced by magnetic flux Φ

$$N_1\, i_1 - N_2\, i_2 = \mathcal{R}\, \Phi \tag{2}$$

N_1 : the number of turns in the primary circuit

N_2 : the number of turns in the secondary circuit

i_1 : the current in the primary circuit (A)

i_2 : the current in the secondary circuit (A)

$\mathcal{R}$: reluctance of the magnetic circuit (manyetik direnç) (1/H)

The induced magnetomotive force (mmf) in the secondary circuit $N_2\, i_2$, opposes the flow of the magnetic flux Φ created by the mmf in the primary circuit $N_1\, i_1$. The reluctance of the ferromagnetic core is

$$\mathcal{R} = \frac{\ell}{\mu S} \tag{3}$$

ℓ : length of the core (m)

S : cross-sectional area of the core (m^2)

μ : permeability (manyetik geçirgenlik) of the core (H/m)

$$N_1\, i_1 - N_2\, i_2 = \mathcal{R}\,\Phi = \frac{\ell}{\mu S}\,\Phi \tag{4}$$

Ideal Transformer

For an ideal transformer we assume that $\mu \to \infty$. So

$$N_1\, i_1 - N_2\, i_2 = 0 \tag{5}$$

$$\boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1}} \tag{6}$$

The ratio of the currents in the primary and secondary windings (sargı) of an ideal transformer is equal to the inverse ratio of the numbers of turns.

From Faraday's law

$$v_1 = -N_1\,\frac{d\Phi}{dt} \tag{7}$$

$$v_2 = -N_2\,\frac{d\Phi}{dt} \tag{8}$$

So, we have

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}} \tag{9}$$

The ratio of the voltages across the primary and secondary windings of an ideal transformer is equal to the turns ratio.

When the secondary winding is terminated in a load resistance R_L, the effective load seen by the source connected to primary winding is

$$(R_1)_{\text{eff}} = \frac{v_1}{i_1} = \frac{v_2\,\frac{N_1}{N_2}}{i_2\,\frac{N_2}{N_1}} = \frac{v_2}{i_2}\left(\frac{N_1}{N_2}\right)^2 = R_L\left(\frac{N_1}{N_2}\right)^2 \tag{10}$$

where

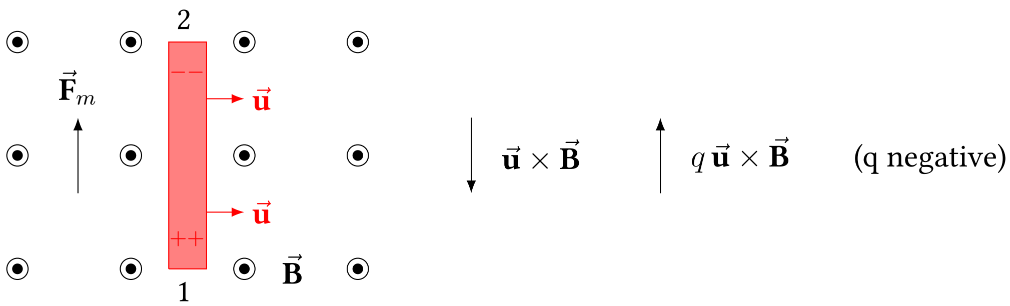
$$R_L = \frac{v_2}{i_2} \tag{11}$$

In a similar way, for a sinusoidal source $v_1(t)$ and a load impedance Z_L, the effective load seen by the source is

$$(Z_1)_{\text{eff}} = Z_L \left(\frac{N_1}{N_2}\right)^2 \tag{12}$$

2.3 A Moving Conductor in a Static Magnetic Field

When a conductor moves with a velocity $\vec{\mathbf{u}}$ in a static (non-time-varying) magnetic field $\vec{\mathbf{B}}$, a force $\vec{\mathbf{F}}_m = q\,\vec{\mathbf{u}} \times \vec{\mathbf{B}}$ will cause the free electrons in the conductor to drift towards one end of the conductor.



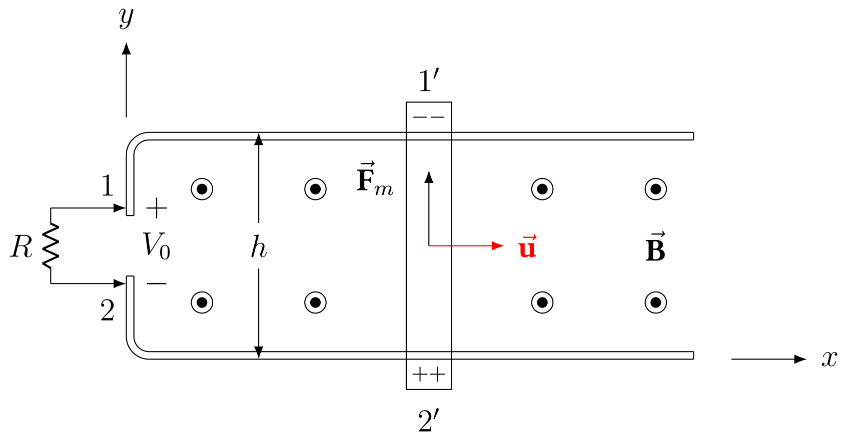
To an observer moving with the conductor there is no apparent motion, and the magnetic force per unit charge $\vec{\mathbf{F}}_m/q = \vec{\mathbf{u}} \times \vec{\mathbf{B}}$ can be interpreted as an induced electric field acting along the conductor and producing a voltage

$$V_{21} = \int_1^2 (\vec{\mathbf{u}} \times \vec{\mathbf{B}}) \cdot d\vec{\ell} \tag{13}$$

If the moving conductor is a part of closed circuit C, then the emf generated around the circuit is

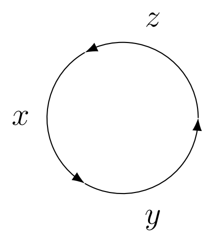
$$V' = \oint_C (\vec{\mathbf{u}} \times \vec{\mathbf{B}}) \cdot d\vec{\ell} \quad \text{(V)} \tag{14}$$

Example 2:
A metal bar slides over a pair of conducting rails in a uniform magnetic field $\vec{\mathbf{B}} = \hat{\mathbf{a}}_z\, B_0$ with a constant velocity $\vec{\mathbf{u}}$, as shown in figure.
a) Determine the open circuit-voltage V_0 that appears across the terminals 1 and 2.
b) Assuming that a resistance R is connected between the terminals, find the electric power dissipated in R.
c) Show that this electric power is equal to the mechanical power required to move the sliding bar with a velocity $\vec{\mathbf{u}}$.



a) <u>Solution I</u>

$$V_0 = V_1 - V_2 = \oint_C (\vec{\mathbf{u}} \times \vec{\mathbf{B}}) \cdot d\vec{\ell} = \int_{2'}^{1'} (\vec{\mathbf{u}} \times \vec{\mathbf{B}}) \cdot d\vec{\ell} \tag{15}$$



$$\vec{\mathbf{u}} \times \vec{\mathbf{B}} = \hat{\mathbf{a}}_x\, u \times \hat{\mathbf{a}}_z\, B_0 = -\hat{\mathbf{a}}_y\, u\, B_0 \tag{16}$$

$$d\vec{\ell} = \hat{\mathbf{a}}_y\, dy \tag{17}$$

$$V_0 = \int_0^h (-)\, u\, B_0\, dy = -u\, B_0\, y \Big|_0^h = -u\, B_0\, h \qquad \text{(V)} \tag{18}$$

<u>Solution II</u>

$$\Phi = \int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} \tag{19}$$

$$\vec{\mathbf{B}} = \hat{\mathbf{a}}_z\, B_0 \tag{20}$$

$$d\vec{\mathbf{s}} = \hat{\mathbf{a}}_z\, dx\, dy \tag{21}$$

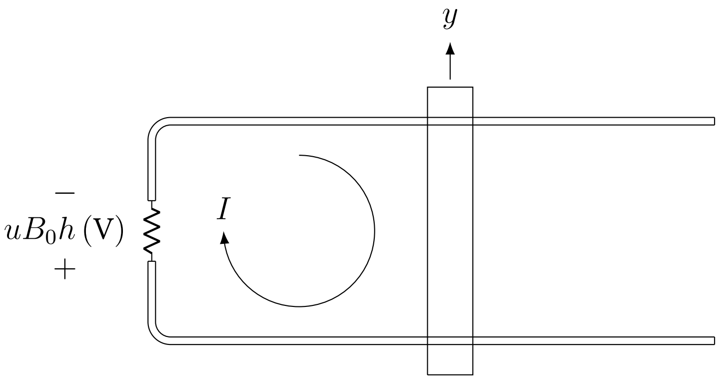
$$\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B_0\, dx\, dy \tag{22}$$

$$\Phi = \int_0^h \int_0^x B_0\, dx\, dy = B_0\, h\, x \tag{23}$$

$$x = u\, t \qquad (\text{distance} = \text{velocity} \times \text{time}) \tag{24}$$

$$\Phi = B_0\, h\, u\, t \tag{25}$$

$$V = -\frac{d\Phi}{dt} = -B_0\, h\, u \qquad \text{(V)} \tag{26}$$



$$I = \frac{V_0}{R} = \frac{B_0 h u}{R}$$

b)

$$Pe = I\,V_0 = \frac{V_0}{R}\,(V_0) = \frac{(V_0)^2}{R} = \frac{(u\,B_0\,h)^2}{R} \qquad \text{(Watt)} \tag{27}$$

c)

$$\text{Work} = \text{Force} \times \text{Distance} \tag{28}$$

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \times \text{Distance}}{\text{Time}} = \text{Force} \times \frac{\text{Distance}}{\text{Time}} \tag{29}$$

$$\text{Power} = \text{Force} \times \text{Velocity} \tag{30}$$

Mechanical power

$$P_{\text{mech}} = \vec{\mathbf{F}}_{\text{mech}} \cdot \vec{\mathbf{u}} \tag{31}$$

$\vec{\mathbf{F}}_{\text{mech}}$ is the mechanical force required to counteract the magnetic force $\vec{\mathbf{F}}_m$, which the magnetic field exerts on the current carrying metal bar.

$$\vec{\mathbf{F}}_m = I \oint_C d\vec{\ell} \times \vec{\mathbf{B}} \tag{32}$$

$d\vec{\ell}$ is in the direction of current flow.

$$d\vec{\ell} = \hat{\mathbf{a}}_y\,dy \tag{33}$$

$$\vec{\mathbf{B}} = \hat{\mathbf{a}}_z\,B_0 \tag{34}$$

$$d\vec{\ell} \times \vec{\mathbf{B}} = (\hat{\mathbf{a}}_y \times \hat{\mathbf{a}}_z)\,B_0\,dy = \hat{\mathbf{a}}_x\,B_0\,dy \tag{35}$$

I is in clockwise direction.

$$\vec{\mathbf{F}}_m = -I \int_{2'}^{1'} \hat{\mathbf{a}}_x\,B_0\,dy = -\hat{\mathbf{a}}_x\,I\,B_0\,y\Big|_0^h = -\hat{\mathbf{a}}_x\,I\,B_0\,h \qquad \text{(magnetic force)} \tag{36}$$

$$\vec{\mathbf{F}}_{\text{mech}} = -\vec{\mathbf{F}}_m = \hat{\mathbf{a}}_x\,I\,B_0\,h \tag{37}$$

$$P_{\text{mech}} = \vec{\mathbf{F}}_{\text{mech}} \cdot \vec{\mathbf{u}} = (\hat{\mathbf{a}}_x\,I\,B_0\,h) \cdot (\hat{\mathbf{a}}_x\,u) = I\,B_0\,h\,u \tag{38}$$

$$I = \frac{V_0}{R} = \frac{(B_0\,h\,u)}{R} \tag{39}$$

$$P_{\text{mech}} = \left(\frac{B_0\,h\,u}{R}\right)(B_0\,h\,u) = \frac{(B_0\,h\,u)^2}{R} \qquad \text{(Watt)} \tag{40}$$

$$P_{\text{mech}} = P_e \tag{41}$$