ENM 202
OPERATIONS RESEARCH (I)
OR (I)
5

LECTURE NOTES

SPECIAL CASES in the SIMPLEX METHOD APPLICATIONS

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1)DEGENERACY	2)ALTERNATIVE OPTIMA	3)UNBOUNDED SOLUTION	4)INFEASIBLE SOLUTION
1)An LP is degenerate if it has at least one bfs in which a BV is equal to zero 2) A degenerate optimal solution occurs when the model has at least one redundant constraint 3) Degeneracy can occur temporarily or permanently. When degeneracy occurs permanently, the objective function value will never improve and the procedure will repeat a sequence of iterations.we call this as cycling or circling. When degeneracy occurs temporarily, the simplex iterations must always be continued until the last iteration satisfies the optimality condition	1) When the objective function is parallel to a binding constraint, alternative optimal solution case occurs. 2) In the simplex tableau of the final iteration, if the coefficient of a NBV in z-row is equal to zero then this indicates that alternative optimal solution exists.	1) In some LPs the values of variables may be increased indefinetely without violating any of the constraints, meaning that the solution space is unbounded. 2) As a result, the objective value may increase (max. case) or decrease (min. case) indefinately. In this case both the solution space and the optimal objective value are unbounded. The general rule for recognizing unboundedness a) If at any iteration, all the constraint coefficients of any NBV are zero or negative, then the solution space is unbounded in that direction. b) In addition, If the objective coefficient of that variable is negative in max problem, or positive in min problem then the objective value is also unbounded.	1) LP with inconsistent constraints has no feasible solutions 2) This situation never occur if all the constraints are of the type ≤ (assumming nonnegative RHSs) since the slack variables always provide a solution. 3)For other type of constraints (=, ≥) we use artificial variables. If the model has one artificial variable with a positive value in the optimum iteration then the model has no solution and infeasible solution case occur

1)DEGENERACY

- 1)An LP is degenerate if it has at least one bfs in which a BV is equal to zero
- 2) A degenerate optimal solution occurs when the model has at least one redundant constraint
- But there are no reliable techniques for identifying the redundant constraints directly from the simplex tableau.

- Example
- (Degenerate Optimal Solution- degeneracy occurs temporarily))

- LP: Max z= 3x1+9x2
- St
- x1+4x2 ≤ 8
- x1+2x2 ≤ 4
- x1, x2 ≥ 0

LP in Standard Form: Max z= 3x1+9x2

st

x1+4x2 +x3= 8

x1+2x2 +x4=4

x1, x2, x3, x4 ≥ 0

Degenerate Optimal Solution

Iteration	Basic	x1	x2	x3	x4	RHS	MRT
(0)	z	-3	-9	0	0	0	
x2 enters x3 leaves	x3	1	4	1	0	8	(8/4)=**2***
	x4	1	2	0	1	4	(4/2)=**2***
(1)	z	-3/4	0	9/4	0	18	
	x2	1/4	1	1/4	0	2	2/(1/4)=8
x1 enters x4 leaves	x4	1/2	0	-1/2	1	0	0/(1/2)=0*
(2)	z	0	0	3/2	3/2	18	
optimum	x2	0	1	1/2	-1/2	2	
	x1	1	0	-1	2	0	

Grafical Representation of Degenerate Optimal Solution

Max $z = 3x_1 + 9x_2$

s.t.

$x_1 + 4x_2 \leq 8$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

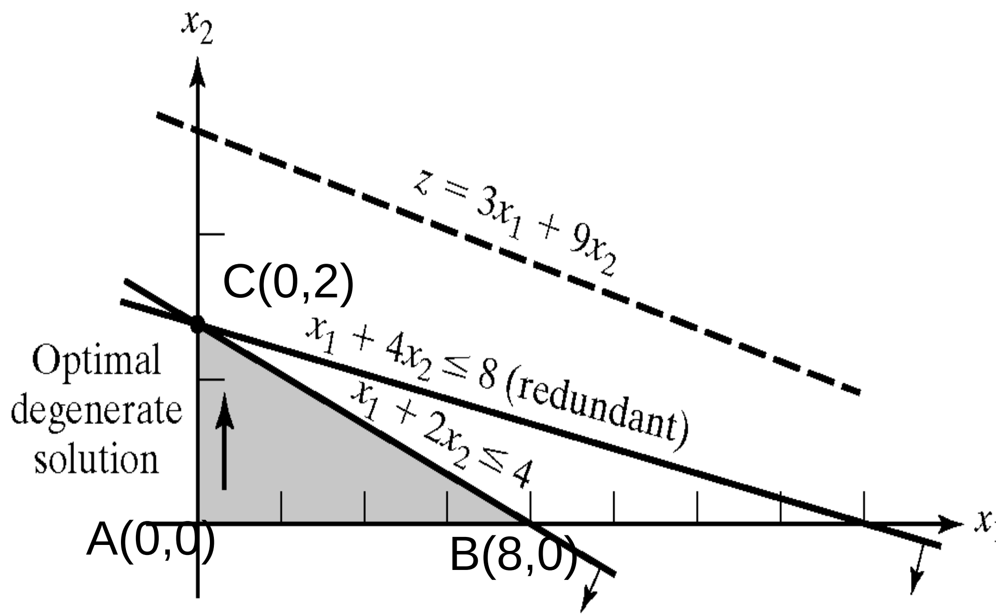

Figure 3.7

LP degeneracy in Example 3.5-1.

CYCLING (CIRCLING)

- **Example**
- (Taha OR Book 7th ed., Set 3.5A, Problem 4, pp 106,)
- LP:
- Max $z = (3/4)x_1 - 20x_2 + (1/2)x_3 - 6x_4$
- St
- $(1/4)x_1 - 8x_2 - x_3 + 9x_4 \leq 0$
- $(1/2)x_1 - 12x_2 - (1/3)x_3 + 3x_4 \leq 0$
- $x_3 \leq 1$
- $x_1, x_2, x_3, x_4 \geq 0$

- You will notice that the starting all-slack bfs at iteration 1 will appear identically in iteration 7.

2)ALTERNATIVE OPTIMA INFINITY of SOLUTIONS

- Example

- LP: Max $z= 2x1+4x2$
- St
- $x1+2x2 \leq 5$
- $x1+x2 \leq 4$
- $x1, x2 \geq 0$

LP in Standard Form: Max $z= 2x1+4x2$

st

$x1+2x2 +x3= 5$

$x1+x2 +x4=4$

$x1, x2, x3, x4 \geq 0$

2)ALTERNATIVE OPTIMA INFINITY of SOLUTIONS

Iteration	Basic	x1	x2	x3	x4	RHS	MRT
(0)	z	-2	-4	0	0	0	
x2 enters x3 leaves	x3	1	2	1	0	5	5/2*
	x4	1	1	0	1	4	4/1
(1) **optimum**	z	0	0	2	0	10	
	x2	1/2	1	1/2	0	5/2	5
x1 enters x4 leaves	x4	1/2	0	-1/2	1	3/2	3*
(1’) **alt. optimum**	z	0	0	2	0	10	
	x2	0	1	1	-1	1	
	x1	1	0	-1	2	3	

2. Alternative Optimal Solutions (Infinity of Solutions) (Multiple Optimal Solutions)

Example (Taha OR Book pp 106)

LP: Max $Z=2x_1+4x_2$

St

$x_1+\ x_2 \leq 4$ (1)

$x_1+2x_2 \leq 5$ (2)

$x_1 \geq 0,\ x_2 \geq 0$ (3,4)

B(0,5/2)

C(3,1)

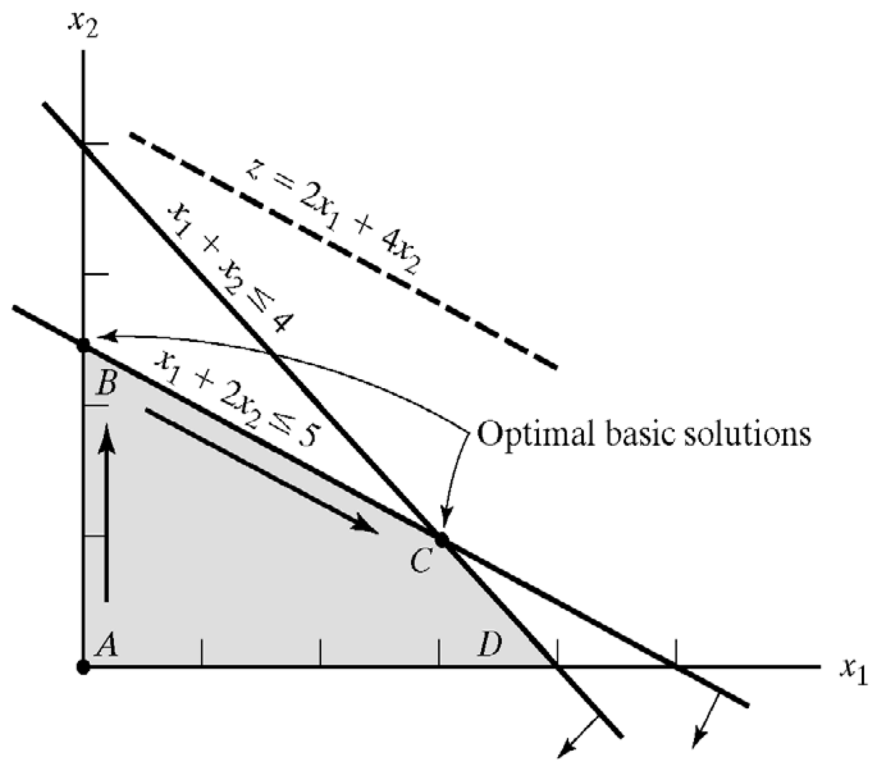

Figure 3.9

LP alternative optima in Example 3.5-2.

3)UNBOUNDED SOLUTION

- Example (Unbounded Objective Value)
- Example

LP: Max z= 2x1+x2	LP in Standard Form: Max z= 2x1+x2
St	st
$x1-x2 \le 10$	$x1-x2 +x3= 10$
$2x1 \le 40$	$2x1 +x4=40$
$x1, x2 \ge 0$	$x1, x2, x3, x4 \ge 0$

(Unbounded Objective Value)

Iteration	Basic	x1	x2	x3	x4	RHS	MRT
(0)	z	-2	-1	0	0	0	
x1 enters x3 leaves	x3	1	-1	1	0	10	10/1*
	x4	2	0	0	1	40	40/2

•In the starting Tableau both x1 and x2 are candidates for entering the solution.

•Because x1 has the most negative coefficient, it is selected as the entering variable.

•However, all the constraint coefficients under x2 are negative or zero, meaning that x2 can be increased indefinitely without violating any of the constraints.

Each unit increase in x2 will increase z by 1, an infinite increase in x2 will also result in an infinite increase in z. Thus the problem has no bounded solution.

(Unbounded Objective Value)

Iteration	Basic	x1	x2	x3	x4	RHS	MRT
(0)	z	-2	-1	0	0	0	
x1 enters x3 leaves	x3	1	-1	1	0	10	10/1*
	x4	2	0	0	1	40	40/2
(1)	z	0	-3	2	0	20	
	x1	1	-1	1	0	10	ignore
x2 enters x4 leaves	x4	0	2	-2	1	20	20/2*
(2)	z	0	0	-1	3/2	50	
	x1	1	0	0	1/2	20	
	x2	0	1	-1	1/2	10	

Graphical Representation of Unbounded Solution Case

- Example (Taha OR Book pp 109)

Max Z=2x1+x2
St
x1- x2 ≤10 (1)
2x1 ≤40 (2)
x1, x2 ≥0 (3,4)

IN A CORRECTLY MODELLED LP, IT SHOULD NOT OCCUR

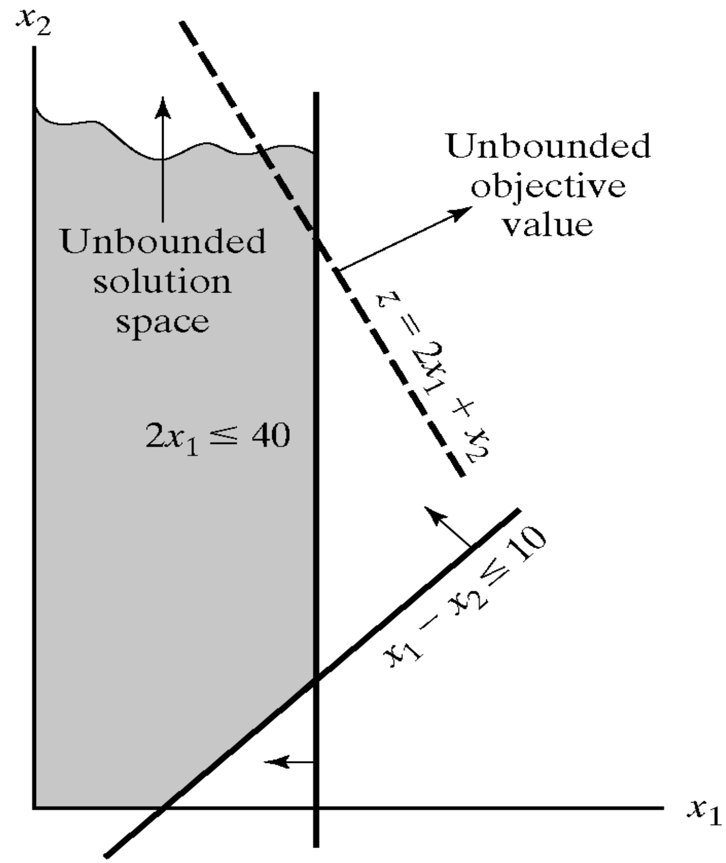

Figure 3.10
LP unbounded solution in Example 3.5-3.

UNBOUNDED SOLUTION SPACE

- Example (Unbounded Solution Space but Finite Optimum Objective Value)

- LP: Max $z= 6x1-2x2$
- St
- $2x1-x2 \leq 2$
- $x1 \leq 4$
- $x1, x2 \geq 0$

LP in Standard Form: Max $z= 6x1-2x2$

st

$2x1-x2 +x3= 2$

$x1 +x4=40$

$x1, x2, x3, x4 \geq 0$

(Unbounded Solution Space but Finite Optimum Objective Value)

Iteration	Basic	x1	x2	x3	x4	RHS	MRT
(0)	z	-6	2	0	0	0	
x1 enters x3 leaves	x3	2	-1	1	0	2	2/2=1*
	x4	1	0	0	1	4	4/1=4
(1)	z	0	-1	3	0	6	
	x1	1	-1/2	1/2	0	1	ignore
x1 enters x3 leaves	x4	0	1/2	-1/2	1	3	3/(1/2)=6*
(2)	z	0	0	2	2	12	
optimum	x1	1	0	0	1	4	
	x2	0	1	-1	2	6	

4)INFEASIBLE SOLUTION

- LP models with inconsistent constraints have no feasible solutions

- Example (Taha OR Book pp 111)

- LP:Max $Z=3x1+2x2$
  St
  $2x1+x2 \quad \leq 2$
- $3x1+4x2 \quad \geq 12$
- $x1\geq 0$, $x2\geq 0$

- LP in Standard Form with Artificials:

- Max $Z=3x1+2x2$
  St
  $2x1+x2 \quad +x3=2$
- $3x1+4x2 \quad -x4+x5=12$
  $x1, x2, x3, x4, x5 \geq 0$

Example for INFEASIBLE SOLUTION

Iteration	Basic	x1	x2	x4	x3	x5	RHS	MRT
Inconsistent tableau	z	-3	-2	0	0	M	0	
	x3	2	1	0	1	0	2	
	x5	3	4	-1	0	1	12	
(0)	z	-3-3M	-2-4M	M	0	0	-12M	
x2 enters x3 leaves	x3	2	1	0	1	0	2	2/1*
	x5	3	4	-1	0	1	12	12/4
(1)	z	1+5M	0	M	2+4M	0	4-4M	
PSEUDO OPTİMUM	x2	2	1	0	1	0	2	
	x5	-5	0	-1	-4	1	4	

Graphical Representation of Infeasible solution case

Example (Taha OR Book pp 111)

Max Z=3x1+2x2
St
3x1+4x2 ≥12 (1)
2x1+x2 ≤2 (2)
x1≥0, x2≥0 (3,4)

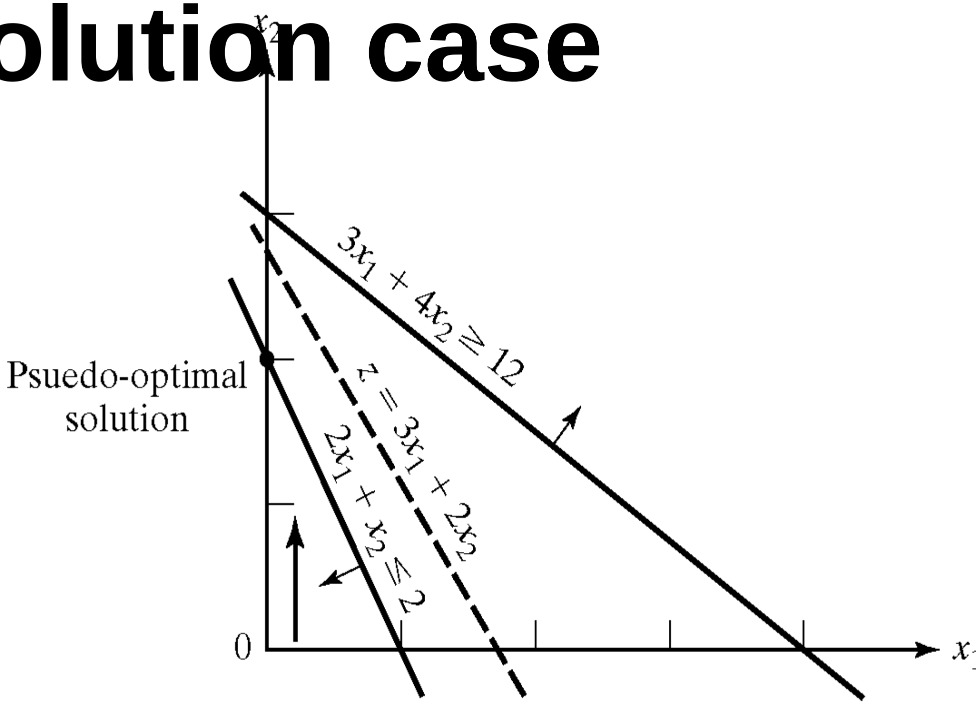

Figure 3.11
Infeasible solution of Example 3.5-4.