

### YÖNEYLEM ARAŞTIRMASI-III

- 1)**  $X, \lambda$  parametresi ile üstel dağılıma uygun bir rassal değişken ise,
- $P(X = t) = f(t) = \lambda e^{-\lambda t}$
  - $P(X > t) = 1 - F(t) = e^{-\lambda t}$
  - $E[X] = \frac{1}{\lambda}$
- 2)** Üstel dağılımin unutkanlık özelliği vardır. Buna göre,  $P(X > t + s | X > t) = P(X > s)$ ,
- 3)**  $T, \lambda$  parametresi ile poisson dağılıma uygun bir rassal değişken ise,  $t$  döneminde  $n$  olay olma olasılığı  
 $p_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$
- 4)** Kuyruk uzunluğu  $N$  ve hizmet sıklığı  $\mu$  olan, saf ölüm modeline uygun bir kuyruk sisteminde,  $t$  anında,  $n$  müşteri kalma olasılığı,  $p_n(t) = \frac{(\mu t)^{N-n} e^{-\mu t}}{(N-n)!}$  dir.
- 5)** Genelleştirilmiş Poisson Kuyruk Modelinde;
- Sistemde  $n$  müşteri olma olasılığı;  

$$p_n = \left( \frac{\lambda_{n-1} \lambda_{n-2} \cdots \lambda_1 \lambda_0}{\mu_n \mu_{n-1} \cdots \mu_2 \mu_1} \right) p_0$$
  - Sistemde hiç müşteri olmama olasılığı;  

$$p_0 = 1 - \sum_{n=1}^{\infty} p_n$$
- 6)** Kararlılık durumu performans ölçütleri;
- $L_s = \sum_{n=1}^{\infty} n p_n = \lambda_{eff} W_s$
  - $L_q = \sum_{n=(c+1)}^{\infty} (n - c) p_n = \lambda_{eff} W_q$
  - $W_s = W_q + \frac{1}{\mu}$
  - $L_s = L_q + \frac{\lambda_{eff}}{\mu}$
  - $\bar{c} = L_s - L_q = \frac{\lambda_{eff}}{\mu}$
  - $\rho = \frac{\lambda}{\mu}$
  - $\lambda = \lambda_{eff} + \lambda_{lost}$
- 7)** **(M/M/1):(GD/∞/∞) ;**
- $p_n = \left( \frac{\lambda}{\mu} \right)^n p_0 = \rho^n p_0$
  - $p_0 = 1 - \rho$
  - $L_s = \frac{\rho}{1-\rho}$
  - $W_s = \frac{L_s}{\lambda} = \frac{1}{\mu(1-\rho)}$
  - $W_q = W_s - \frac{1}{\mu} = \frac{\rho}{\mu(1-\rho)}$
  - $L_q = \lambda W_q = \frac{\rho^2}{(1-\rho)}$
  - $\bar{c} = L_s - L_q = \rho$

- 8)** **(M/M/1):(GD/N/∞) ;**
- $p_n = \begin{cases} \rho^n p_0 & n \leq N \\ 0 & n > N \end{cases}$
  - $p_0 = \begin{cases} \frac{(1-\rho)\rho^n}{1-\rho^{N+1}} & \rho \neq 1 \\ \frac{1}{N+1} & \rho = 1 \end{cases}$
  - $\lambda_{lost} = \lambda p_N$
  - $\lambda_{eff} = \lambda(1 - p_N)$
  - $L_s = \begin{cases} \sum_{n=0}^N n p_n = \frac{\rho \{1-(N+1)\rho^n+N\rho^{n+1}\}}{(1-\rho)(1-\rho^{n+1})} & \rho \neq 1 \\ \frac{N}{2} & \rho = 1 \end{cases}$
  - $W_s = \frac{L_s}{\lambda_{eff}}$
  - $W_q = W_s - \frac{1}{\mu}$
  - $L_q = \lambda_{eff} W_q$
  - $\bar{c} = L_s - L_q = \rho$
- 9)** **(M/M/c):(GD/∞/∞) ;**
- $p_n = \begin{cases} \frac{\lambda^n}{n! \mu^n} p_0 & n \leq c \\ \frac{\lambda^n}{c! c^{n-c} \mu^n} p_0 & n > c \end{cases}$
  - $p_0 = \left\{ \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left( \frac{1}{1-\frac{\rho}{c}} \right) \right\}^{-1} \quad \frac{\rho}{c} < 1$
  - $L_q = \sum_{n=c}^{\infty} (n - c) p_n = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} p_0$
  - $L_s = L_q + \rho$
  - $\lambda_{eff} = \lambda, \quad W_s = \frac{L_s}{\lambda}, \quad W_q = \frac{L_q}{\lambda}$
- 10)** **(M/M/c):(GD/N/∞) ; c≤N**
- $p_n = \begin{cases} \frac{\rho^n}{n!} p_0 & 0 \leq n \leq c \\ \frac{\rho^n}{c! c^{n-c}} p_0 & c \leq n \leq N \end{cases}$
  - $p_0 = \begin{cases} \left[ \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c \left( 1 - \left( \frac{\rho}{c} \right)^{N-c+1} \right)}{c! \left( 1 - \frac{\rho}{c} \right)} \right]^{-1} & \frac{\rho}{c} \neq 1 \\ \left[ \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c (N - c + 1)}{c!} \right]^{-1} & \frac{\rho}{c} = 1 \end{cases}$
  - $L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} \left\{ 1 - \left( \frac{\rho}{c} \right)^{N-c+1} - (N - c + 1) \left( 1 - \frac{\rho}{c} \right) \left( \frac{\rho}{c} \right)^{N-c} \right\} p_0$
  - $\frac{\rho}{c} = 1$  için  $L_q = \frac{\rho^c (N - c) (N - c + 1)}{2c!} p_0$
  - $\lambda_{lost} = \lambda p_N$
  - $\lambda_{eff} = \lambda - \lambda_{lost}, \quad W_s = \frac{L_s}{\lambda_{eff}}, \quad W_q = \frac{L_q}{\lambda_{eff}}$
- 11)** **(M/M/∞):(GD/∞/∞) – Self Servis Modeli**
- $p_n = \frac{e^{-\rho} \rho^n}{n!} \quad n = 0, 1, 2, \dots$
  - $L_s = \rho, \quad L_q = W_q = 0$