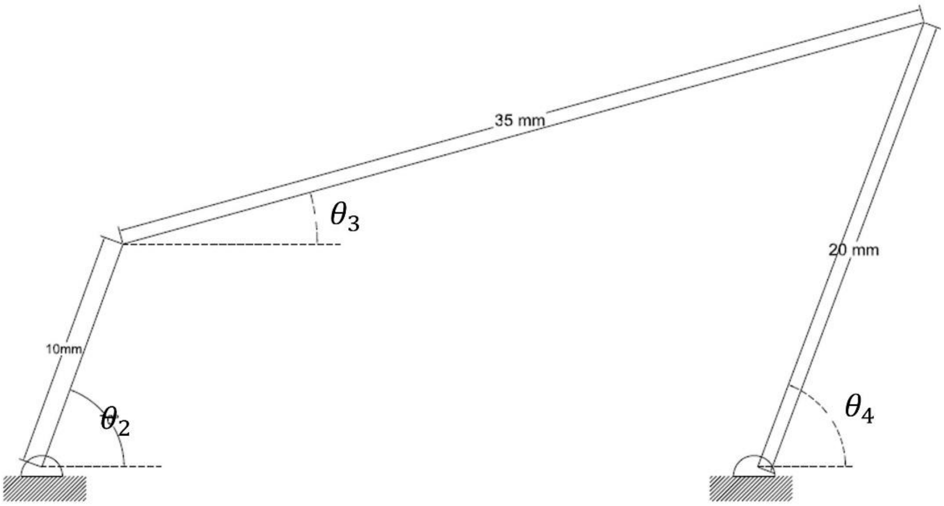
FOUR BAR MECHANISM



$a_1 = 30\ mm$

$a_2 = 10\ mm$

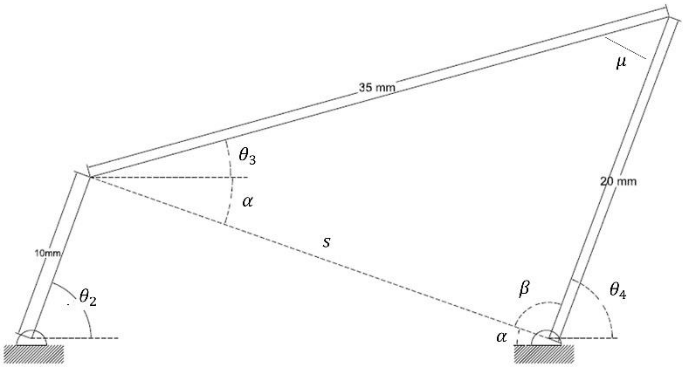
$a_3 = 35\ mm$

$a_4 = 20\ mm$

$\theta_2 = 45°$

Find θ_3, θ_4 for $\theta_2 = 60°$

Step-wise Solution



$$s^2 = a_1^2 + a_2^2 - 2a_1a_2\cos\theta_2$$

$$\alpha = \cos^{-1}\left(\frac{s^2 + a_1^2 - a_2^2}{2sa_1}\right)$$

$$\mu = \cos^{-1}\left(\frac{a_3^2 + a_4^2 - s^2}{2a_3a_4}\right)$$

$$\beta = \cos^{-1}\left(\frac{a_4^2 + s^2 - a_3^2}{2a_4s}\right)$$

$$\theta_3 = 180 - \mu - \beta - \alpha$$

Hazırlayan: Arş. Gör. Görkem AĞÖREN
Arş. Gör. Mustafa Alper SARIİPEK

İnceleyen: Prof. Dr. Tuncay KARAÇAY

$\theta_4 = 180 - \beta - \alpha$

For $\theta_2 = 60$; $s^2 = 30^2 + 10^2 - 2.30.10.\cos 60 \rightarrow s = 26{,}46\,mm$

$$\alpha = \cos^{-1}\left(\frac{26{,}46^2 + 30^2 - 10^2}{2x26{,}46x30}\right) \rightarrow \alpha = 19{,}1°$$

$$\mu = \cos^{-1}\left(\frac{35^2 + 20^2 - 26{,}46^2}{2x35x20}\right) \rightarrow \mu = 48{,}65°$$

$$\beta = \cos^{-1}\left(\frac{20^2 + 26{,}46^2 - 35^2}{2x20x26{,}46}\right) \rightarrow \beta = 96{,}78°$$

$\theta_3 = 180 - 48{,}65 - 96{,}78 - 19{,}1 = 15{,}7°$

$\theta_4 = 64{,}12°$

<u>Cartesian Solution</u>

$\overline{|O_1A|} + |\overrightarrow{AB}| = \overline{|O_1O_2|} + \overline{|O_2B|}$

$$a_2\cos\theta_2\vec{\imath} + a_2\sin\theta_2\vec{\jmath} + a_3\cos\theta_3\vec{\imath} + a_3\sin\theta_3\vec{\jmath} = a_1\vec{\imath} + a_4\cos\theta_4\vec{\imath} + a_4\sin\theta_4\vec{\jmath}$$

$\vec{\imath} \rightarrow a_2\cos\theta_2 + a_3\cos\theta_3 = a_1 + a_4\cos\theta_4$

$\vec{\jmath} \rightarrow a_2\sin\theta_2 + a_3\sin\theta_3 = a_4\sin\theta_4$

$a_3\cos\theta_3 = a_1 + a_4\cos\theta_4 - a_2\cos\theta_2 \ldots\ldots\ldots\ldots 1$

$a_3\sin\theta_3 = a_4\sin\theta_4 - a_2\sin\theta_2 \ldots\ldots\ldots\ldots\ldots\ldots 2$

When we Square equation 1, 2 and add them together;

$$a_3^2 = a_1^2 + a_4^2\cos^2\theta_4 + a_2^2\cos^2\theta_2 + 2[a_1a_4\cos\theta_4 - a_1a_2\cos\theta_2 - a_2a_4\cos\theta_2\cos\theta_4] + a_4^2\sin^2\theta_4 + a_2^2\sin^2\theta_4 - 2a_2a_4\sin\theta_2\sin\theta_4$$

$a_4^2 = a_4^2(\sin^2\theta_4 + \cos^2\theta_4)$

$a_2^2 = a_2^2(\sin^2\theta_2 + \cos^2\theta_2)$

$$a_1^2 + a_2^2 - a_3^2 + a_4^2 + 2a_1a_4\cos\theta_4 - 2a_1a_2\cos\theta_2 = 2a_2a_4(\cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4)$$

Dividing both side by $2a_2a_4$;

$$\frac{a_1^2 + a_2^2 - a_3^2 + a_4^2}{2a_2a_4} + \frac{2a_1a_4}{2a_2a_4}\cos\theta_4 - \frac{2a_1a_2}{2a_2a_4}\cos\theta_2 = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4$$

$$K_1 = \frac{a_1^2 + a_2^2 - a_3^2 + a_4^2}{2a_2a_4}$$

$K_2 = \frac{a_1}{a_2}$

$K_3 = \frac{a_1}{a_4}$

$K_1 + K_2 \cos\theta_4 - K_3 \cos\theta_2 = \cos\theta_2 \cos\theta_4 + \sin\theta_2 \sin\theta_4$

If $\tan\frac{\theta_4}{2} = u \rightarrow \cos\theta_4 = \frac{1-u^2}{1+u^2} \quad \sin\theta_4 = \frac{2u}{1+u^2}$

$K_1 + K_2 \left(\frac{1-u^2}{1+u^2}\right) - K_3 \cos\theta_2 = \cos\theta_2 \left(\frac{1-u^2}{1+u^2}\right) + \sin\theta_2 \left(\frac{2u}{1+u^2}\right)$

If we multiply both sides with $(1+u^2)$

$K_1(1+u^2) + K_2(1-u^2) - K_3 \cos\theta_2 (1+u^2) - \cos\theta_2 (1-u^2) - \sin\theta_2 (2u) = 0$

$u^2[K_1 - K_2 - K_3 \cos\theta_2 + \cos\theta_2] + u[-2\sin\theta_2] + [K_1 + K_2 - K_3 \cos\theta_2 - \cos\theta_2] = 0$

$A = [K_1 - K_2 - K_3 \cos\theta_2 + \cos\theta_2]$

$B = [-2\sin\theta_2]$

$C = [K_1 + K_2 - K_3 \cos\theta_2 - \cos\theta_2]$

$u_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\frac{\theta_4}{2} = \tan^{-1}(u_{1,2})$

$a_1 = 30\ mm$

$a_2 = 10\ mm$

$a_3 = 35\ mm$

$a_4 = 20\ mm$

$\theta_2 = 60°$

$K_1 = \frac{a_1^2 + a_2^2 - a_3^2 + a_4^2}{2a_2a_4} = \frac{30^2 + 10^2 - 35^2 + 20^2}{2x10x20} = 0{,}4375$

$K_2 = \frac{a_1}{a_2} = \frac{30}{10} = 3$

$K_3 = \frac{a_1}{a_4} = \frac{30}{20} = 1{,}5$

$A = [K_1 - K_2 - K_3 \cos\theta_2 + \cos\theta_2] = 0{,}4375 - 3 - 1{,}5x \cos 60° + \cos 60° = -2{,}8175$

$B = [-2\sin\theta_2] = -2x \sin 60° = -1{,}7321$

$C = [K_1 + K_2 - K_3 \cos\theta_2 - \cos\theta_2] = 0{,}4375 + 3 - 1{,}5\text{x}\cos 60° - \cos 60° = 2{,}1875$

$$u_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \frac{1{,}7321 + \sqrt{(-1{,}7321)^2 - 4x(-2{,}8175)x(2{,}1875)}}{2x(-2{,}8175)} = 0{,}6262$$

$$u_2 = \frac{-B - \sqrt{B^2 - 4AC}}{2A} = \frac{1{,}7321 - \sqrt{(-1{,}7321)^2 - 4x(-2{,}8175)x(2{,}1875)}}{2x(-2{,}8175)} = -1{,}2420$$

$\theta_4 = 2\tan^{-1}(u_1) = 64{,}1101°$

$\theta_4 = 2\tan^{-1}(u_2) = -102{,}3233°$

To find θ_3 we can use equation 2;

$a_3 \sin\theta_3 = a_4 \sin\theta_4 - a_2 \sin\theta_2$

$$\theta_3 = \sin^{-1}\left(\frac{a_4 \sin\theta_4 - a_2 \sin\theta_2}{a_3}\right)$$

$$\theta_3 = \sin^{-1}\left(\frac{20\sin(64{,}1101) - 10\sin(60)}{35}\right) = 15{,}47°$$

<u>Complex Solution</u>

$a_2 e^{i\theta_2} + a_3 e^{i\theta_3} = a_1 + a_4 e^{i\theta_4}$ $(Loop\ Closure\ Equation)$

$a_2 e^{-i\theta_2} + a_3 e^{-i\theta_3} = a_1 + a_4 e^{-i\theta_4}$ $(Complex\ Conjugate\ of\ Loop\ Closure\ Equation)$

$a_4 e^{i\theta_4} = a_2 e^{i\theta_2} + a_3 e^{i\theta_3} - a_1$

$a_4 e^{-i\theta_4} = a_2 e^{-i\theta_2} + a_3 e^{-i\theta_3} - a_1$

Multiply two sides;

$$a_4^2 = a_2^2 + a_3^2 + a_1^2 + a_2 a_3 e^{i(\theta_2-\theta_3)} - a_1 a_2 e^{i\theta_2} + a_2 a_3 e^{i(\theta_3-\theta_2)} - a_1 a_3 e^{i\theta_3} - a_1 a_2 e^{-i\theta_2} - a_1 a_3 e^{-i\theta_3}$$

$$a_4^2 = a_2^2 + a_3^2 + a_1^2 + a_2 a_3\left(e^{i(\theta_2-\theta_3)} + e^{-i(\theta_2-\theta_3)}\right) - a_1 a_2\left(e^{i\theta_2} + e^{-i\theta_2}\right) - a_1 a_3\left(e^{i\theta_3} + e^{-i\theta_3}\right)$$

$e^{i\theta} + e^{-i\theta} = 2\cos\theta$

$e^{i\theta} + e^{-i\theta} = 2i\sin\theta$

So, the equation will be;

$a_1^2 + a_2^2 + a_3^2 - a_4^2 + 2a_2 a_3 \cos(\theta_2 - \theta_3) - 2a_1 a_2 \cos\theta_2 - 2a_1 a_3 \cos\theta_3 = 0$

$2a_2 a_3 \cos(\theta_2 - \theta_3) = a_1^2 + a_2^2 + a_3^2 - a_4^2 + 2a_1 a_2 \cos\theta_2 + 2a_1 a_3 \cos\theta_3$

$$\cos(\theta_2 - \theta_3) = \frac{a_1^2 + a_2^2 + a_3^2 - a_4^2}{2a_2 a_3} + \frac{a_1}{a_3}\cos\theta_2 + \frac{a_1}{a_2}\cos\theta_3$$

$$K_1 = \frac{a_1^2 + a_2^2 + a_3^2 - a_4^2}{2a_2a_3}$$

$$K_2 = \frac{a_1}{a_3}$$

$$K_3 = \frac{a_1}{a_2}$$

As seen, the equation is same with Cartesian Solution.

This equation is called Freudenstein's Equation.

If $\tan\frac{\theta_3}{2} = u \ \rightarrow \ \cos\theta_3 = \frac{1-u^2}{1+u^2} \quad \sin\theta_3 = \frac{2u}{1+u^2}$

$$\cos\theta_2\left(\frac{1-u^2}{1+u^2}\right) + \sin\theta_2\left(\frac{2u}{1+u^2}\right) = K_1 + K_2\cos\theta_2 + K_3\left(\frac{1-u^2}{1+u^2}\right)$$

$$\cos\theta_2(1-u^2) + \sin\theta_2\,(2u) = K_1(1+u^2) + K_2\cos\theta_2\,(1+u^2) + K_3(1-u^2)$$

$$u^2[K_1 + K_2\cos\theta_2 - K_3 + \cos\theta_2] + u[-2\sin\theta_2] + [K_1 + K_2\cos\theta_2 + K_3 - \cos\theta_2] = 0$$

$$A = [K_1 + K_2\cos\theta_2 - K_3 + \cos\theta_2]$$

$$B = [-2\sin\theta_2]$$

$$C = [K_1 + K_2\cos\theta_2 + K_3 - \cos\theta_2]$$

$$u_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\theta_3 = 2\tan^{-1}(u_{1,2}) \rightarrow \theta_3 = 15{,}47°$$

From Loop Closure Equations;

$$\theta_4 = 64{,}1101°$$

VELOCITY ANALYSIS

1 <u>Graphical Method:</u>

From displacement analysis;

When $\theta_2 = 60°, \theta_3 = 15{,}47°, \theta_4 = 64{,}12°$

$$\omega_2 = 2rad/s$$

$$\overrightarrow{V_A} = \omega_2|O_1A| = 2\frac{rad}{s}x10mm = 20\ \frac{mm}{s}$$

$$\overrightarrow{V_A} = 20\ \frac{mm}{s} \quad (\perp\ O_1A)$$

$$\overrightarrow{V_B} = \overrightarrow{V_A} + \overrightarrow{V_{B/A}}$$

2 unknowns

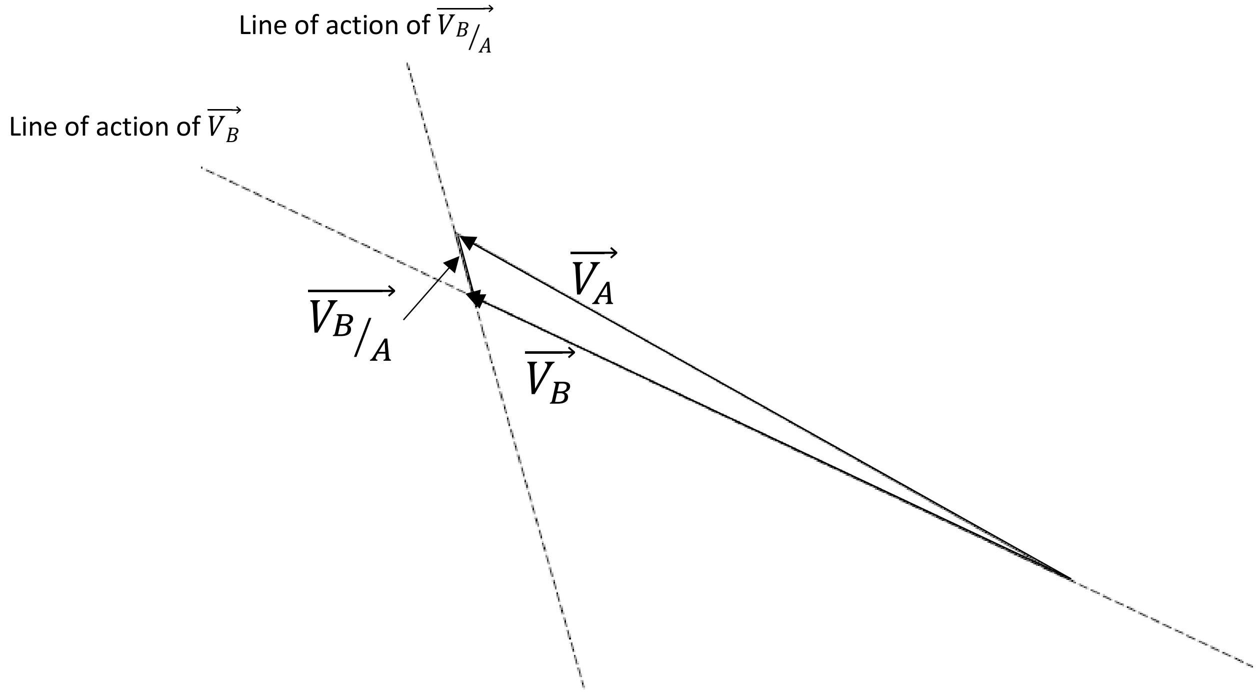
- Magnitude of V_B
- Magnitude of $V_{B/A}$ Solution is possible

1 vector equation

$\perp AB = \overrightarrow{V_B}$

$\perp O_1A = W_2|O_1A| = \overrightarrow{V_A}$

$\perp BA = \overrightarrow{V_{B/A}}$



As a result of the measurements, $\overrightarrow{V_B}$ is found to be 18.6 mm/s and $\overrightarrow{V_{B/A}}$ is 1.44 mm/s. So the angular velocities:

$$\overrightarrow{V_B} = \omega_4 a_4 \rightarrow \omega_4 = \frac{\overrightarrow{V_B}}{a_4} = \frac{18.6}{20} = 0.93\, rad/s$$

$$\overrightarrow{V_{B/A}} = \omega_3 a_3 \rightarrow \omega_3 = \frac{\overrightarrow{V_{B/A}}}{a_3} = \frac{1.44}{35} = 0.041\, rad/s\ (Clock - wise)$$

2 <u>Analytical Method:</u>

<u>a. Cartesian Coordinates:</u>

$a_2e^{i\theta_2} + a_3e^{i\theta_3} = a_1 + a_4e^{i\theta_4}$ $(Loop\ Closure\ Equation)$

First derivative of loop closure equation with respect to time;

$\underbrace{a_2\dot{\theta}_2e^{i\theta_2}}_{\overrightarrow{V_A}} + \underbrace{a_3\dot{\theta}_3e^{i\theta_3}}_{\overrightarrow{V_{B/A}}} = \underbrace{a_4\dot{\theta}_4e^{i\theta_4}}_{\overrightarrow{V_B}}$ where $\dot{\theta}_2 = \omega_2,\ \dot{\theta}_3 = \omega_3,\ \dot{\theta}_4 = \omega_4$.

$\overrightarrow{\omega_4}x\ \overrightarrow{r_{0_2B}} = \overrightarrow{\omega_2}x\ \overrightarrow{r_{0_1A}} + \overrightarrow{\omega_3}x\ \overrightarrow{r_{AB}}$ $\quad(\overrightarrow{\omega_2}, \overrightarrow{r_{0_2B}}, \overrightarrow{r_{0_1A}}, \overrightarrow{r_{AB}}\ known, \overrightarrow{\omega_3}\ and\ \overrightarrow{\omega_4}\ unknowns.)$

1 vector equation with two unknowns can be solved.

$\overrightarrow{r_{0_2B}} = a_4.(\cos 64.12\vec{\imath} + \sin 64.12\vec{\jmath} = 8.730\vec{\imath} + 17.994\vec{\jmath}$

$\overrightarrow{r_{0_1A}} = a_2.(\cos 60\vec{\imath} + \sin 60\vec{\jmath} = 5\vec{\imath} + 8.66\vec{\jmath}$

$\overrightarrow{r_{AB}} = a_3.(\cos 15.47\vec{\imath} + \sin 15.47\vec{\jmath} = 33.732\vec{\imath} + 9.33\vec{\jmath}$

$\omega_4\vec{k}\ x\ (8.730\vec{\imath} + 17.994\vec{\jmath}) = 2\vec{k}\ x\ (5\vec{\imath} + 8.66\vec{\jmath}) + \omega_3\vec{k}\ x\ (33.732\vec{\imath} + 9.33\vec{\jmath})$

$8.730\omega_4\vec{\jmath} - 17.994\omega_4\vec{\imath} = 10\vec{\jmath} - 17.32\vec{\imath} + 33.72\omega_3\vec{\jmath} - 9.33\omega_3\vec{\imath}$

$From\ \vec{\imath}\ equation \longrightarrow \quad -17.994\omega_4 = -17.32 - 9.33\omega_3$

$From\ \vec{\jmath}\ equation \longrightarrow \quad 8.730\omega_4 = 10 - 33.72\omega_3$

Rearrange terms;

$9.33\omega_3 - 17.994\omega_4 = -17.32$

$\underbrace{33.72\omega_3 + 8.730\omega_4}_{unknowns} = -\underbrace{10}_{knowns}$

So we can write this equation;

$$\begin{bmatrix} 9.33 & -17.994 \\ 33.72 & -8.730 \end{bmatrix}\begin{bmatrix} \omega_3 \\ \omega_4 \end{bmatrix} = \begin{bmatrix} -17.32 \\ -10 \end{bmatrix}$$

Using Cramer's Rule:

$$\omega_3 = \frac{\begin{vmatrix} -17.32 & -17.994 \\ -10 & -8.730 \end{vmatrix}}{\begin{vmatrix} 9.33 & -17.994 \\ 33.72 & -8.730 \end{vmatrix}} = -0.0547\ rad/s$$

$$\omega_4 = \frac{\begin{vmatrix} 9.33 & -17.32 \\ 33.72 & -10 \end{vmatrix}}{\begin{vmatrix} 9.33 & -17.994 \\ 33.72 & -8.730 \end{vmatrix}} = 0.9342\ rad/s$$

b. Complex Solution:

$a_2 e^{i\theta_2} + a_3 e^{i\theta_3} = a_1 + a_4 e^{i\theta_4}$ $(Loop\ Closure\ Equation)$

First derivative of the loop closure equation with respect to time;

$$\underbrace{i a_2 \dot{\theta}_2 e^{i\theta_2}}_{\overrightarrow{V_A}} + \underbrace{i a_3 \dot{\theta}_3 e^{i\theta_3}}_{\overrightarrow{V_{B/A}}} = \underbrace{i a_4 \dot{\theta}_4 e^{i\theta_4}}_{\overrightarrow{V_B}}$$

Rearrange terms;

$$i a_3 \dot{\theta}_3 e^{i\theta_3} - i a_4 \dot{\theta}_4 e^{i\theta_4} = -i a_2 \dot{\theta}_2 e^{i\theta_2}$$

$$-i a_3 \dot{\theta}_3 e^{-i\theta_3} + i a_4 \dot{\theta}_4 e^{-i\theta_4} = i a_2 \dot{\theta}_2 e^{-i\theta_2} \quad (Complex\ conjugate)$$

$$\begin{bmatrix} i a_3 e^{i\theta_3} & -i a_4 e^{i\theta_4} \\ -i a_3 e^{-i\theta_3} & i a_4 e^{-i\theta_4} \end{bmatrix} \begin{bmatrix} \omega_3 \\ \omega_4 \end{bmatrix} = \begin{bmatrix} -i a_2 e^{i\theta_2} \\ i a_2 e^{-i\theta_2} \end{bmatrix} \dot{\theta}_2$$

Using Cramer's Rule;

$$\omega_3 = \frac{\begin{vmatrix} -i a_2 e^{i\theta_2} & -i a_4 e^{i\theta_4} \\ i a_2 e^{-i\theta_2} & i a_4 e^{-i\theta_4} \end{vmatrix}}{\begin{vmatrix} i a_3 e^{i\theta_3} & -i a_4 e^{i\theta_4} \\ -i a_3 e^{-i\theta_3} & i a_4 e^{-i\theta_4} \end{vmatrix}} \dot{\theta}_2 = \frac{a_2 a_4 \left[e^{i(\theta_2-\theta_4)} - e^{-i(\theta_2-\theta_4)} \right]}{-a_3 a_4 \left[e^{i(\theta_3-\theta_4)} - e^{-i(\theta_3-\theta_4)} \right]} \dot{\theta}_2 = -\frac{a_2}{a_3} \frac{\sin(\theta_2-\theta_4)}{\sin(\theta_3-\theta_4)} \dot{\theta}_2$$

$$\omega_3 = -\frac{10}{35} \frac{\sin(60-64.12)}{\sin(15.47-64.12)} 2 = -0.547\ rad/s$$

$$\omega_4 = \frac{\begin{vmatrix} i a_3 e^{i\theta_3} & -i a_2 e^{i\theta_2} \\ -i a_3 e^{-i\theta_3} & i a_2 e^{-i\theta_2} \end{vmatrix}}{\begin{vmatrix} i a_3 e^{i\theta_3} & -i a_4 e^{i\theta_4} \\ -i a_3 e^{-i\theta_3} & i a_4 e^{-i\theta_4} \end{vmatrix}} \dot{\theta}_2 = \frac{a_2}{a_3} \frac{\sin(\theta_3-\theta_2)}{\sin(\theta_3-\theta_4)} \dot{\theta}_2 = \frac{10}{20} \frac{\sin(15.47-60)}{\sin(15.47-64.12)} 2$$
$$= 0.934\ rad/s$$

ACCELERATION ANALYSIS

1 Graphical Method:

$$\overrightarrow{V_B} = \overrightarrow{V_A} + \overrightarrow{V_{B/A}}$$

$$\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{B/A}}$$

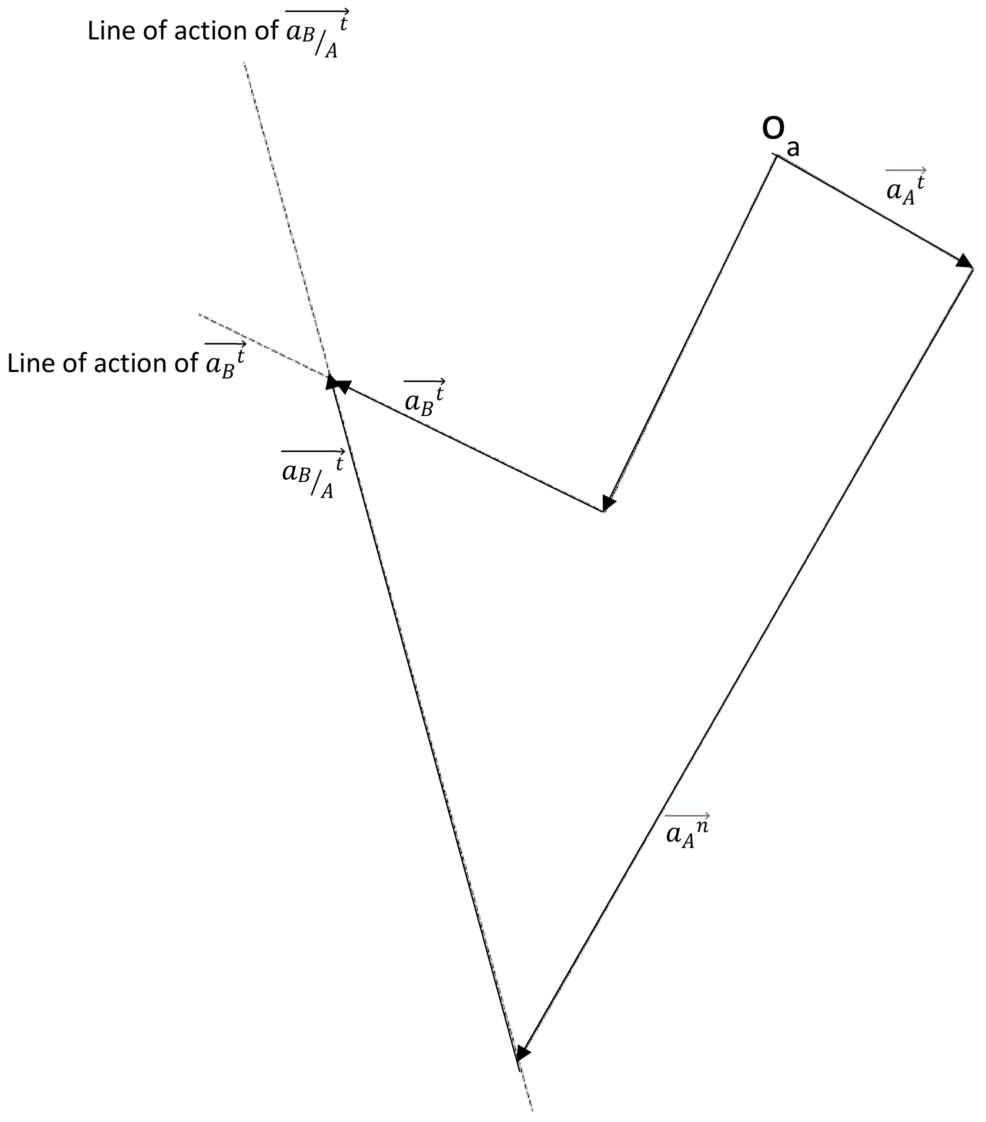
$$\underbrace{\overbrace{\overrightarrow{a_B}^t}^{\alpha_4.a_4}}_{\perp O_2B} + \underbrace{\overbrace{\overrightarrow{a_B}^n}^{\omega_4{}^2.a_4}}_{\parallel O_2B} = \underbrace{\overbrace{\overrightarrow{a_A}^t}^{\alpha_2.a_2}}_{\perp O_1A} + \underbrace{\overbrace{\overrightarrow{a_A}^n}^{\omega_2{}^2.a_2}}_{\parallel O_1A} + \underbrace{\overbrace{\overrightarrow{a_{B/A}}^t}^{\alpha_3.a_3}}_{\perp AB} + \underbrace{\overbrace{\overrightarrow{a_{B/A}}^n}^{\omega_3{}^2.a_3}}_{\parallel AB}$$

$\overrightarrow{a_B}^n = \omega_4{}^2 . a_4 \ = \ (0.934)^2 . 20 \ = \ 17.45\, mm/s^2$ ($\parallel O_2B$)

$\overrightarrow{a_A}^t = \alpha_2 . a_2 \ = \ (-1).10 \ = \ -10\, mm/s^2$ ($\perp O_1A$)

$\overrightarrow{a_A}^n = \omega_2{}^2 . a_2 \ = \ (2)^2 . 10 \ = \ 40\, mm/s^2$ ($\parallel O_1A$)

$\overrightarrow{a_{B/A}}^n = \omega_3{}^2 . a_3 \ = \ (-0.0546)^2 . 35 = \ 0.104\, mm/s^2$ ($\parallel AB$)



Since $\overrightarrow{a_{B/A}}^n$ is very small compared to other accelerations, it is not seen on the polygon. As a result of the measurements, $\overrightarrow{a_B}^t$ is found to be 13.42 mm/s^2 and $\overrightarrow{a_{B/A}}^t$ is 31 mm/s^2. So the angular accelerations:

$$\overrightarrow{a_B}^t \ = \ \alpha_4 . a_4 \ \rightarrow \ \alpha_4 = \frac{a_B{}^t}{a_4} = \frac{13.42}{20} = 0.671\, rad/s^2$$

$$\overrightarrow{a_{B/A}}^t \ = \ \alpha_3 . a_3 \ \rightarrow \ \alpha_3 = \frac{a_{B/A}{}^t}{a_3} = \frac{31}{35} = 0.886\, rad/s^2$$

2 Analytical Method:

a. Cartesian Coordinates:

$$\underbrace{\overbrace{\overrightarrow{a_B{}^t}}^{\alpha_4.a_4}}_{\perp O_2B} + \underbrace{\overbrace{\overrightarrow{a_B{}^n}}^{\omega_4{}^2.a_4}}_{\parallel O_2B} = \underbrace{\overbrace{\overrightarrow{a_A{}^t}}^{\alpha_2.a_2}}_{\perp O_1A} + \underbrace{\overbrace{\overrightarrow{a_A{}^n}}^{\omega_2{}^2.a_2}}_{\parallel O_1A} + \underbrace{\overbrace{\overrightarrow{a_{B/A}{}^t}}^{\alpha_3.a_3}}_{\perp AB} + \underbrace{\overbrace{\overrightarrow{a_{B/A}{}^n}}^{\omega_3{}^2.a_3}}_{\parallel AB}$$

$$\alpha_4\ x\ \overrightarrow{r_{0_2B}} + \omega_4 x(\omega_4 x\ \overrightarrow{r_{0_2B}}) = \alpha_2\ x\ \overrightarrow{r_{0_1A}} + \omega_2 x(\omega_2 x\ \overrightarrow{r_{0_1A}}) + \alpha_3\ x\ \overrightarrow{r_{AB}} + \omega_3 x(\omega_3 x\ \overrightarrow{r_{AB}})$$

$$\overrightarrow{r_{0_2B}} = a_4.(\cos 64.12\vec{\imath} + \sin 64.12\,\vec{\jmath} = (8.730\vec{\imath} + 17.994\vec{\jmath})$$

$$\overrightarrow{r_{0_1A}} = a_2.(\cos 60\vec{\imath} + \sin 60\,\vec{\jmath} = (5\vec{\imath} + 8.66\vec{\jmath})$$

$$\overrightarrow{r_{AB}} = a_3.(\cos 15.47\vec{\imath} + \sin 15.47\,\vec{\jmath} = (33.732\vec{\imath} + 9.33\vec{\jmath})$$

$\omega_2 = 2\vec{k}\,rad/s \quad \omega_3 = -0.0547\vec{k}\,rad/s \quad \omega_4 = 0.9342\vec{k}\,rad/s \quad \alpha_2 = -1\vec{k}\ rad/s^2$

$\alpha_3\ and\ \alpha_4\ are\ unknowns.$

$$\begin{aligned}\alpha_4\vec{k}\ x\ (8.730\vec{\imath} + 17.994\vec{\jmath}) + 0.9342\vec{k}\ x\ [0.9342\vec{k}\ x\ (8.730\vec{\imath} + 17.994\vec{\jmath})]\\ = 1\vec{k}\ x\ (5\vec{\imath} + 8.66\vec{\jmath}) + 2\vec{k}\ x\ [2\vec{k}\ x\ (5\vec{\imath} + 8.66\vec{\jmath})]\\ + \alpha_3\vec{k}\ x\ (33.732\vec{\imath} + 9.33\vec{\jmath})\\ + -0.0547\vec{k}\ x\ [-0.0547\vec{k}\ x\ (33.732\vec{\imath} + 9.33\vec{\jmath})]\end{aligned}$$

$$\begin{aligned}8.730\alpha_4\vec{\jmath} - 17.994\alpha_4\vec{\imath} - 7.619\vec{\imath} - 15.704\vec{\jmath}\\ = -5\vec{\jmath} + 8.66\vec{\imath} - 20\vec{\imath} - 34.64\vec{\jmath} + 33.72\alpha_3\vec{\jmath} - 9.33\alpha_3\vec{\imath} - 0.1\vec{\imath} - 0.28\vec{\jmath}\end{aligned}$$

$From\ \vec{\imath}\ equation\ \rightarrow 17.994\alpha_4 - 7.619 = 8.66 - 20 - 9.33\alpha_3 - 0.1$

$17.994\alpha_4 = -9.33\alpha_3 - 3.821$

$\alpha_4 = 0.518\alpha_3 + 0.212 \qquad (1)$

$From\ \vec{\jmath}\ equation\ \rightarrow 8.730\alpha_4 - 15.704 = -5 - 34.64 + 33.72\alpha_3 - 0.028$

$8.730\alpha_4 = 33.72\alpha_3 - 23.964$

$\alpha_4 = 3.863\alpha_3 - 2.745 \qquad (2)$

From equation (1) and (2);

$$\alpha_3 = 0.884\,rad/s^2$$

$$\alpha_4 = 0.670\,rad/s^2$$

b. Complex Solution:

Loop Closure Equation:

$$\overrightarrow{|O_1A|} + \left|\overrightarrow{AB}\right| = \overrightarrow{|O_1O_2|} + \overrightarrow{|O_2B|}$$

$$a_2e^{i\theta_2} + a_3e^{i\theta_3} = a_1 + a_4e^{i\theta_4} \ (Loop\ Closure\ Equation)$$

Derivation of the loop closure equation with respect to time:

$$\underbrace{ia_2\dot{\theta}_2e^{i\theta_2}}_{\overrightarrow{V_A}} + \underbrace{ia_3\dot{\theta}_3e^{i\theta_3}}_{\overrightarrow{V_{B/A}}} = \underbrace{ia_4\dot{\theta}_4e^{i\theta_4}}_{\overrightarrow{V_B}}$$

Second derivative of the loop closure equation with respect to time:

$$\underbrace{ia_2\ddot{\theta}_2e^{i\theta_2}}_{\overrightarrow{a_A}^t} - \underbrace{a_2\dot{\theta}_2^{\ 2}e^{i\theta_2}}_{\overrightarrow{a_A}^n} + \underbrace{ia_3\ddot{\theta}_3e^{i\theta_3}}_{\overrightarrow{a_{B/A}}^t} - \underbrace{a_3\dot{\theta}_3^{\ 2}e^{i\theta_3}}_{\overrightarrow{a_{B/A}}^n} = \underbrace{ia_4\ddot{\theta}_4e^{i\theta_4}}_{\overrightarrow{a_B}^t} - \underbrace{a_4\dot{\theta}_4^{\ 2}e^{i\theta_4}}_{\overrightarrow{a_B}^n}$$

Rearrange terms:

$$\overbrace{ia_4\ddot{\theta}_4e^{i\theta_4} - ia_3\ddot{\theta}_3e^{i\theta_3}}^{unknowns} = \overbrace{\underbrace{ia_2\ddot{\theta}_2e^{i\theta_2} - a_2\dot{\theta}_2^{\ 2}e^{i\theta_2} - a_3\dot{\theta}_3^{\ 2}e^{i\theta_3} + a_4\dot{\theta}_4^{\ 2}e^{i\theta_4}}_{s}}^{knowns}$$

Complex conjugate:

$$\overbrace{-ia_4\ddot{\theta}_4e^{-i\theta_4} + ia_3\ddot{\theta}_3e^{-i\theta_3}}^{unknowns} = \overbrace{\underbrace{-ia_2\ddot{\theta}_2e^{-i\theta_2} - a_2\dot{\theta}_2^{\ 2}e^{-i\theta_2} - a_3\dot{\theta}_3^{\ 2}e^{-i\theta_3} + a_4\dot{\theta}_4^{\ 2}e^{-i\theta_4}}_{p}}^{knowns}$$

Using Cramer's rule:

$$\ddot{\theta}_3 = \frac{\begin{vmatrix} s & ia_4e^{i\theta_4} \\ p & -ia_4e^{-i\theta_4} \end{vmatrix}}{\begin{vmatrix} -ia_3e^{i\theta_3} & ia_4e^{i\theta_4} \\ ia_3e^{-i\theta_3} & -ia_4e^{-i\theta_4} \end{vmatrix}}$$

$$\begin{aligned} \begin{vmatrix} s & ia_4e^{i\theta_4} \\ p & -ia_4e^{-i\theta_4} \end{vmatrix} &= a_2a_4\ddot{\theta}_2e^{i(\theta_2-\theta_4)} + ia_2a_4\dot{\theta}_2^{\ 2}e^{i(\theta_2-\theta_4)} + ia_3a_4\dot{\theta}_3^{\ 2}e^{i(\theta_3-\theta_4)} + ia_4\dot{\theta}_4^{\ 2} \\ &- \left[a_2a_4\ddot{\theta}_2e^{-i(\theta_2-\theta_4)} + ia_2a_4\dot{\theta}_2^{\ 2}e^{-i(\theta_2-\theta_4)} + ia_3a_4\dot{\theta}_3^{\ 2}e^{-i(\theta_3-\theta_4)} + ia_4\dot{\theta}_4^{\ 2}\right] \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} s & ia_4e^{i\theta_4} \\ p & -ia_4e^{-i\theta_4} \end{vmatrix} &= a_2a_4\ddot{\theta}_2\left[e^{i(\theta_2-\theta_4)} - e^{-i(\theta_2-\theta_4)}\right] + ia_2a_4\dot{\theta}_2^{\ 2}\left[e^{i(\theta_2-\theta_4)} + e^{-i(\theta_2-\theta_4)}\right] \\ &+ ia_3a_4\dot{\theta}_3^{\ 2}\left[e^{i(\theta_3-\theta_4)} + e^{-i(\theta_3-\theta_4)}\right] - 2ia_4^{\ 2}\dot{\theta}_4^{\ 2} \end{aligned}$$

NOTE:

$e^{i\theta} - e^{-i\theta} = cos\theta + isin\theta - cos\theta + isin\theta = 2isin\theta$

$e^{i\theta} + e^{-i\theta} = cos\theta + isin\theta + cos\theta - isin\theta = 2cos\theta$

$$\begin{vmatrix} s & ia_4e^{i\theta_4} \\ p & -ia_4e^{-i\theta_4} \end{vmatrix} = a_2a_4\ddot{\theta}_2 2i\sin(\theta_2 - \theta_4) + 2ia_2a_4\dot{\theta}_2{}^2\cos(\theta_2 - \theta_4) + ia_3a_4\dot{\theta}_3{}^2\cos(\theta_3 - \theta_4) - 2ia_4{}^2\dot{\theta}_4{}^2$$

$$\begin{vmatrix} -ia_3e^{i\theta_3} & ia_4e^{i\theta_4} \\ ia_3e^{-i\theta_3} & -ia_4e^{-i\theta_4} \end{vmatrix} = -a_3a_4e^{i(\theta_3-\theta_4)} + a_3a_4e^{-i(\theta_3-\theta_4)}$$

$$\begin{vmatrix} -ia_3e^{i\theta_3} & ia_4e^{i\theta_4} \\ ia_3e^{-i\theta_3} & -ia_4e^{-i\theta_4} \end{vmatrix} = a_3a_4\left[e^{i(\theta_4-\theta_3)} + e^{-i(\theta_4-\theta_3)}\right]$$

$$\begin{vmatrix} -ia_3e^{i\theta_3} & ia_4e^{i\theta_4} \\ ia_3e^{-i\theta_3} & -ia_4e^{-i\theta_4} \end{vmatrix} = a_3a_4 2i\sin(\theta_4 - \theta_3)$$

$$\ddot{\theta}_3 = \frac{\frac{a_2}{a_3}\ddot{\theta}_2\sin(\theta_2 - \theta_4) + \frac{a_2}{a_3}\dot{\theta}_2{}^2\cos(\theta_2 - \theta_4) + \dot{\theta}_3{}^2\cos(\theta_3 - \theta_4) - \frac{a_4}{a_3}\dot{\theta}_4{}^2}{\sin(\theta_4 - \theta_3)}$$

$$a_2 = 10mm \quad a_3 = 35\,mm \quad a_4 = 20\,mm \quad \theta_2 = 60° \quad \theta_3 = 15.47° \quad \theta_4 = 64.12° \quad \dot{\theta}_2 = 2\,rad/s \quad \dot{\theta}_3 = -0.0547\,rad/s \quad \dot{\theta}_4 = 0.934\,rad/s \quad \ddot{\theta}_2 = -1\,rad/s^2$$

$$\ddot{\theta}_3 = \frac{\frac{10}{35}(-1)\sin(60 - 64.12) + \frac{10}{35}(2)^2\cos(60 - 64.12) + (-0.0547)^2\cos(15.47 - 64.12) - \frac{20}{35}(0.934)^2}{\sin(64.12 - 15.47)}$$

$$\ddot{\theta}_3 = 0.884\,rad/s^2$$

Similarly:

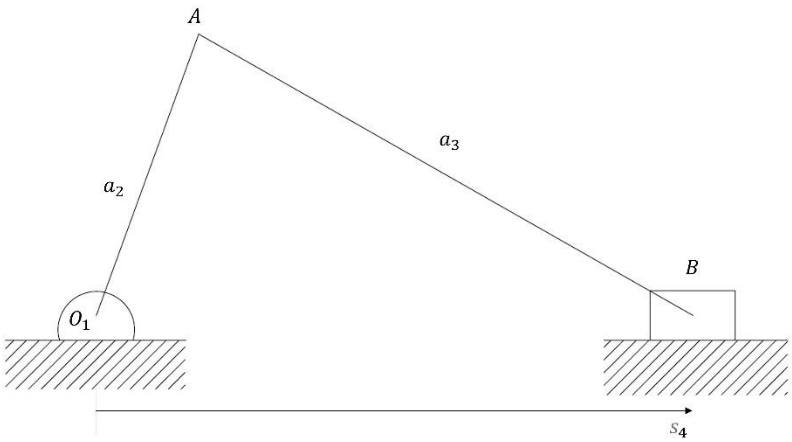
$$\ddot{\theta}_4 = \frac{-\frac{a_2}{a_4}\ddot{\theta}_2\sin(\theta_2 - \theta_3) + \frac{a_2}{a_4}\dot{\theta}_2{}^2\cos(\theta_2 - \theta_3) + \dot{\theta}_4{}^2\cos(\theta_3 - \theta_4) - \frac{a_3}{a_4}\dot{\theta}_3{}^2}{\sin(\theta_4 - \theta_3)}$$

$$\ddot{\theta}_4 = 0.670\,rad/s^2$$

Hazırlayan: Arş. Gör. Görkem AĞÖREN
Arş. Gör. Mustafa Alper SARIİPEK

İnceleyen: Prof. Dr. Tuncay KARAÇAY

SLIDER-CRANK MECHANISM

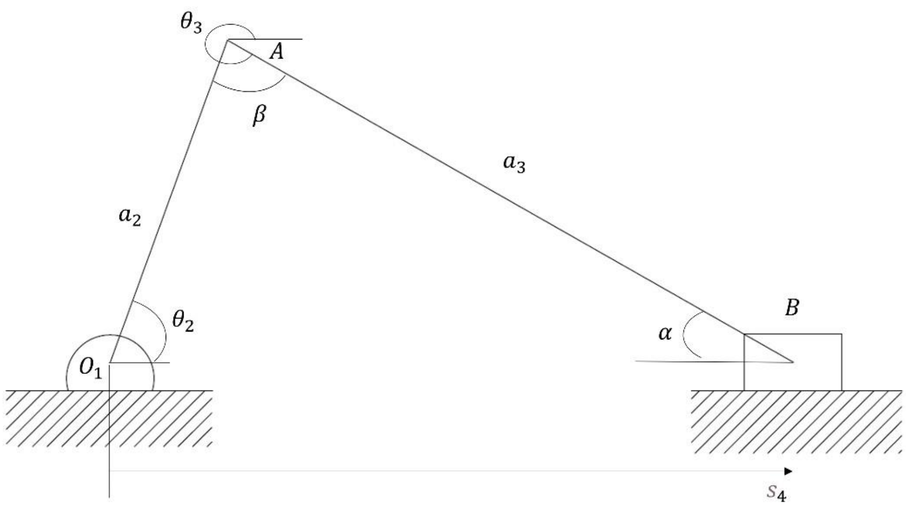


$a_2 = 50\ mm$

$a_3 = 95\ mm$

$\theta_2 = 70°$

<u>Step-wise Solution</u>



$$\frac{\sin 70°}{95} = \frac{\sin \alpha}{50} \quad \rightarrow \quad \alpha = 29.65°$$

$$\beta = 180° - 29.65° - 70° = 80.35°$$

$$\theta_3 = \beta + \theta_1 + 180° = 330.35°$$

$$\frac{\sin \beta}{s_4} = \frac{\sin 70\,°}{a_3} \quad \rightarrow \quad s_4 = 99.66\ mm$$

Loop closure equation:

$$|O_1A| + |AB| = s_4$$

$$a_2.e^{i\theta_2} + a_3.e^{i\theta_3} = s_4$$

$a_2.(\cos\theta_2\,\vec{\imath} + \sin\theta_2\,\vec{\jmath}) \ + a_3.(\cos\theta_3\,\vec{\imath} + \sin\theta_3\,\vec{\jmath}) \ = s_4$

$50.(\cos 70\,\vec{\imath} + \sin 70\,\vec{\jmath}) \ + 95.(\cos\theta_3\,\vec{\imath} + \sin\theta_3\,\vec{\jmath}) \ = s_4$

$From\ \vec{\jmath}\ equation \ \rightarrow \ 50.\sin 70 \ + 95.\sin\theta_3 = 0 \ \rightarrow \ \theta_3 = -29.65° = 330.35°$

$From\ \vec{\imath}\ equation \ \rightarrow \ 50.\cos 70 \ + 95.\cos\theta_3 = s_4 \ \rightarrow \ s_4 = 99.66\ mm$

<u>**Complex Solution**</u>

Loop closure equation:

$|O_1A| + |AB| = s_4$

$a_2.e^{i\theta_2} + a_3.e^{i\theta_3} = s_4$

$a_2.e^{-i\theta_2} + a_3.e^{-i\theta_3} = s_4 \qquad (Complex\ Conjugate\ of\ the\ Loop\ Closure\ Equation)$

Rearrange terms,

$a_3.e^{i\theta_3} = s_4 - a_2.e^{i\theta_2}$

$a_3.e^{-i\theta_3} = s_4 - a_2.e^{-i\theta_2}$

multiplying the two sides together to eliminate θ_3:

$a_3.e^{i\theta_3}.a_3.e^{-i\theta_3} = \left(s_4 - a_2.e^{i\theta_2}\right)\left(s_4 - a_2.e^{-i\theta_2}\right)$

$a_3^2 = s_4^2 - s_4.a_2e^{i\theta_2} - s_4.a_2e^{-i\theta_2} + a_2^2$

$s_4^2 - s_4.a_2.\underbrace{(e^{i\theta_2} + e^{-i\theta_2})}_{2\cos\theta_2} + (a_2^2 - a_3^2) = 0$

$s_4^2 - \underbrace{2.a_2.\cos\theta_2.}_{B}s_4 + \underbrace{(a_2^2 - a_3^2)}_{C} \ = \ 0$

$$s_{4_{1,2}} = \frac{-B \pm \sqrt{B^2 - 4.A.C}}{2.A}$$

$A = 1$

$B = 2.a_2.\cos\theta_2 \ = \ -34.202$

$C = a_2^2 - a_3^2 \ = \ -6525$

$$s_{4_{1,2}} = \frac{34.202 \pm \sqrt{34.202^2 + 4.1.6525}}{2} = \frac{34.202 \pm 165.135}{2}$$

$s_{4_1} = 99.66, s_{4_2} = -65.467$

Since the crank is in positive side, positive root will be used.

VELOCITY ANALYSIS

Graphical Solution

$|O_1A| + |AB| = s_4$

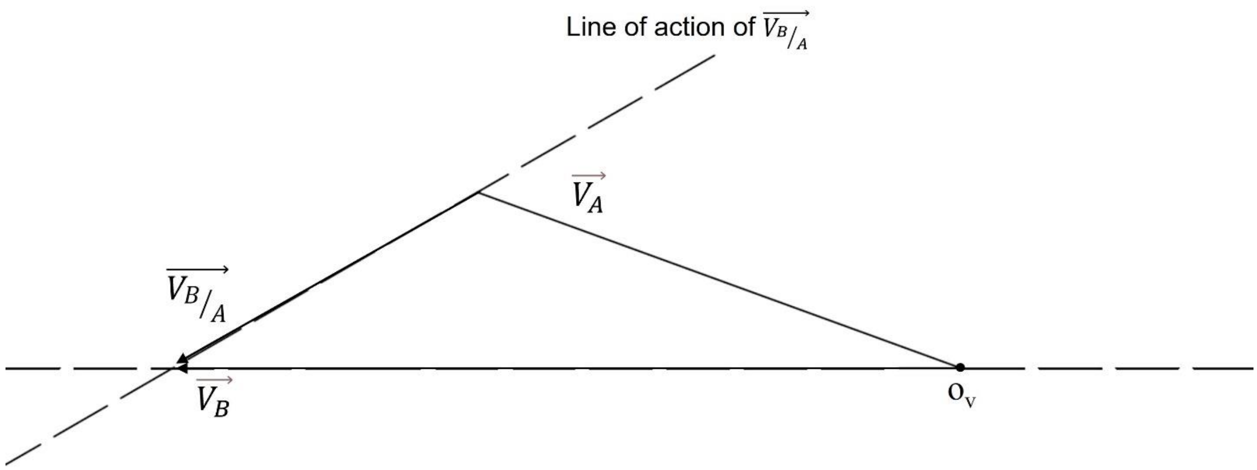
First derivative of the loop closure equation with respect to time:

$\underbrace{a_2.\dot{\theta}_2.e^{i\theta_2}}_{V_A} + \underbrace{a_3.\dot{\theta}_3.e^{i\theta_3}}_{V_{B/A}} = \underbrace{\dot{s_4}}_{V_B}$

$\overrightarrow{V_A} = a_2.\dot{\theta}_2 \ (\perp O_1A)$

$\overrightarrow{V_{B/A}} = a_3.\dot{\theta}_3 \ (\parallel AB)$

$\overrightarrow{V_B} = \dot{s_4} \ (\parallel Slider\ axis)$



As a result of the measurements, $\overrightarrow{V_B}$ is found to be -110 mm/s and $\overrightarrow{V_{B/A}}$ is 39 mm/s. So the angular velocity of link 3:

$$\overrightarrow{V_{B/A}} = \omega_3 a_3 \ \rightarrow \ \omega_3 = \frac{\overrightarrow{V_{B/A}}}{a_3} = \frac{39}{95} = 0.41\ rad/s\ (Clock - wise)$$

Analytical Solution

a) Cartesian Solution

$$\overrightarrow{V_A} + \overrightarrow{V_{B/A}} = \overrightarrow{V_B}$$

$$\overrightarrow{\omega_2} x\, \overrightarrow{r_{0_1A}} + \overrightarrow{\omega_3} x\, \overrightarrow{r_{AB}} = \overrightarrow{\dot{s}_4}$$

$$\overrightarrow{r_{0_1A}} = a_2.(\cos\theta_2 \vec{\imath} + \sin\theta_2 \vec{\jmath} = 17.101\vec{\imath} + 46.985\vec{\jmath}$$

$$\overrightarrow{r_{AB}} = a_3.(\cos\theta_3 \vec{\imath} + \sin\theta_3 \vec{\jmath} = 82.805\vec{\imath} - 46.564\vec{\jmath}$$

$$\overrightarrow{\omega_2} = 2\vec{k}\ rad/s$$

$$2\vec{k}\ x\ (17.101\vec{\imath} + 46.985\vec{\jmath}) + \omega_3\vec{k}\ x\ (82.805\vec{\imath} - 46.564\vec{\jmath}) = \dot{s}_4\vec{\imath}$$

$$34.202\vec{\jmath} - 93.97\vec{\imath} + 82.805\omega_3\vec{\jmath} + 46.56\omega_3\vec{\imath} = \dot{s}_4\vec{\imath}$$

$$From\ \vec{\jmath}\ equation \quad \rightarrow \quad 34.202 + 82.805\omega_3 = 0 \quad \rightarrow \quad \omega_3 = -0.413\ rad/s$$

$$From\ \vec{\imath}\ equation \quad \rightarrow \quad -93.97 + 46.56\omega_3 = \dot{s}_4 \quad \rightarrow \quad \dot{s}_4 = -113.199\ mm/s$$

b) Complex Solution

$$|O_1A| + |AB| = s_4$$

First derivative of the loop closure equation with respect to time:

$$\underbrace{i.a_2.\dot{\theta}_2.e^{i\theta_2}}_{V_A} + \underbrace{i.a_3.\dot{\theta}_3.e^{i\theta_3}}_{V_{B/A}} = \underbrace{\dot{s}_4}_{V_B}$$

Rearrange terms:

$$\overbrace{i.a_3.\dot{\theta}_3.e^{i\theta_3} - \dot{s}_4}^{unknowns} = \overbrace{-i.a_2.\dot{\theta}_2.e^{i\theta_2}}^{knowns}$$

Complex conjugate:

$$-i.a_3.\dot{\theta}_3.e^{-i\theta_3} - \dot{s}_4 = i.a_2.\dot{\theta}_2.e^{-i\theta_2}$$

$$\begin{bmatrix} ia_3e^{i\theta_3} & -1 \\ -ia_3e^{-i\theta_3} & -1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_3 \\ \dot{s}_4 \end{bmatrix} = \begin{bmatrix} -ia_2e^{i\theta_2} \\ ia_2e^{-i\theta_2} \end{bmatrix} \dot{\theta}_2$$

Using Cramer's Rule:

$$\dot{\theta}_3 = \frac{\begin{vmatrix} -ia_2e^{i\theta_2} & -1 \\ ia_2e^{-i\theta_2} & -1 \end{vmatrix}}{\begin{vmatrix} ia_3e^{i\theta_3} & -1 \\ -ia_3e^{-i\theta_3} & -1 \end{vmatrix}}.\dot{\theta}_2 = \frac{ia_2e^{i\theta_2} + ia_2e^{-i\theta_2}}{-ia_3e^{i\theta_3} - ia_3e^{-i\theta_3}}.\dot{\theta}_2 = \frac{ia_2\cos\theta_2}{-ia_3\cos\theta_3}.\dot{\theta}_2 = \frac{-50}{90}\frac{\cos 70}{\cos 330.65}.2$$

$$= -0.413\ rad/s$$

To find $\dot{s_4}$, we can use Cramer's rule again:

$$\dot{s_4} = \frac{\begin{vmatrix} ia_3e^{i\theta_3} & -ia_2e^{i\theta_2} \\ -ia_3e^{-i\theta_3} & ia_2e^{-i\theta_2} \end{vmatrix}}{\begin{vmatrix} ia_3e^{i\theta_3} & -1 \\ -ia_3e^{-i\theta_3} & -1 \end{vmatrix}}.\dot{\theta}_2 = \frac{-a_2a_3\left[\overbrace{e^{i(\theta_3-\theta_2)} - e^{-i(\theta_3-\theta_2)}}^{2i.sin(\theta_3-\theta_2)}\right]}{-i.a_3\underbrace{[e^{i\theta_3} + e^{-i\theta_3}]}_{cos(\theta_3)}}.\dot{\theta}_2 = a_2.\frac{sin(\theta_3-\theta_2}{cos(\theta_3)}.\dot{\theta}_2$$

$$\dot{s_4} = 50.\frac{sin(330.35-70)}{cos(330.35)}.2 = -113.438\, mm/s$$

ACCELERATION ANALYSIS ($\alpha_2 = -1\, rad/s^2$)

<u>Graphical Solution</u>

$|O_1A| + |AB| = s_4$

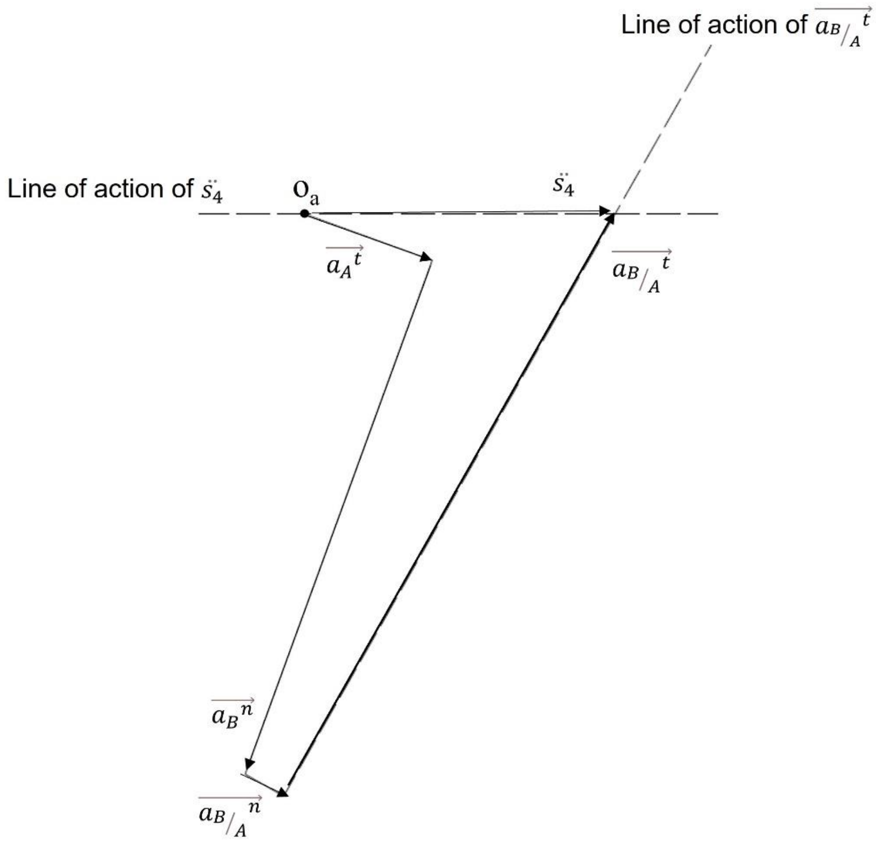
First derivative of the loop closure equation with respect to time:

$$\underbrace{\overbrace{\overrightarrow{a_A{}^t}}^{\alpha_2.a_2}}_{\perp O_1A} + \underbrace{\overbrace{\overrightarrow{a_A{}^n}}^{\omega_2{}^2.a_2}}_{\parallel O_1A} + \underbrace{\overbrace{\overrightarrow{a_{B/A}{}^t}}^{\alpha_3.a_3}}_{\perp AB} + \underbrace{\overbrace{\overrightarrow{a_{B/A}{}^n}}^{\omega_3{}^2.a_3}}_{\parallel AB} = \underbrace{\ddot{s_4}}_{\parallel Slider\ Axis}$$

$$\overrightarrow{a_A{}^t} = \alpha_2.a_2 = -50\, mm/s^2$$

$$\overrightarrow{a_A{}^n} = \omega_2{}^2.a_{2"} = (2\, rad/s^2)^2.50mm = 200\, mm/s^2$$

$$\overrightarrow{a_{B/A}{}^n} = \omega_3{}^2.a_3 = (-0.413)^2.95 = 16.20\, mm/s^2$$



As a result of the measurements, $\ddot{s_4}$ is found to be 75 mm/s^2 and $\overrightarrow{a_{B/A}}^t$ is 225 mm/s^2. So the angular acceleration:

$$\overrightarrow{a_{B/A}}^t = \alpha_3 . a_3 \rightarrow \alpha_3 = \frac{a_{B/A}{}^t}{a_3} = \frac{225}{95} = 2.37\, rad/s^2$$

Cartesian Coordinates:

$$\overrightarrow{\alpha_2}\, x\, \overrightarrow{r_{0_1A}} + \overrightarrow{\omega_2} x(\omega_2 x\, \overrightarrow{r_{0_1A}}) + \overrightarrow{\alpha_3}\, x\, \overrightarrow{r_{AB}} + \overrightarrow{\omega_3} x(\omega_3 x\, \overrightarrow{r_{AB}}) = \ddot{s_4}$$

$$\overrightarrow{r_{0_1A}} = a_2 .(\cos\theta_2 \vec{\imath} + \sin\theta_2 \vec{\jmath} = 17.101\vec{\imath} + 46.985\vec{\jmath}$$

$$\overrightarrow{r_{AB}} = a_3 .(\cos\theta_3 \vec{\imath} + \sin\theta_3 \vec{\jmath} = 82.805\vec{\imath} - 46.564\vec{\jmath}$$

$$\overrightarrow{\omega_2} = 2\vec{k}\, rad/s$$

$$\overrightarrow{\omega_3} = -0.413\vec{k}\, rad/s$$

$$\alpha_2 = -1\, rad/s^2$$

$$-1\vec{k}\ x\,(17.101\vec{\imath}\ +\ 46.985\vec{\jmath})\ +\ 2\vec{k}\ x\,[2\vec{k}\ x\,(17.101\vec{\imath}\ +\ 46.985\vec{\jmath})]$$
$$+\ \alpha_3\vec{k}\ x\,(82.805\vec{\imath}\ -\ 46.564\vec{\jmath})$$
$$+\ \ -0.413\vec{k}\ x\,[-0.413\vec{k}\ x\,(82.805\vec{\imath}\ -\ 46.564\vec{\jmath})] = \ddot{s_4}\vec{\imath}$$

$$-17.101\vec{\jmath}\ +\ 46.985\vec{\imath} - 68.404\vec{\imath}\ - 187.92\vec{\jmath} + 82.805\alpha_3\vec{\jmath} + 45.56\alpha_3\vec{\imath} - 14.124\vec{\imath} + 7.942\vec{\jmath}$$
$$= \ddot{s_4}\vec{\imath}$$

From j equation:

$$-17.101\vec{\jmath} - 187.92\vec{\jmath} + 82.805\alpha_3\vec{\jmath} + 7.942\vec{\jmath} = 0\ \rightarrow\ \alpha_3 = 2.38\,rad/s^2$$

From i equation:

$$46.985\vec{\imath} - 68.404\vec{\imath} + 45.56\alpha_3\vec{\imath} - 14.124\vec{\imath} = \ddot{s_4}\vec{\imath}\ \rightarrow\ \ \ddot{s_4} = 75.270\,mm/s^2$$

Complex Coordinates:

$$|O_1A| + |AB| = s_4$$

$$a_2.e^{i\theta_2} + a_3.e^{i\theta_3} = s_4$$

First derivative of the loop closure equation with respect to time:

$$i.a_2.\dot{\theta}_2.e^{i\theta_2} + i.a_3.\dot{\theta}_3.e^{i\theta_3} = \dot{s}_4$$

Second derivative of the loop closure equation with respect to time:

$$\underbrace{ia_2\ddot{\theta}_2e^{i\theta_2}}_{\overrightarrow{a_A}^t} - \underbrace{a_2\dot{\theta}_2{}^2e^{i\theta_2}}_{\overrightarrow{a_A}^n} + \underbrace{ia_3\ddot{\theta}_3e^{i\theta_3}}_{\overrightarrow{a_{B/A}}^t} - \underbrace{a_3\dot{\theta}_3{}^2e^{i\theta_3}}_{\overrightarrow{a_{B/A}}^n} = \ddot{s}_4$$

Rearrange terms:

$$ia_3\ddot{\theta}_3e^{i\theta_3} - \ddot{s}_4 = \underbrace{a_2\dot{\theta}_2{}^2e^{i\theta_2} - ia_2\ddot{\theta}_2e^{i\theta_2} + a_3\dot{\theta}_3{}^2e^{i\theta_3}}_{s}$$

Complex conjugate:

$$-ia_3\ddot{\theta}_3e^{-i\theta_3} - \ddot{s}_4 = \underbrace{a_2\dot{\theta}_2{}^2e^{-i\theta_2} + ia_2\ddot{\theta}_2e^{-i\theta_2} + a_3\dot{\theta}_3{}^2e^{-i\theta_3}}_{p}$$

In order to find $\ddot{\theta}_3$ *and* $\ddot{s}_4$ we can use Cramer's Rule:

$$\ddot{\theta}_3 = \frac{\begin{vmatrix} s & -1 \\ p & -1 \end{vmatrix}}{\begin{vmatrix} ia_3e^{i\theta_3} & -1 \\ -ia_3e^{-i\theta_3} & -1 \end{vmatrix}} = \frac{-s+p}{-ia_3[e^{i\theta_3} + e^{-i\theta_3}]}$$

$$\ddot{\theta}_3 = \frac{a_2\dot{\theta}_2{}^2(e^{i\theta_2} - e^{-i\theta_2}) - a_3\dot{\theta}_3{}^2(e^{i\theta_3} - e^{-i\theta_3}) + ia_2\ddot{\theta}_2(e^{i\theta_2} + e^{-i\theta_2})}{-ia_3(e^{i\theta_3} + e^{-i\theta_3})}$$

$$\ddot{\theta}_3 = \frac{a_2\dot{\theta}_2{}^2\, 2i.\sin\theta_2 - a_3\dot{\theta}_3{}^2\, 2i.\sin\theta_3 + 2ia_2\ddot{\theta}_2\cos\theta_2}{-2ia_3\cos\theta_3}$$

$$\ddot{\theta}_3 = \frac{\frac{a_2}{a_3}\dot{\theta}_2{}^2\sin\theta_2 + \dot{\theta}_3{}^2\sin\theta_3 - \frac{a_2}{a_3}\ddot{\theta}_2\cos\theta_2}{\cos\theta_3}$$

$$\ddot{\theta}_3 = \frac{\frac{50}{95}2^2\sin 70 + (-0.413)^2\sin(330.35) - \frac{50}{95}(-1)\cos 70}{\cos 330.35} = 2.386\ rad/s^2$$

Similarly, we can find $\ddot{s}_4$;

$$\ddot{s}_4 = \frac{a_2\dot{\theta}_2{}^2\cos(\theta_3 - \theta_2) + a_3\dot{\theta}_3{}^2 - a_2\ddot{\theta}_2\sin(\theta_2 - \theta_3)}{-\cos\theta_3}$$

$$\ddot{s}_4 = \frac{50(2)^2\cos(330.35 - 70) + 95(-0.413)^2 - 50.-1.\sin(70 - 330.35)}{-\cos 330.35} = 76.65\ mm/s^2$$