POWER SYSTEM DYNAMICS (STABILITY) AND CONTROL

Small-Signal Stability Analysis of Power Systems
Lecture Notes 3

Prof. Dr. Saffet AYASUN

Department of Electrical and Electronics Engineering

Gazi University

STABILITY

- In general terms, power system stability refers to that property of the power system which enables the system to maintain an equilibrium operating point under normal conditions and to attain a state of equilibrium after being subjected to a disturbance.
- As primarily synchronous generators are used for generating power in grid, power system stability is generally implied by the ability of the synchronous generators to remain in ’synchronism’ or ’in step’.
- On the other hand, if the synchronous generators loose synchronism after a disturbance, then the system is called unstable.

LOSS OF SYNCHRONISM

- In the normal equilibrium condition, all the synchronous generators run at a constant speed and the difference between the rotor angles of any two generators is constant.
- Under any disturbance, small or large, the speed of the machines will deviate from the steady state values due to mismatch between mechanical and electrical powers (torque) and therefore, the difference of the rotor angles would also change.
- If these rotor angle differences (between any pair of generators) attain steady state values (not necessarily the same as in the pre-disturbance condition) after some finite time, then the synchronous generators are said to be in **’synchronism**’.
- On the other hand, if the rotor angle differences keep on increasing indefinitely, then the machines are considered to have lost **’synchronism’.**
- Under this **’out of step’** condition, the output power, voltage etc. of the generator continuously drift away from the corresponding pre-disturbance values until the protection system trips the machine.

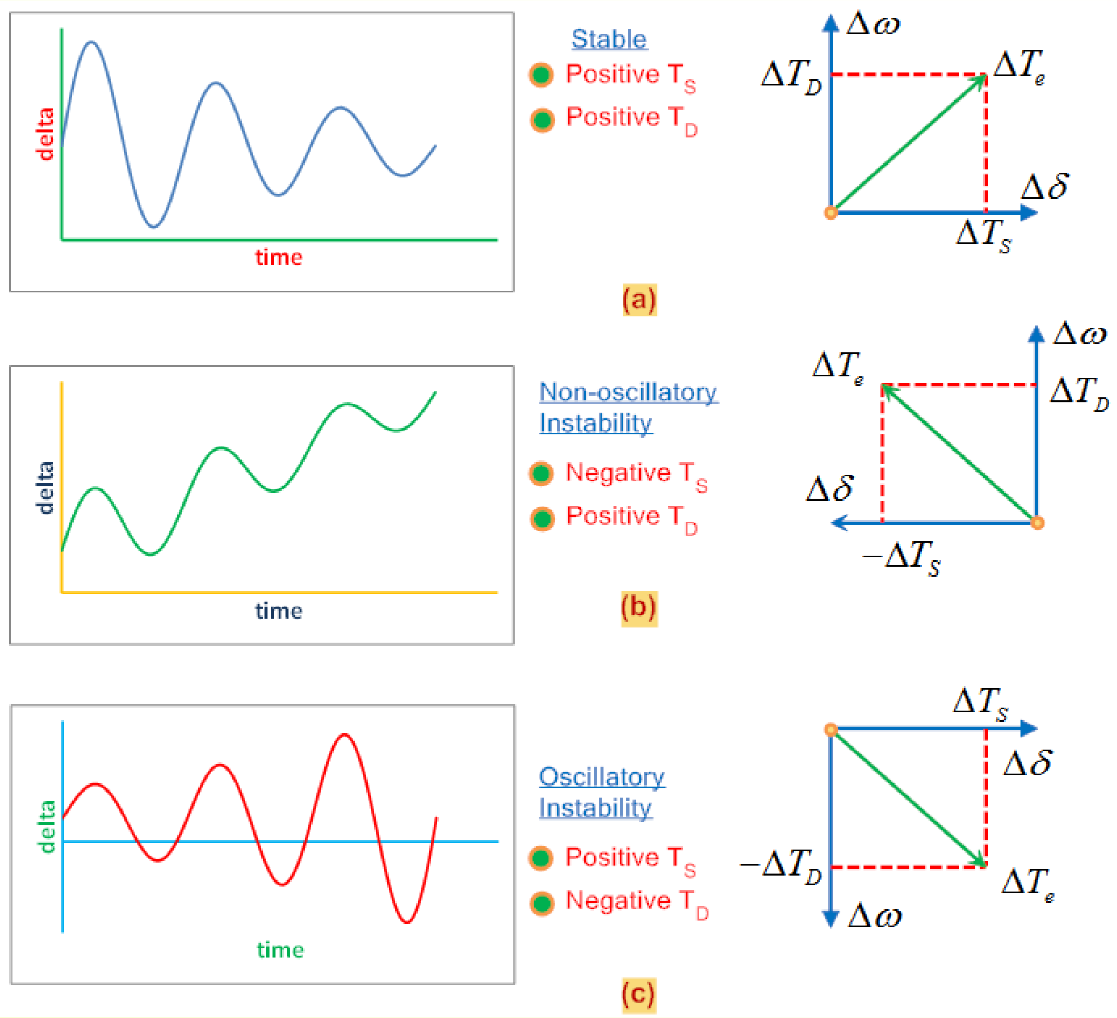
SMALL-SIGNAL STABILITY

- In this case, the disturbance occurring in the system is small. Such kind of small disturbances always take place in the system due to random variations of the loads and the generation.
- Under small perturbation (or disturbance), the change in the electrical torque of a synchronous generator can be resolved into two components, namely,
  - synchronizing torque (T_s) - which is proportional to the change in the rotor angle
  - damping orque (T_d), which is proportional to the change in the speed

$$\Delta T_e = \Delta T_s \, \Delta\delta + \Delta T_D \, \Delta\omega$$

- As a result, depending on the amounts of synchronizing and damping torques, small signal instability can manifest itself in two forms.
- When there is insufficient amount of synchronizing torque, the rotor angle increases steadily. On the other hand, for inadequate amount of damping torque, the rotor angle undergoes oscillations with increasing amplitude.

SMALL-SIGNAL STABILITY



Influence of synchronous and damping torque

TYPES OF SMALL-SIGNAL INSTABILITY

- **Local mode:** In this type, the units within a generating station oscillate with respect to the rest of the system. The term **’local’** is used because the oscillations are localized in a particular generating station.
- **Inter-area mode:** In this case, the generators in one part of the system oscillate with respect to the machines in another part of the system.
- **Control mode:** This type of instability is excited due to poorly damped control systems such as exciter, speed governor, static var compensators, HVDC converters etc.
- **Torsional mode:** This type is associated with the rotating turbine-governor shaft. This type is more prominent in a series compensated transmission system in which the mechanical system resonates with the electrical system.

STABILITY

- **The ability of the power system to remain in synchronism and maintain the state of equilibrium following a disturbing force**
  - Steady-state stability: analysis of small and slow disturbances
    - gradual power changes
  - Transient stability: analysis of large and sudden disturbances
    - faults, outage of a line, sudden application or removal of load

GENERATOR DYNAMIC MODEL

- **Under normal conditions, the relative position of the rotor axis and the stator magnetic field axis is fixed**
  - the angle between the two is the power angle or torque angle, δ
  - during a disturbance, the rotor will accelerate or decelerate w.r.t. the rotating stator field
  - acceleration or deceleration causes a change in the power angle

$$T_e = \frac{P_e}{\omega_e} = \frac{P_e}{2\pi(60Hz)} \qquad \frac{P_m}{\omega_{rotor}} = T_m$$

$$T_{accelation} = \Delta T = T_m - T_e$$

$$J\frac{d^2\theta_m}{dt^2} = \Delta T = T_m - T_e \qquad \theta_m = \omega_{ms}t + \delta_m \qquad \frac{\omega_{rotor}}{\omega_{ms}} = \frac{poles}{2}$$

where θ_m is the angular displacement of the rotor with respect to the stationary reference axis on the stator. Since we are interested in the rotor speed relative to synchronous speed, the angular reference is chosen relative to a synchronously rotating reference frame moving with constant angular velocity ω_{sm}, that is

GENERATOR DYNAMIC MODEL

$$\omega_m = \frac{d\theta_m}{dt} = \omega_{ms} + \frac{d\delta_m}{dt} \qquad \alpha_m = \frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$J\frac{d^2\theta_m}{dt^2} = J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

$$J\omega_m \frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e = P_m - P_e$$

$$W_{KE} = \tfrac{1}{2} J\omega_m^2 = \tfrac{1}{2} M\omega_m \qquad M = \frac{2W_{KE}}{\omega_m} = J\omega_m$$

$$\omega_m \approx \omega_{ms} \rightarrow M \approx \frac{2W_{KE}}{\omega_{ms}} = J\omega_{ms}$$

GENERATOR DYNAMIC MODEL

$$\frac{2W_{KE}}{\omega_s S_B}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)}$$

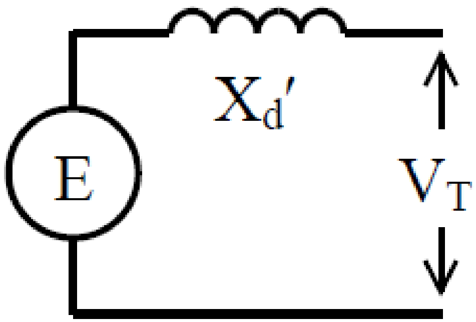
$$\frac{W_{KE}}{S_B} = \frac{\text{kinetic energy in MJ at rated speed}}{\text{machine power rating in MVA}} = H$$

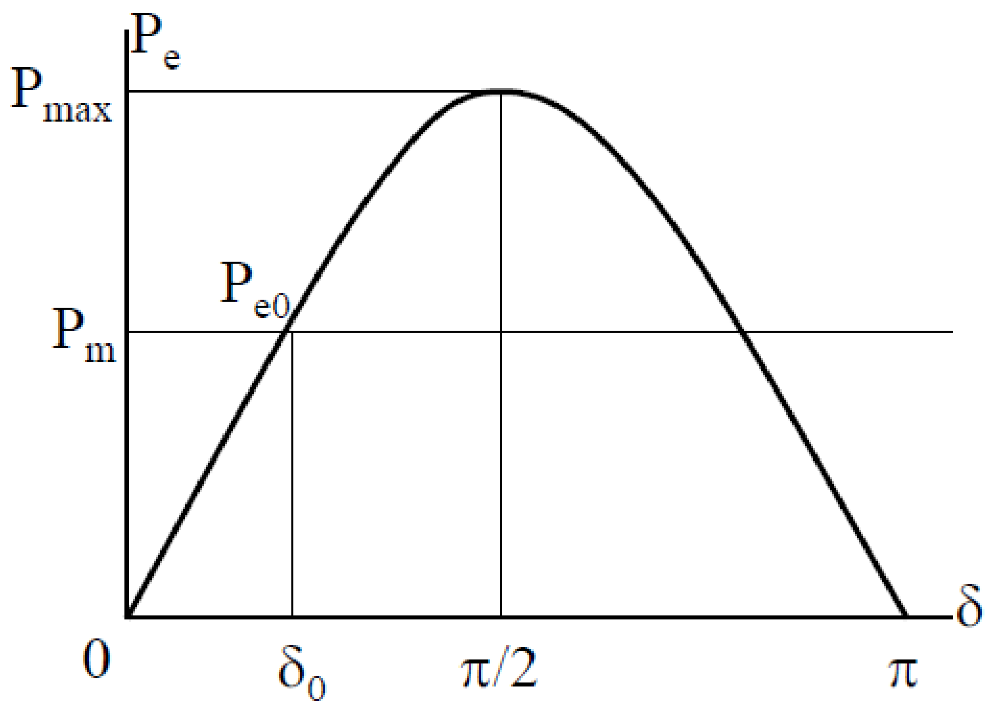
$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)}$$

$$\rightarrow \quad \frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} \quad (radians)$$

$$\rightarrow \quad \frac{H}{180 f}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} \quad (degrees)$$

SYNCHRONNOUS MACHINE MODEL





power angle curve

Round Rotor Machine Model

$$E' = |E'|\angle\delta$$

$$V_G = |V_G|\angle 0°$$

$$B = 1/X_d'$$

$$P_e = |E'||V_G||B|\cos(\delta - 90°) = \frac{|E'||V_G|}{X_d'}\sin\delta = P_{max}\sin\delta$$

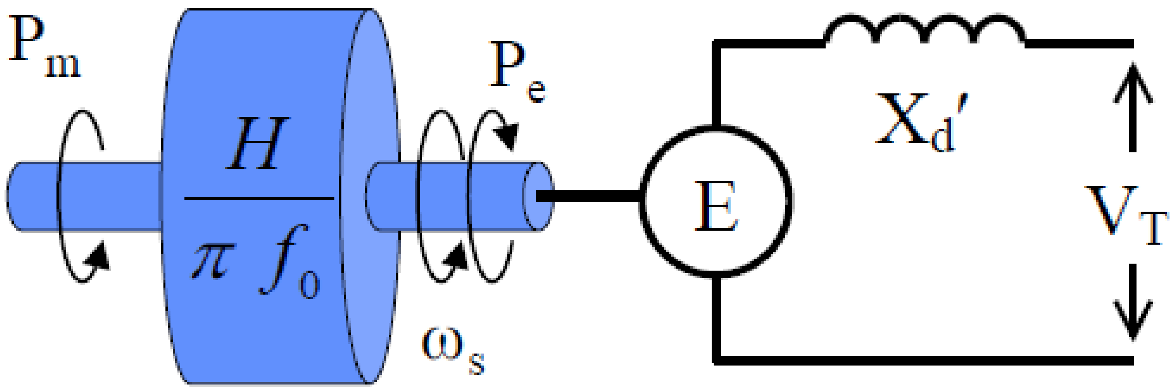
Derive this equation on the board

SWING EQUATION

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$ Dynamic Generator Model

$$P_e = P_{\max}\sin\delta$$ Synchronous Machine Model

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_{\max}\sin\delta$$ Forming the Swing Equation



SMALL-SIGNAL STABILITY

- **The ability of the power system to remain in synchronism when subject to small disturbances**
- **Stability is assured if the system returns to its original operating state (voltage magnitude and angle profile)**
- **The behavior can be determined with a linear system model**
- **Assumption:**
  - the automatic controls are not active
  - the power shift is not large
  - the voltage angles changes are small

SMALL-SIGNAL STABILITY

- **Swing Equation**

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{max}\sin\delta$$

- **Small disturbance modeling**

$$\delta = \delta_0 + \Delta\delta \qquad \text{Consider a small deviation}$$

$$\frac{H}{\pi f_0}\frac{d^2(\delta_0 + \Delta\delta)}{dt^2} = P_m - P_{max}\sin(\delta_0 + \Delta\delta)$$

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} + \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} = P_m - P_{max}\left[\sin\delta_0\cos\Delta\delta + \cos\delta_0\sin\Delta\delta\right]$$

SMALL-SIGNAL STABILITY

- **Simplification of the swing equation**

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}+\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}=P_m-P_{max}\left[\sin\delta_0\cos\Delta\delta+\cos\delta_0\sin\Delta\delta\right]$$

Substitute the following approximations

$$\Delta\delta << \delta \qquad \cos\Delta\delta\approx 1 \qquad \sin\Delta\delta\approx\Delta\delta$$

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}+\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}=P_m-P_{max}\sin\delta_0-P_{max}\cos\delta_0\cdot\Delta\delta$$

Group steady state and transient terms

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}-P_m+P_{max}\sin\delta_0=-\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}-P_{max}\cos\delta_0\cdot\Delta\delta$$

SMALL-SIGNAL STABILITY

- **Simplification of the swing equation**

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} - P_m + P_{max}\sin\delta_0 = -\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} - P_{max}\cos\delta_0\cdot\Delta\delta$$

$$0 = \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_{max}\cos\delta_0\cdot\Delta\delta$$

Steady state term is equal to zero

$$\left.\frac{dP_e}{d\delta}\right|_{\delta_0} = \frac{d}{d\delta}P_{max}\sin\delta\Big|_{\delta_0} = P_{max}\cos\delta_0 = P_s$$

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_s\cdot\Delta\delta = 0$$

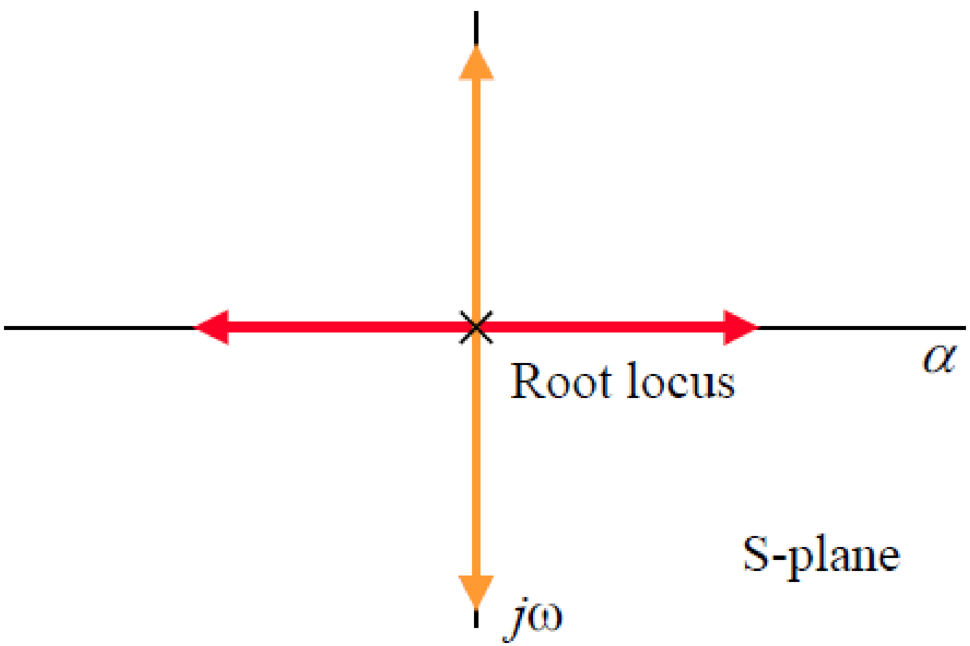
Second order equation.
The solution depends on the roots of the characteristic equation

SMALL-SIGNAL STABILITY

- **Stability Assessment**
  - When P_s is negative, one root is in the right-half s-plane, and the response is exponentially increasing and stability is lost
  - When P_s is positive, both roots are on the $j\omega$ axis, and the motion is oscillatory and undamped, the natural frequency is:

$$s^2 = -\frac{\pi f_0}{H} P_S$$

$$\omega_n = \sqrt{\frac{\pi f_0}{H} P_S}$$



DAMPING TORQUE

$$P_D = D\frac{d\delta}{dt}$$ Damping force is due to air-gap interaction

$$\frac{H}{\pi\ f_0}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_S\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi\ f_0}{H}D\frac{d\Delta\delta}{dt} + \frac{\pi\ f_0}{H}P_S\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\ \omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi\ f_0}{H\ P_S}}$$

CHARACTERISTIC EQUATION

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{H P_S}} < 1$$ for normal operation conditions

$$s_1, s_2 = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$$ complex roots

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$ the damped frequency of oscillation

LAPALACE TRANSFORM ANALYSIS

$$x_1 = \Delta\delta, \quad x_2 = \frac{d\Delta\delta}{dt}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

$$\mathcal{L}\{\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}\} \rightarrow s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s)$$

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(0)$$

$$(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -1 \\ \omega_n^2 & s + 2\zeta\omega_n \end{bmatrix}$$

$$\mathbf{X}(s) = \frac{\begin{bmatrix} s + 2\zeta\omega_n & 1 \\ -\omega_n^2 & s \end{bmatrix}}{s^2 + 2\zeta\omega_n s + \omega_n^2}\mathbf{x}(0)$$

LAPALACE TRANSFORM ANALYSIS

$$\Delta\delta(s) = \frac{(s + 2\zeta\omega_n)\Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Delta\omega(s) = \frac{-\omega_n^2\Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Delta\delta(t) = \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_d t + \theta), \quad \theta = \cos^{-1}\zeta$$

$$\Delta\omega(t) = -\frac{\omega_n\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_d t)$$

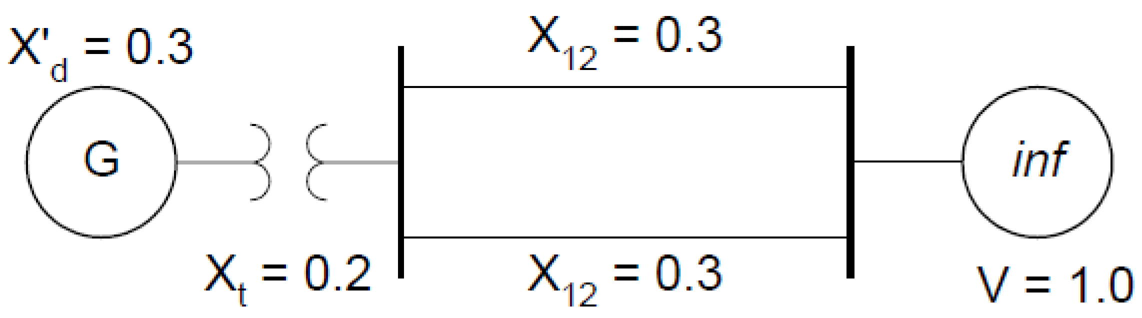
$$\delta(t) = \delta_0 + \Delta\delta(t), \quad \omega(t) = \omega_0 + \Delta\omega(t)$$

Power Systems I

Derive this equation on the board

EXAMPLE 1

- A 60 Hz synchronous generator having inertia constant H = 9.94 MJ/MVA and a transient reactance X'_d = 0.3 pu is connected to an infinite bus through the following network. The generator is delivering 0.6 *pu* real power at 0.8 power factor lagging to the infinite bus at a voltage of 1 *pu*. Assume the damping power coefficient is *D* = 0.138 *pu*. Consider a small disturbance of 10° or 0.1745 radians. Obtain equations of rotor angle and generator frequency motion.



Solution

The transfer reactance between the generated voltage and the infinite bus is

$$X = 0.3 + 0.2 + \frac{0.3}{2} = 0.65$$

The per unit apparent power is

$$S = \frac{0.6}{0.8}\angle\cos^{-1} 0.8 = 0.75\angle 36.87^\circ$$

The current is

$$I = \frac{S^*}{V^*} = \frac{0.75\angle -36.87^\circ}{1.0\angle 0^\circ} = 0.75\angle -36.87^\circ$$

The excitation voltage is

$$E' = V + jXI = 1.0\angle 0^\circ + (j0.65)(0.75\angle -36.87^\circ) = 1.35\angle 16.79^\circ$$

Solution

Thus, the initial operating power angle is $16.79° = 0.2931$ radian. The synchronizing power coefficient given by (11.39) is

$$P_s = P_{max} \cos \delta_0 = \frac{(1.35)(1)}{0.65} \cos 16.79° = 1.9884$$

The undamped angular frequency of oscillation and damping ratio are

$$\omega_n = \sqrt{\frac{\pi f_0}{H} P_s} = \sqrt{\frac{(\pi)(60)}{9.94} 1.9884} = 6.1405 \quad \text{rad/sec}$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{H P_s}} = \frac{0.138}{2}\sqrt{\frac{(\pi)(60)}{(9.94)(1.9884)}} = 0.2131$$

Solution

The linearized force-free equation which determines the mode of oscillation given by (11.46) with δ in radian is

$$\frac{d^2\Delta\delta}{dt^2} + 2.62\frac{d\Delta\delta}{dt} + 37.7\Delta\delta = 0$$

From (11.50), the damped angular frequency of oscillation is

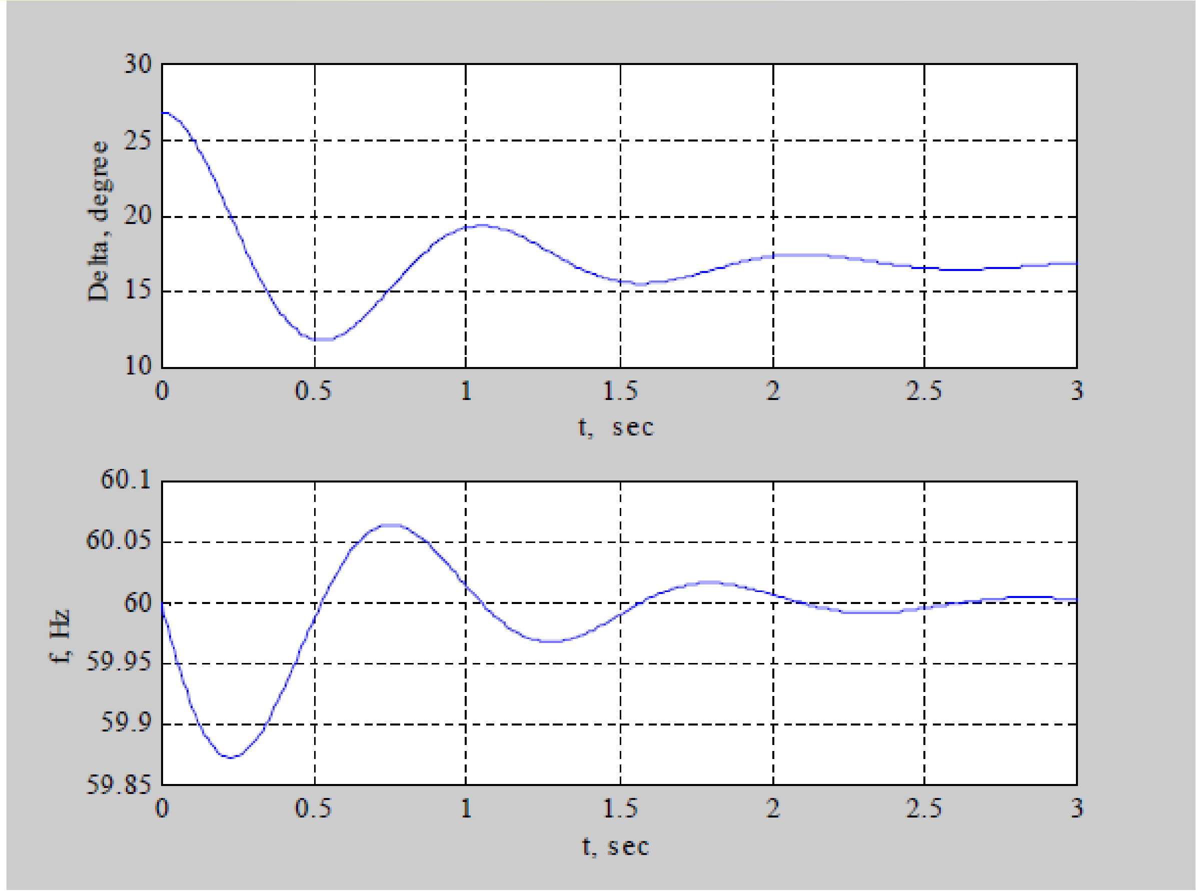
$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6.1405\sqrt{1-(0.2131)^2} = 6.0 \ \ \text{rad/sec}$$

corresponding to a damped oscillation frequency of

$$f_d = \frac{6.0}{2\pi} = 0.9549 \ \ \text{Hz}$$

Solution

$$\delta = 16.79^\circ + 10.234e^{-1.3t}\sin(6.0t + 77.6966^\circ)$$
$$f = 60 - 0.1746e^{-1.3t}\sin 6.0t$$



MATLAB M-File

```
E=1.35; V=1.0; H=9.94; X=0.65; Pm=0.6; D= 0.138; f0 = 60;
Pmax = E*V/X, d0 = asin(Pm/Pmax)                    % Max. power
Ps = Pmax*cos(d0)           % Synchronizing power coefficient
wn = sqrt(pi*60/H*Ps)% Undamped frequency of  oscillation
z = D/2*sqrt(pi*60/(H*Ps))                     % Damping ratio
wd=wn*sqrt(1-z^2), fd=wd/(2*pi) %Damped frequency oscill.
tau = 1/(z*wn)                                 % Time constant
th = acos(z)                               % Phase angle theta
Dd0 = 10*pi/180;                     % Initial angle in radian
t = 0:.01:3;
Dd = Dd0/sqrt(1-z^2)*exp(-z*wn*t).*sin(wd*t + th);
d = (d0+Dd)*180/pi;                    % Power angle in degree
Dw = -wn*Dd0/sqrt(1-z^2)*exp(-z*wn*t).*sin(wd*t);
f = f0 + Dw/(2*pi);                           % Frequency in Hz
subplot(2,1,1), plot(t, d), grid
xlabel('t  sec'), ylabel('Delta    degree')
subplot(2,1,2), plot(t,f), grid
xlabel('t  sec'), ylabel('Frequency    Hz')
subplot(111)
```

MATLAB initial Command

$$\dot{\mathbf{x}}(t) = \mathbf{AX}(t) + \mathbf{Bu}(t)$$
$$\mathbf{y} = \mathbf{Cx}(t) + \mathbf{Du}(t)$$

$$[\mathrm{y}, \mathrm{x}] = \mathbf{initial}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{x_0}, \mathbf{t})$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -37.705 & -2.617 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

```
A = [0  1; -37.705  -2.617];
B = [0; 0];                              %  Column B zero-input
C=[1 0; 0 1];%Unity matrix defining output y as x1 and x2
D = [0; 0];
Dx0 = [0.1745; 0];                        % Initial conditions
[y, x] = initial(A, B, C, D, Dx0, t);
Dd = x(:, 1); Dw = x(:, 2);   % State variables x1 and x2
d = (d0 + Dd)*180/pi;                   % Power angle in degree
f = f0 + Dw/(2*pi);                         % Frequency in Hz
subplot(2,1,1), plot(t, d), grid
xlabel('t  sec'), ylabel('Delta    Degree')
subplot(2,1,2), plot(t, f), grid
xlabel('t  sec'), ylabel('Frequency    Hz'),subplot(111)
```

Small Disturbance in Input Power

- **Assume that the input power is increased by a small amount**
- **The linearized swing equation**

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_s\,\Delta\delta = \Delta P$$

or

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi f_0}{H}D\frac{d\Delta\delta}{dt} + \frac{\pi f_0}{H}P_s\,\Delta\delta = \frac{\pi f_0}{H}\Delta P$$

or in terms of the standard second-order differential equation, we have

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\,\Delta\delta = \Delta u$$

where

$$\Delta u = \frac{\pi f_0}{H}\Delta P$$

Small Disturbance in Input Power

$$x_1 = \Delta\delta \quad \text{and} \quad x_2 = \Delta\omega = \dot{\Delta\delta} \quad \text{then}$$
$$\dot{x}_1 = x_2 \quad \text{and} \quad \dot{x}_2 = -\omega_n^2 x_1 - 2\zeta\omega_n x_2$$

Writing the above equations in matrix, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta u$$

or

$$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\Delta u(t)$$

Small Disturbance in Input Power: Laplace Transform

- Take Laplace transform

$$s\mathbf{X}(s) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\,\Delta U(s)$$

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\,\Delta U(s)$$

$$\Delta U(s) = \frac{\Delta u}{s}$$

- The state will be

$$\mathbf{X}(s) = \frac{\begin{bmatrix} s + 2\zeta\omega_n & 1 \\ -\omega_n^2 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{\Delta u}{s}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Delta\delta(s) = \frac{\Delta u}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$\Delta\omega(s) = \frac{\Delta u}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

Small Disturbance in Input Power: Laplace Transform

- Take inverse Laplace transform

$$\Delta\delta = \frac{\Delta u}{\omega_n^2}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_d t + \theta)\right]$$

$$\theta = \cos^{-1}\zeta$$

$$\Delta\omega = \frac{\Delta u}{\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\omega_d t$$

- Substituting Δu we obtain angle (in radian) and angular frequency

$$\delta = \delta_0 + \frac{\pi f_0 \Delta P}{H\omega_n^2}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_d t + \theta)\right]$$

$$\omega = \omega_0 + \frac{\pi f_0 \Delta P}{H\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\omega_d t$$

Example 2

The generator of Example 11.2 is operating in the steady state at $\delta_0 = 16.79°$ when the input power is increased by a small amount $\Delta P = 0.2$ per unit. The generator excitation and the infinite bus bar voltage are the same as before, i.e., $E' = 1.35$ per unit and $V = 1.0$ per unit.

(a) Using (11.75) and (11.76), obtain the step response for the rotor angle and the generator frequency.

(b) Obtain the response using the *MATLAB* **step** function.

(c) Obtain a *SIMULINK* block diagram representation of the state-space model and simulate to obtain the response.

Solution

Substituting for H, δ_0, ζ, and ω_n

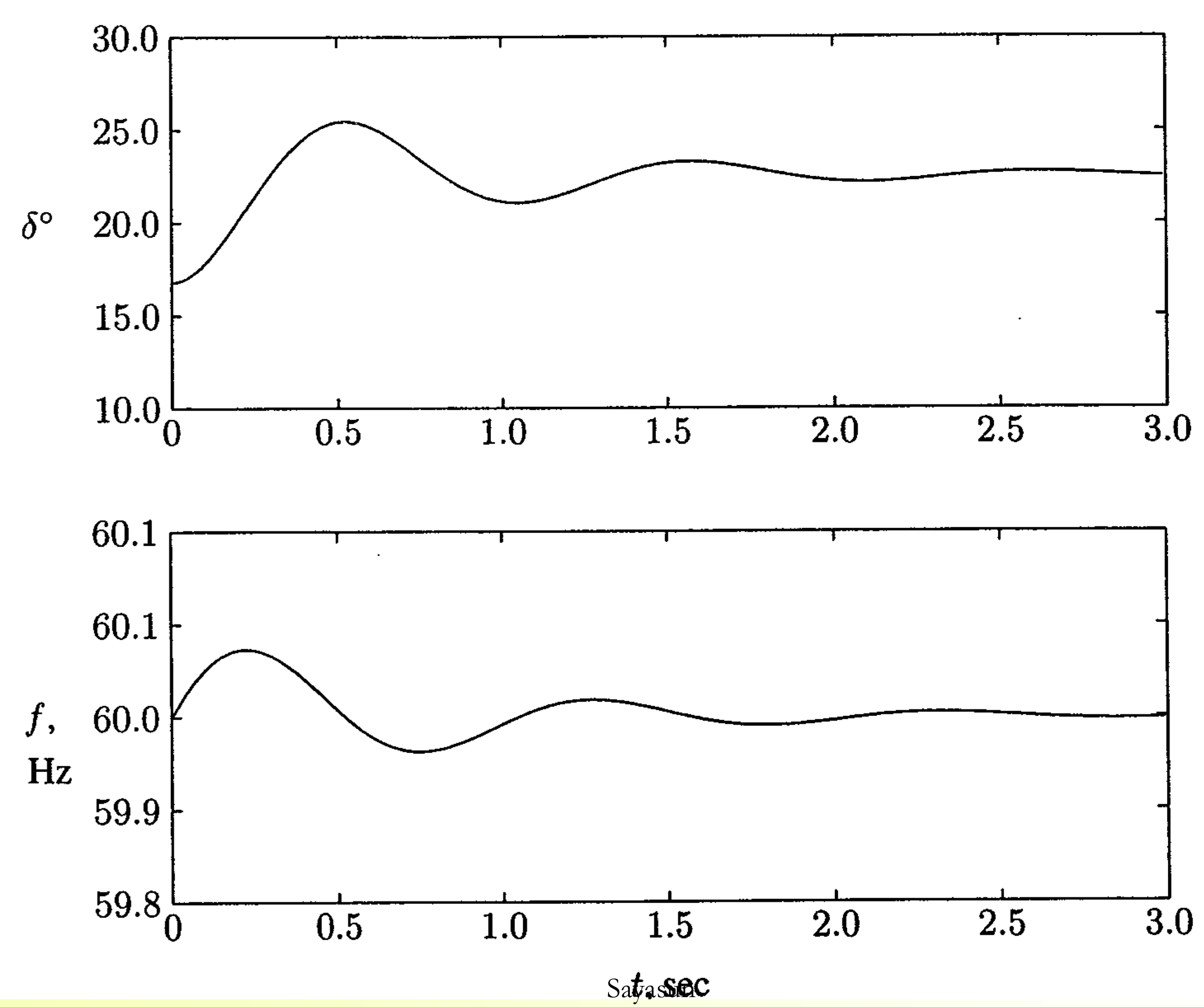
$$\delta = 16.79^\circ + \frac{(180)(60)(0.2)}{(9.94)(6.1405)^2}[1 - \frac{1}{\sqrt{1-(0.2131)^2}}e^{-1.3t}\sin(6t + 77.6966^\circ)]$$

$$\delta = 16.79^\circ + 5.7631[1 - 1.0235e^{-1.3t}\sin(6t + 77.6966^\circ)]$$

$$f = 60 + \frac{(60)(0.2)}{2(9.94)(6.1405)\sqrt{1-(0.2131)^2}}e^{-1.3t}\sin 6t$$

$$f = 60 + 0.10e^{-1.3t}\sin 6t$$

Solution



30.0
25.0
20.0
15.0
10.0
$\delta°$
0
0.5
1.0
1.5
2.0
2.5
3.0
60.1
60.1
60.0
59.9
59.8
f,
Hz
t, sec

MATLAB step Command

$$\dot{\mathbf{x}}(t) = \mathbf{AX}(t) + \mathbf{Bu}(t)$$
$$\mathbf{y} = \mathbf{Cx}(t) + \mathbf{Du}(t)$$

$$[\mathbf{y}, \mathbf{x}] = \mathbf{step}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{iu}, \mathbf{t})$$

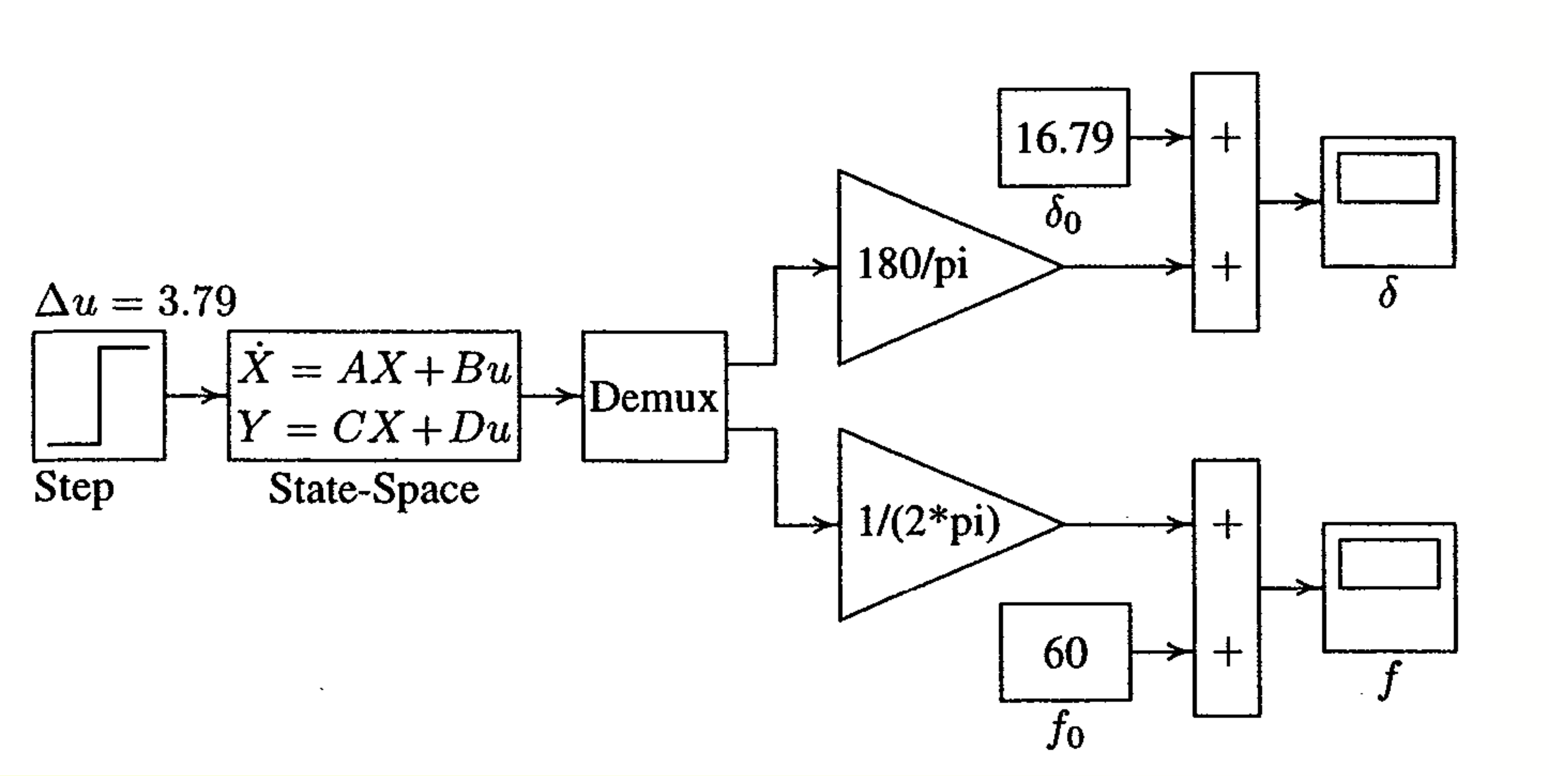
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -37.705 & -2.617 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta u$$

$$\Delta u = (60\pi/9.94)(0.2) = 3.79$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

```
A = [0  1; -37.705  -2.617];
Dp = 0.2; Du = 3.79;    % Small step change in power input
B = [0; 1]*Du;
C=[1 0; 0 1];%Unity matrix defining output y as x1 and x2
D = [0; 0];
[y, x] = step(A, B, C, D, 1, t);
Dd = x(:, 1); Dw = x(:, 2);    % State variables x1 and x2
d = (d0 + Dd)*180/pi;                % Power angle in degree
f = f0 + Dw/(2*pi);                        % Frequency in Hz
subplot(2,1,1), plot(t, d), grid
xlabel('t  sec'), ylabel('Delta    degree')
subplot(2,1,2), plot(t, f), grid
xlabel('t  sec'), ylabel('Frequency    Hz'),subplot(111)
```

Simulink Model



Homework 1 (20 points)

A two-pole, 60-Hz synchronous generator has a rating of 250 MVA, 0.8 power factor lagging. The kinetic energy of the machine at synchronous speed is 1080 MJ. The machine is running steadily at synchronous speed and delivering 60 MW to a load at a power angle of 8 electrical degrees. The load is suddenly removed.

a) Determine the acceleration of the rotor.
b) If the acceleration computed for the generator is constant for a period of 12 cycles, determine the value of the power angle and the rpm at the end of this time.

Homework 2 (20 points)

The swing equations of two interconnected synchronous machines are written as.

$$\frac{H_1}{\pi f_0}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$

$$\frac{H_2}{\pi f_0}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Obtain a swing equation equivalent to that of a single machine in terms of δ, and show that

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$

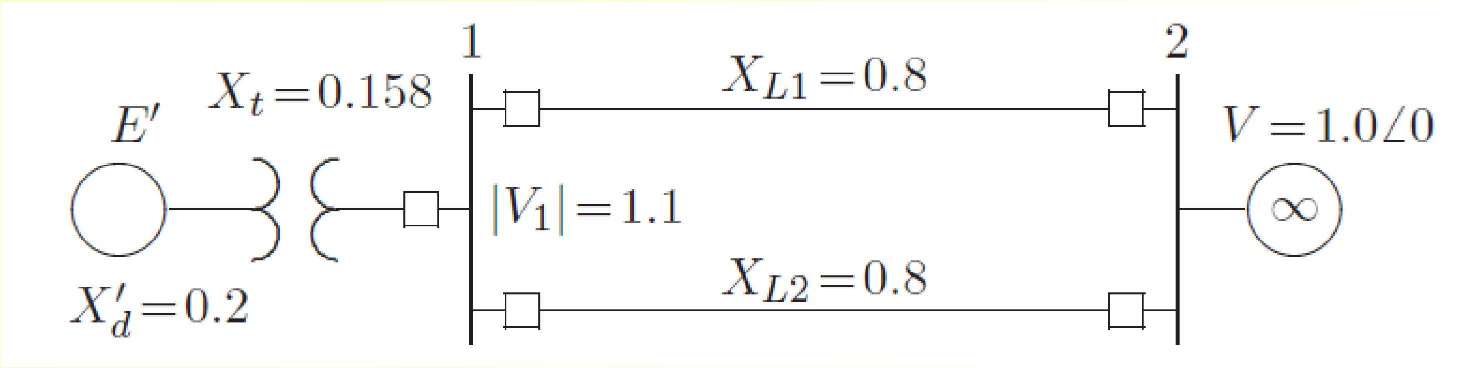
where

$$H = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_m = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2} \quad \text{and} \quad P_e = \frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2}$$

Homework 3 (60 points)

A 60-Hz synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 5.66 MJ/MVA. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown below. Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator is delivering a real power of 0.77 per unit to bus bar 1. Voltage magnitude at bus 1 is $|V_1| = 1.1 pu$ pu. The infinite bus voltage $V_2 = 1.0\angle 0^0 \, pu$ pu.



Homework 3 (cont'nd)

a) Determine the generator excitation voltage and the swing equation

b) The machine has a pu damping coefficient of D = 0.15. The generator excitation voltage is E' = 1.25 pu and the generator is delivering a real power of 0.77 pu to the infinite bus at a voltage of V = 1.0 pu. Write the linearized swing equation for this power system annd to find the equations describing the motion of the rotor angle and the generator frequency for a small disturbance of $\Delta\delta = 15^0$. Use MATLAB to obtain the plots of rotor angle and frequency.

c) Write the linearized swing equation found in part b) in state variable form. Use $[y, x] = \textbf{initial}(A, B, C, D, x_0, t)$ and **Simulink** commands to obtain the zero-input response for the initial conditions $\delta_0 = 27.835^\circ$

Homework 3 (cont'nd)

d) The generator is operating in the steady state at when the input power is increased by a small amount $\Delta P = 0.15$ pu. The generator excitation and the infinite bus voltage are the same as before. Find the equations describing the motion of the rotor angle and the generator frequency for a small disturbance of $\Delta P = 0.15$pu. Use MATLAB to obtain the plots o rotor angle and frequency.

e) Determine the linearized state-space equation for the case in part d). Use $[y, x]$ = **step**$(A, B, C, D, 1, t)$ or Simulink to to obtain the zero-state response when the input power is increased by a small amount $\Delta P = 0.15$