ENM 202
OPERATIONS RESEARCH (I)
OR (I)
9

LECTURE NOTES

SENSITIVITY ANAYSIS

(POSTOPTIMALITY ANAYSIS)

SENSITIVITY ANAYSIS (POSTOPTIMALITY ANAYSIS)

- Sensitivity analysis investigate the change in the optimum solution resulting from making changes in parameters of the LP model.
- The following table lists all possible cases that can arise in sensitivity analysis and the actions needed to obtain the new solution.

Condition Resulting from the changes	**Recommended Action**
Current Solution remains optimal and feasible	Nothing
Current solution becomes **infeasible**	Use the Dual Simplex Algorithm to recover feasibility
Current solution becomes **non-optimal**	Use The Primal Simplex Algorithm to recover optimality

- **Q. When the current optimum solution becomes infeasible**?
- Changes affecting feasibility:
- 1) The RHSs of the current constraints are changed or
- 2) A new constraint is added to the model
- In both cases, infeasibility occurs when at least one element of the RHS of the optimal Tablaeu becomes negative (ie. one or more of the current BV become negative)

- **Q. When the solution becomes non-optimal?**
- 1) The original objective coefficients are changed.
- 2) A new activity (variable) is added to the model

SENSITIVITY ANAYSIS (POSTOPTIMALITY ANAYSIS)

1) Changes affecting feasibility

- Changes in the RHSs
- Feasibility Range for the elements of the RHSs
- Addition of a New Constraint

2) Changes affecting optimality

- Change in the c_j of a NBV
- Change in the c_j of a BV
- Optimality Range of the objective coefficents
- Addition of a New Variable
    - Changes in the Technological Coefficients

SENSITIVITY ANAYSIS
(POSTOPTIMALITY ANAYSIS)

- In sensitivity analysis, after finding the optimal solution we try to investigate the changes of the data affecting the solution.
- We will talk about 2 main cases resulting from the changes

- 1) CHANGES AFFECTING FEASIBILITY
- Infeasibility occurs when at least one element of the RHS of the optimal tablaeu becomes negative (ie. One or more of the current BV become negative)

  a) Feasibility range of the elements of the RHS

  b) The RHS of the current constraints is changed

  c) A new constraint is added to the model

- 2) CHANGES AFFECTING OPTIMALITY
- Nonoptimality occurs when one or more z-row coefficients becomes negative <u>in max problem</u> or becomes positive <u>in min problem</u>

  a) Optimality Range of the objective coefficents for a BV

  b) Optimality Range of the objective coefficents for a NBV

  c) Change in the c_j of a BV

  d) Change in the c_j of a NBV

  e) Addition of new activity

  f) Changes in the Technological Coefficients

Toyco Example, Taha OR Book PP 135 for sensitivity analysis

- Toyco assembles three types of toys: trains, trucks and cars using three operations. The daily limits on the available times for the three operations are 430,460, and 420 minutes, respectively; and the profits per toy train, truck and car are \$3, \$2, and \$5, respectively. The assembly times per train at the three operations are 1, 3 and 1 minutes, respectively. The corresponding times per truck and per car are (2, 0, 4) and (1, 2, 0) minutes (a zero time indicates that the operation is not used.)

Mathematical Model

Decision variables

$x1$: The daily number of units assembled of trains

$x2$: The daily number of units assembled of trucks

$x3$: The daily number of units assembled of cars

LP: (P)

Max $z = 3x1+2x2+5x3$

St

$$
\begin{aligned}
x1+2x2+x3 &\le 430\\
3x1+2x3 &\le 460\\
x1+4x2 &\le 420\\
x1,x2,x3 &\ge 0
\end{aligned}
$$

(D):

Min $w = 430y1+460y2+420y3$

St

$$
\begin{aligned}
y1+3y2+y3 &\ge 3\\
2y1+4y3 &\ge 2\\
y1+2y2 &\ge 5\\
y1,y2,y3 &\ge 0
\end{aligned}
$$

The associated optimal primal tableau

Basis	x1	x2	x3	x4	x5	x6	RHS
z	4	0	0	1	2	0	1350
x2	-1/4	1	0	1/2	-1/4	0	100
x3	3/2	0	1	0	1/2	0	230
x6	2	0	0	-2	1	1	20

1.CHANGES AFFECTING FEASIBILITY

The feasibility of the current solution may be affected only if

a) The RHS of the current constraints is changed

b) A new constraint is added to the model

1-a) Changes in the right-Hand Side

If a change occur at the right-hand side of the original problem, we need to recompute the right-hand side of the tableau

$$\begin{pmatrix} \text{New right - hand side} \\ \text{of tableau in iteration } \text{i} \end{pmatrix} = \begin{pmatrix} \text{Inverse in} \\ \text{iteration} \\ \text{i} \end{pmatrix} \times \begin{pmatrix} \text{New right -} \\ \text{hand side of} \\ \text{the constraints} \end{pmatrix}$$

$$X_B = B^{-1} b$$

Recall that the right-hand side of the tableau gives the values of the basic variables.

Example: Suppose that TOYCO wants to expand its assembly lines by increasing the daily capacity of each line by 40 %, 602, 644 and 588 minutes, respectively.

$$\begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 602 \\ 644 \\ 588 \end{pmatrix} = \begin{pmatrix} 140 \\ 322 \\ 28 \end{pmatrix}$$

$$Z = c_B B^{-1} b = \begin{pmatrix} 2 & 5 & 0 \end{pmatrix} \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 602 \\ 644 \\ 588 \end{pmatrix} = 1890$$

Thus, current basic variables – x_2, x_3, and x_6 – remain feasible at the new values 140, 322, and 28. The associated optimum profit is $1890.

Although the new solution is attractive from the standpoint of increased profit, TOYCO recognizes that its implementation will take time. Another proposal was thus made to shift the slack capacity of operation 3 (x_6=20 minutes) to the capacity of operation 1, which changes the capacity mix of three operations to 450, 460 and 400 minutes, respectively.

$$\begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 450 \\ 460 \\ 400 \end{pmatrix} = \begin{pmatrix} 110 \\ 230 \\ -40 \end{pmatrix}$$

$$Z = c_B B^{-1} b = \begin{pmatrix} 2 & 5 & 0 \end{pmatrix} \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 450 \\ 460 \\ 400 \end{pmatrix} = \$1370$$

The resulting solution is infeasible because x_6=-40. We apply the dual simplex method to recover feasibility. First, we modify the right-hand side of the tableau as shown by the shaded column.

(** Don’t forget to modify the objective function value)

Iteration	Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	Z	4	0	0	1	2	0	1370
	x_2	-1/4	1	0	1/2	-1/4	0	110
x_6 leaves x_4 enters	x_3	3/2	0	1	0	1/2	0	230
	X_6	2	0	0	-2	1	1	-40
	Z	5	0	0	0	5/2	1/2	1350
	x_2	1/4	1	0	0	0	1/4	100
	x_3	3/2	0	1	0	1/2	0	230
	x_4	-1	0	0	1	-1/2	-1/2	20

The optimum solution (in terms of x_1, x_2, and x_3) remains the same as in the original model. It also shows that the additional capacity for operation 1 was not used (x_4=20). The only conclusion then is that operation 2 is the bottleneck.

Feasibility Range of the Elements of the Right-Hand Side

Another way of looking at the affect of changing the availability of resources (i.e. the right hand side vector) is to determine the range for which the current solution remains feasible.

Example: In the TOYCO model, suppose that we are interested in determining the feasibility range of the capacity of operation 1. We can do so by replacing the right-hand side with

$$\begin{pmatrix} 430 + D_1 \\ 460 \\ 420 \end{pmatrix}$$

The amount of D_1 represents the change in the capacity of operation1 above and below the present level of 430 minutes.

- Answer:

The current bfs remains feasible if all the BVs are nonnegative, ie:

$$x = B^{-1}b$$

$$\begin{pmatrix} x2 \\ x3 \\ x6 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 430 + D_1 \\ 460 \\ 420 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 100 + D_1/2 \\ 230 \\ 20 - 2D_1 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

These conditions lead to the following bounds on D_1:

$(x_2 \geq 0): 100 + D_1/2 \geq 0 \quad \Rightarrow \quad D_1 \geq -200$

$(x_3 \geq 0)$: x_3 is independent of D_1

$(x_6 \geq 0): 20 - 2D_1 \geq 0 \quad \Rightarrow \quad D_1 \leq 10$

Thus, the current basic solution remains feasible for

$$-200 \leq D_1 \leq 10$$

Feasibility Range for b_1

$$230 \leq (\text{operation 1 capacity}) \leq 440$$

• The change in the optimal objective value associated with D_1 is $D_1 y_1$, where y_1 is the unit worth per unit (dual price) in dollars per minute of operation 1.

• To illustrate the use of the determined range, suppose that the capacity of operation 1 is changed from the current level of 430 minutes to 400 minutes.

•The current basic solution remains feasible because the new capacity falls within the feasible range.

To compute the new values of the variables we use

$$D_1=400-430=-30$$

Thus,

$$\begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 100+(-30)/2 \\ 230 \\ 20-2(-30) \end{pmatrix} = \begin{pmatrix} 85 \\ 230 \\ 80 \end{pmatrix}$$

• To compute the associated change in the optimal value of the objective function, we first compute the dual prices using the formula

$$y = c_B B^{-1} \qquad\qquad (Method\ I)$$

Thus,

$$\begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix} = \begin{pmatrix} 2 & 5 & 0 \end{pmatrix} \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix}$$

This means that the worth per unit of operation 1 is y_1 = \$1 per minute, and the change in the optimal profit is D_1y_1=-30*1=-\$30

Remember that the given worth per unit, y_1=1, remains valid only within the specified range -200 ≤ D_1 ≤ 10.

Any change outside this range causes infeasibility – hence we need to use the dual simplex method to determine the new solution, if one exists.

1-b) Addition of a New Constraint

The addition of a new constraint can lead to one of two cases:

1) The new constraint is redundant, meaning that it is satisfied by the current optimum solution and, hence, can be dropped from the model altogether.
2) The current solution violates the new constraint, in which case the dual simplex method can be used to recover feasibility.

Note that: The addition of a new constraint can never improve the current optimum objective value.

<u>Example:</u> Suppose that TOYCO is changing the design of its toys, and that change will require the addition of a fourth operation in the assembly lines. The daily capacity of the new operation is 500 minutes; and the times per unit for the three products on this operation are 3, 1, and 1 minutes, respectively. The resulting constraint is thus constructed as

$$3x_1 + x_2 + x_3 \leq 500$$

This constraint is ***redundant*** because it is satisfied by the current optimum solution $x_1=0$, $x_2=100$, and $x_3=230$. This means that the current optimum solution remains unchanged.

Suppose, instead, that the unit times on the fourth operation are 3, 3, and 1 minutes, respectively. All the remaining data of the model remain unchanged. In this case, the fourth constraint

$$3x_1 + 3x_2 + x_3 \leq 500$$

is not satisfied by the current optimum solution.

We must thus augment the new constraint to the current optimum tableau as follows (x_7 is slack):

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
Z	4	0	0	1	2	0	0	1350
x_2	-1/4	1	0	1/2	-1/4	0	0	100
x_3	3/2	0	1	0	1/2	0	0	230
x_6	2	0	0	-2	1	1	0	20
x_7	3	3	1	0	0	0	1	500

Because the variables x_2 and x_3 are basic, we must substitute out their constraint coefficients in the x_7 row, which can be achieved by performing the following operation:

$$\text{New } x_7\text{-row} = \text{Old } x_7\text{-row} - (3\times(x_2\text{-row}) + 1\times(x_3\text{-row}))$$

The new tableau is thus given as

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
Z	4	0	0	1	2	0	0	1350
x_2	-1/4	1	0	1/2	-1/4	0	0	100
x_3	3/2	0	1	0	1/2	0	0	230
x_6	2	0	0	-2	1	1	0	20
x_7	9/4	0	0	-3/2	1/4	0	1	-30
Z	11/2	0	0	0	13/6	0	2/3	1330
x_2	1/2	1	0	0	-1/6	0	1/3	90
x_3	3/2	0	1	0	1/2	0	0	230
x_6	-1	0	0	0	2/3	1	-4/3	60
x_4	-3/2	0	0	1	-1/6	0	-2/3	20

The new optimum solution is x_1=0, x_2=90, x_3=230, and Z=$1330 (verify!)

2. CHANGES AFFECTING OPTIMALITY

This section considers two particular situations that could affect the optimality of the current solution:

1. Changes in the original objective coefficients
   - Changes in the objective coefficients of the **basic variables**
     - Finding the optimality range of a BV
   - Changes in the objective coefficients of the **nonbasic variables**
     - Finding the optimality range of a NBV
2. Addition of a new activity (variable) to the model
   - Changes in activity’s usage of resources

Changes in the Objective Function Coefficients

These changes only affect the optimalility of the solution.

- **For a Basic Variable**
  1. Compute the new dual values (using Method I)
  2. Use the new dual values to determine the new z-row coefficients

- **For a Nonbasic Variable**
  1. Use the current dual values.
  2. Calculate the z-row coefficient only of that NBV (because only z-row coefficient of that NBV changes)

Two cases will result:

- The new z-row satisfies the optimality condition and the solution remains unchanged (the optimum objective value may change , however).
- The optimality condition is not satisfied, in which case the (primal) simplex method is used to recover optimality.

Example: In the TOYCO model, suppose that the company has a new pricing policy to meet or match the competition. The unit profits under the new policy are \$2, \$3, and \$4 for train, truck, and car toys, respectively. The new objective function is

$$\text{Maximize } Z = 2x_1 + 3x_2 + 4x_3$$

Thus,

(New objective coefficients of basic variables x2, x3 and x6)

$$c_B = \begin{pmatrix} 3 & 4 & 0 \end{pmatrix}$$

Using Method I $\left(y = c_B B^{-1}\right)$ the dual variables are computed as;

$$\begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix} = \begin{pmatrix} 3 & 4 & 0 \end{pmatrix} \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3/2 & 5/4 & 0 \end{pmatrix}$$

- The z-row coefficients are determined as the difference between the left- and right-hand sides of the dual constraints.
- It is not necessary to recompute the objective row coefficients of the basic variables x_2, x_3, and x_6 because they always equal zero regardless of the changes made in the objective coefficients (verify!).

$$x_1 : z_1 - c_1 = y_1 + 3y_2 + y_3 - 2 = \frac{3}{2} + 3\frac{5}{4} + 0 - 2 = \frac{13}{4}$$

$$x_4 : z_4 - c_4 = y_1 - 0 = \frac{3}{2}$$

$$x_5 : z_5 - c_5 = y_2 - 0 = \frac{5}{4}$$

Note that the right-hand side of the dual constraint associated with x_1 is 2, the new coefficient in the modified objective function.

The computations show that the current solution show that the current solution – $x_1=0$ train, $x_2=100$ trucks and $x_3=230$ cars – remain optimal.

$\Rightarrow$The corresponding new profit is computed as (objective function)

$$2\times 0 + 3 \times 100 + 4 \times 230 = \$1220$$

Suppose that the TOYCO objective function is changed to

Maximize Z = $6x_1 + 3x_2 + 4x_3$

Determine if the current solution remains optimal or not.

Dual variables:

$$(y_1 \quad y_2 \quad y_3) = (3 \quad 4 \quad 0)\begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} = (3/2 \quad 5/4 \quad 0)$$

$$x_1 : y_1 + 3y_2 + y_3 - 6 = \frac{3}{2} + 3\left(\frac{5}{4}\right) + 0 - 6 = \frac{21}{4} - 6 = -3/4$$

$$x_4 : y_1 - 0 = \frac{3}{2}$$

$$x_5 : y_2 - 0 = \frac{5}{4}$$

The new tableau is given below. The corresponding changes in the optimal z-row are highlighted. The table is not optimal. Primal-simplex is applied to recover optimality

Iteration	Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS
	Z	-3/4	0	0	3/2	5/4	0	1220
x_6 leaves x_1 enters	x_2	-1/4	1	0	1/2	-1/4	0	100
	x_3	3/2	0	1	0	1/2	0	230
	X_6	2	0	0	-2	1	1	20
	Z	0	0	0	3/4	13/8	3/8	1227,5
	x_2	0	1	0	1/4	-1/8	1/8	102,5
	x_3	0	0	1	3/2	-1/4	-3/4	215
	x_1	1	0	0	-1	1/2	1/2	10

The new optimum solution is

x_1=10, x_2=102,5, x_3=215 and Z=$1227,5

A change in the c_j of a nonbasic variable

Let's cosider a change in the objective function coefficient of a nonbasic variable. Note that, in this case, c_B (objective coefficients of the basic variables) is not affected.

Thus, the only impact of such a change is on the single tableau element z_k-c_k.

In the TOYCO example x_1 is a nonbasic variable. Suppose that the new c_1 equals to 5 (old c_1=3).

To determine if the current basis remains optimal or not we recompute z_1-c_1 as follows:

$$z_1 - c_1 = c_B B^{-1} a_j - c_1$$

$$= \begin{pmatrix} 2 & 5 & 0 \end{pmatrix} \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} - 5 = 7 - 5 = 2$$

The computation shows that the current solution remains optimal.

Optimality Range of the Objective Coefficients

Another way to investigate effect of changes in the objective function coefficients is to compute the range for each individual coefficient that will keep the current solution optimal.

This is achieved by replacing the current c_j with c_j+d_j where d_j represents the (positive or negative) amount of change.

Example: Suppose that the objective function of the TOYCO model is changed to

$$\text{Maximize } Z = (3+d_1)x_1 + 2x_2 + 5x_3$$

Find the optimality range for the change d_1.

Note that, **because x_1 is not basic in the optimal tableau**, **the dual values will not be affected by this change** and hence, will remain the same as in the original model (i.e., y_1=1, y_2=2, y_3=0). Indeed because x_1 is nonbasic, only its z-row coefficient will be affected, and all the remaining z-row coefficients remain unchanged.

This means we only need to compute z_1-c_1.

$$x_1 : z_1 - c_1 = y_1 + 3y_2 + y_3 - (3 + d_1) = 1 + 3 \times 2 + 0 - (3 + d_1) = 4 - d_1$$

or

$$= c_B B^{-1} a_1 - c_1' = \begin{pmatrix} 2 & 5 & 0 \end{pmatrix} \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} - (3 + d_1) = 4 - d_1$$

Because TOYCO model is a maximization problem, the original solution will remain optimal so long as

$$4 - d_1 \geq 0 \Rightarrow d_1 \leq 4$$

This is equivalent to saying that the current solution remains optimal so long as the objective coefficient c_1 (= $3 + d_1$) of x_1 does not exceed $3 + 4 = \$7$.

$\Rightarrow$ Next, we consider the change d_2 in the objective coefficient of x_2:

Maximize $Z = 3x_1 + (2 + d_2)x_2 + 5x_3$

In this case, x_2 is basic and its change will affect the dual variables and subsequently, the z-row coefficients of all the nonbasic variables.

$$\begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix} = \begin{pmatrix} 2+d_2 & 5 & 0 \end{pmatrix} \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+\frac{d_2}{2} & 2-\frac{d_2}{4} & 0 \end{pmatrix}$$

We can thus compute the z-row coefficients of the NBVs as:

$$x_1 : z_1 - c_1 = y_1 + 3y_2 + y_3 - 3 = (1+\frac{d_2}{2}) + 3(2-\frac{d_2}{4}) + 0 - 3 = 4 - \frac{d_2}{4} \geq 0 \Rightarrow d_2 \leq 16$$

$$x_4 : z_4 - c_4 = y_1 - 0 = (1+\frac{d_2}{2}) - 0 = 1 + \frac{d_2}{2} \geq 0 \Rightarrow d_2 \geq -2$$

$$x_5 : z_5 - c_5 = y_2 - 0 = (2-\frac{d_2}{4}) - 0 = 2 - \frac{d_2}{4} \geq 0 \Rightarrow d_2 \leq 8$$

$$or \quad -2 \leq d_2 \leq 8$$

Equivalently, given $c_2 = 2 + d_2$, we get $\boxed{0 \leq c_2 \leq 10}$

Addition of a new Activity

- ⇒ The addition of a new activity in an LP model is equivalent to adding a new variable.
- ⇒ Intuitively, the addition of a new activity is desirable only if it is profitable – that is, if it improves the optimal value of the objective function.
- ⇒ Because the new activity is not yet part of the solution, it can be thought of as a nonbasic variable. This means that the dual values associated with the current solution remain unchanged.

Example: TOYCO recognizes that toy trains are not currently in production because they are not profitable. The company wants to replace toy trains with a new product, a toy fire engine, to be assembled on the existing facilities. TOYCO estimates the profit per toy fire engine to be \$4 and the assembly times per unit to be 1 minute on each of operations 1 and 2, and 2 minutes on operation 3.

Let x_7 represent the new fire engine product. Given $(y_1\ y_2\ y_3)=(1\ 2\ 0)$ are the optimal dual values, the reduced cost for x_7 is computed as

$$x_7 : z_7 - c_7 = 1y_1 + 1y_2 + 2y_3 - 4 = 1\times1 + 1\times2 + 2\times0 - 4 = -1$$

or

$$= c_B B^{-1} a_7 - c_7 = \begin{pmatrix}2 & 5 & 0\end{pmatrix} \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - 4 = -1$$

The result shows that it is profitable to include x_7 in the optimal solution.

To obtain the new optimum, we first compute its column constraint as given below.

$$\alpha_7 = B^{-1}a_7 = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/4 \\ 1/2 \\ 1 \end{pmatrix}$$

Basic	x_1	x_2	x_3	x_7	x_4	x_5	x_6	RHS
Z	4	0	0	-1	1	2	0	1350
x_2	-1/4	1	0	1/4	1/2	-1/4	0	100
x_3	3/2	0	1	1/2	0	1/2	0	230
x_6	2	0	0	1	-2	1	1	20

Basic	x_1	x_2	x_3	x_7	x_4	x_5	x_6	RHS
Z	6	0	0	0	-1	3	1	1370
x_2	-3/4	1	0	0	1	-1/2	-1/4	95
x_3	1/2	0	1	0	1	0	-1/2	220
x_7	2	0	0	1	-2	1	1	20
Z	21/4	1	0	0	0	5/2	3/4	1465
x_4	-3/4	1	0	0	1	-1/2	-1/4	95
x_3	5/4	-1	1	0	0	1/2	-1/4	125
x_7	1/2	2	0	1	0	0	1/2	210

The new optimum is determined by letting x_7 enter the basic solution, in which case x_6 must leave. The new solution is x_1=0, x_2=0, x_3=125, x_7=210 and Z=$1465

Changes in Activity's Usage of Resources

This kind of a change can affect only the optimality of the solution, since it affects the LHS of its dual constraint.

$\Rightarrow$ However, we must restrict this statement to activities that are currently nonbasic. A change in the constraint coefficients of basic activities will affect the inverse matrix and could lead to complications in the computations.

$\Rightarrow$ We shall thus restrict our presentation to changes in nonbasic activities.

$\Rightarrow$ The easiest way to handle changes in basic activities is to solve the problem anew.

Let us consider in TOYCO example the constraint column associated with the nonbasic activity x_1 is changed as

$$a_1 = \begin{pmatrix} 1/2 \\ 2 \\ 1/2 \end{pmatrix} \quad \text{(the old} \quad a_1 = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \quad \text{)}$$

Then the corresponding dual constraint is:

$$\frac{1}{2}y_1 + 2y_2 + \frac{1}{2}y_3 \geq 3$$

Since the objective function coefficients and the dual variables are unchanged, the new x_1 coefficient in the z-row is computed as

$$x_1 : z_1 - c_1 = (1/2)\times 1 + 2\times 2 + (1/2)\times 0 - 3 = \frac{3}{2}$$

Since it is ≥0, the proposed change does not affect the optimum solution.

$\Rightarrow$ The addition of a new constraint can never improve the value of the objective function.

$\Rightarrow$ The addition of a new variable can never worsen the value of the objective function.