EEE 472 POWER SYSTEM ANALYSIS II
Symmetrical Fault Analysis
(Balanced 3-Phase Faults)
Z_{bus} Method

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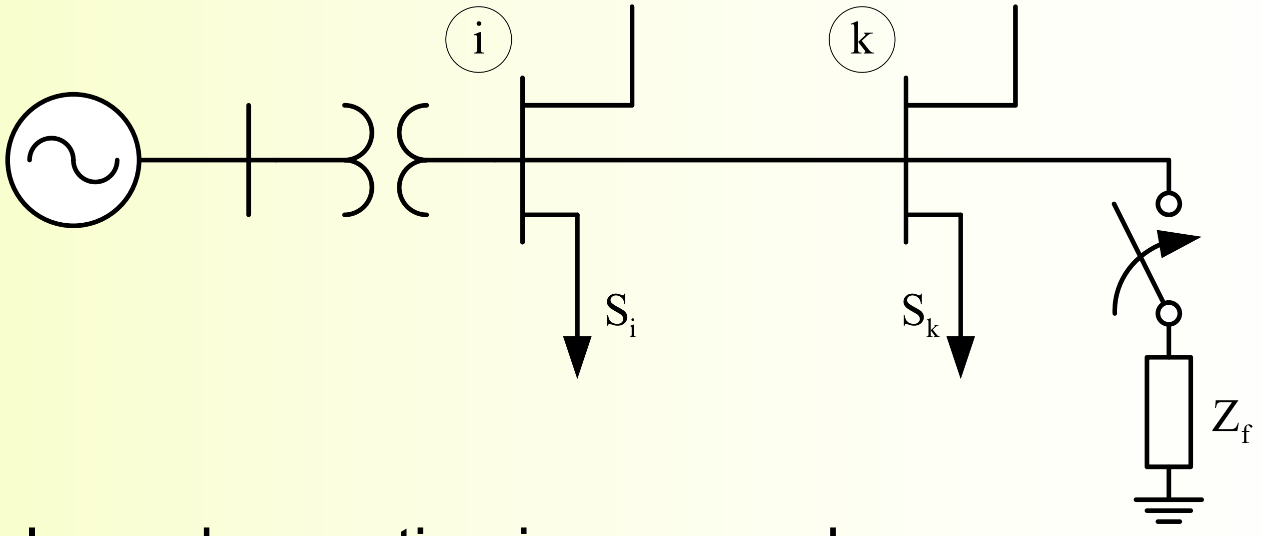
Fault Analysis Using Impedance Matrix

- In a large network, an impedance matrix is readily available
- Matrix algebra formation
  - Seek a matrix where the diagonal elements represent the source impedance for the buses
- Consider the following system



  - Balanced operation is assumed

Fault Analysis Using Impedance Matrix

- Place the prefault voltages into a vector

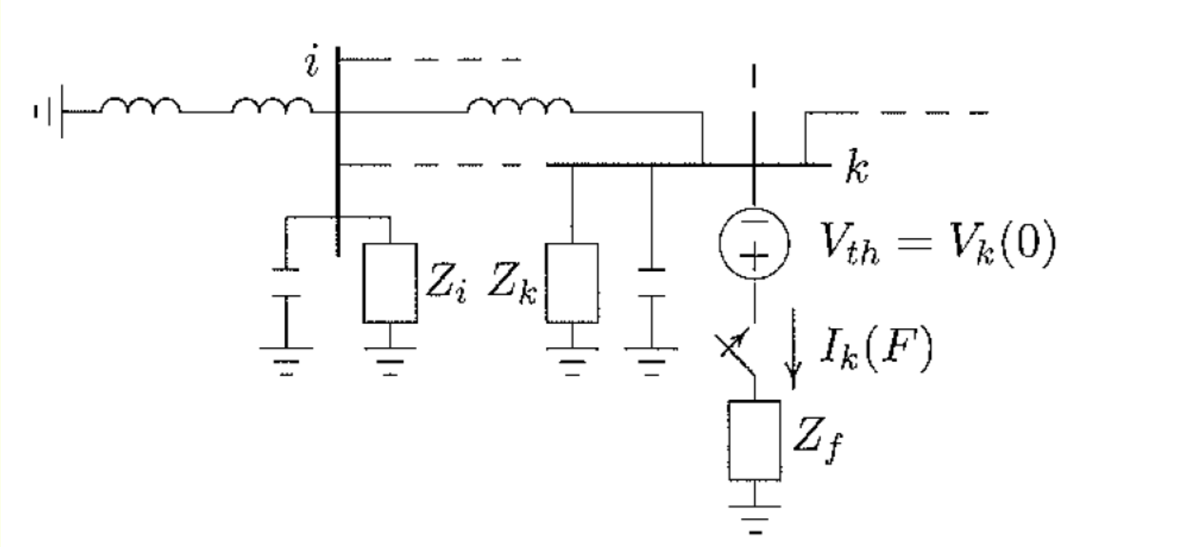
$$V_{bus}(0) = \left[V_1(0) \quad \cdots \quad V_k(0) \quad \cdots \quad V_n(0) \right]^T$$

- Replace the loads by a constant impedance model using the prefault bus voltages

$$Z_i = \frac{\left| V_i(0) \right|^2}{S_i^*}$$

- The voltage change $\left(\Delta \mathrm{V} \right)$ is determined by placing a fault voltage at the faulted bus with all the other sources short-circuited

Fault Analysis Using Impedance Matrix



- Superposition: the fault voltages are calculated from the prefault voltages

$$\mathbf{V}_{bus}^{f} = \mathbf{V}_{bus}(0) + \Delta\mathbf{V}_{bus}$$

where bus voltage changes caused by the fauly

$$\Delta\mathbf{V}_{bus} = \begin{bmatrix} \Delta V_1 & \cdots & \Delta V_k & \cdots & \Delta V_n \end{bmatrix}^T$$

Fault Analysis Using Impedance Matrix

- The change in bus voltages can be calculated from the network matrix

$$\begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1k} & \cdots & Y_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{k1} & \cdots & Y_{kk} & \cdots & Y_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{n1} & \cdots & Y_{nk} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix}$$

$$\mathbf{I}_{bus}(F) = \mathbf{Y}_{bus} \Delta \mathbf{V}_{bus}$$

Fault Analysis Using Impedance Matrix

- Solving for bus voltage changes

$$\Delta \mathbf{V}_{bus} = \mathbf{Z}_{bus}\mathbf{I}_{bus}(F)$$

$$\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1}$$

- The bus voltages during the fault:

$$\mathbf{V}_{bus}(F) = \mathbf{V}_{bus}(0) + \mathbf{Z}_{bus}\mathbf{I}_{bus}(F)$$

Fault Analysis Using Impedance Matrix

$$\begin{bmatrix} V_1(F) \\ \vdots \\ V_k(F) \\ \vdots \\ V_n(F) \end{bmatrix} = \begin{bmatrix} V_1(0) \\ \vdots \\ V_k(0) \\ \vdots \\ V_n(0) \end{bmatrix} + \begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{n1} & \cdots & Z_{nk} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix}$$

$$V_k(F) = V_k(0) - Z_{kk} I_k(F)$$

$$V_k(F) = Z_f I_k(F)$$

$$I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f}$$

Fault Analysis Using Impedance Matrix

- The bus voltage during the fault at bus *i* becomes

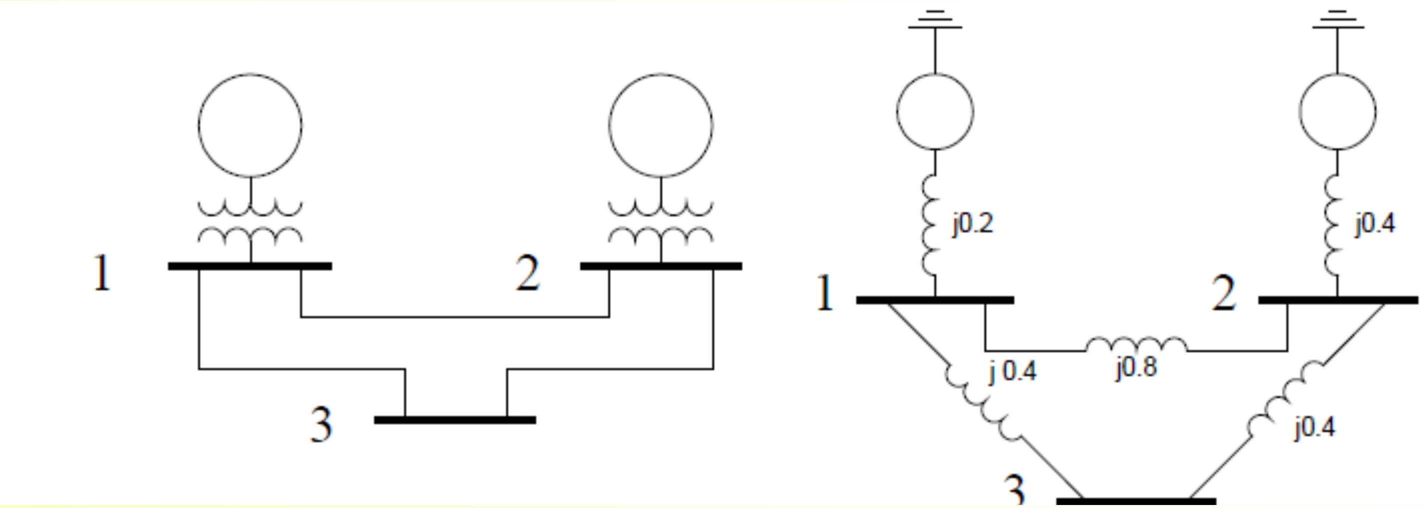
$$V_i(F) = V_i(0) - Z_{ik} I_k(F) \qquad I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f}$$

$$V_i(F) = V_i(0) - \frac{Z_{ik}}{Z_{kk} + Z_f} V_k(0)$$

- Note that for bolted fault $Z_f = 0 \text{ and } V_k(F) = 0$
- With the knowledge of bus voltages during the fault, we can calculate the current in all the lines. For the line connecting bus *i* and *j* with impedance z_{ij}, the short-circuit current in this line

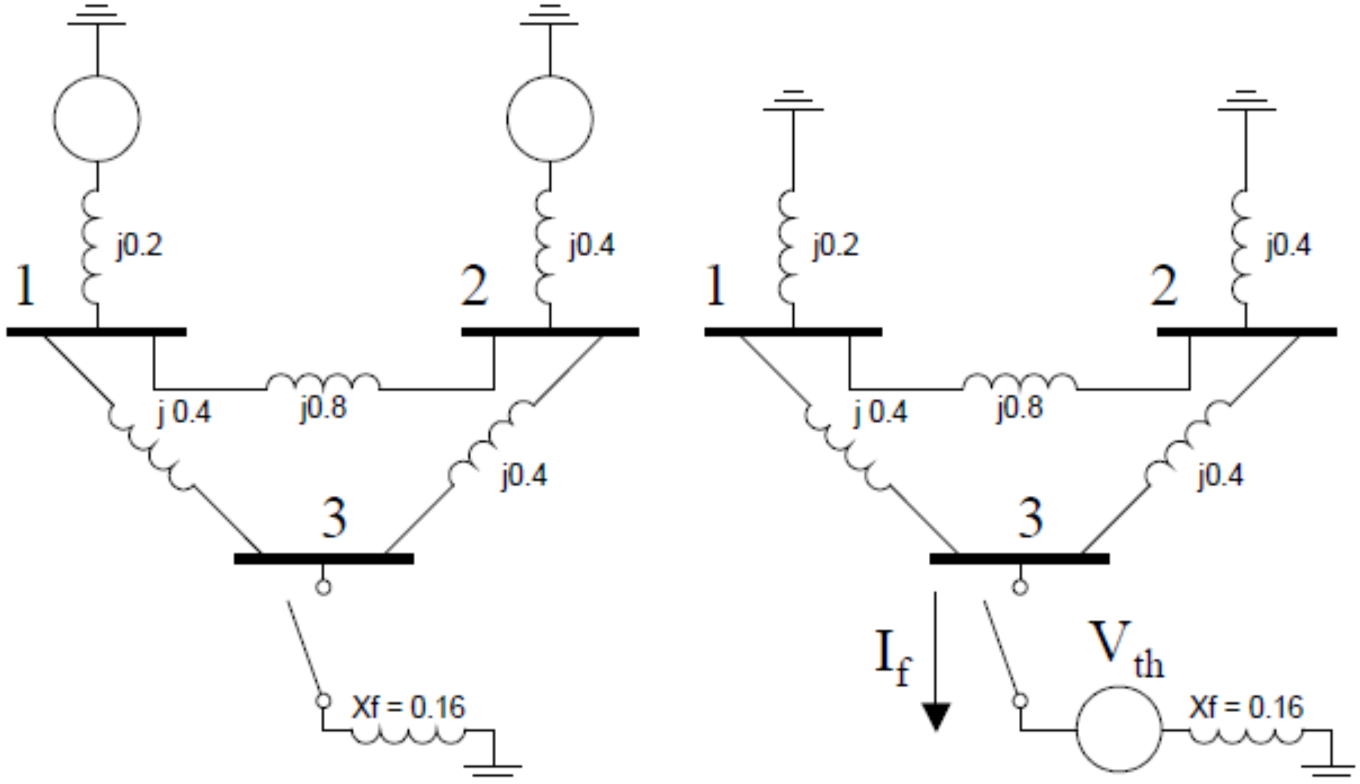
$$I_{ij}(F) = \frac{V_i(F) - V_j(F)}{z_{ij}}$$

Example 1

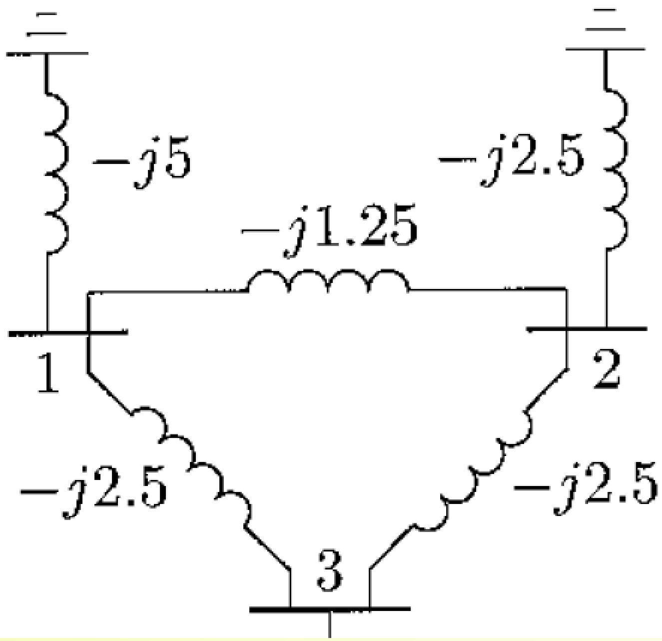


Example 1 (cont'd)

- **3-phase fault with Z_f = j0.16 on**
  - bus 3



Example 1 (cont'd)



$$Y_{bus} = \begin{bmatrix} -j8.75 & j1.25 & j2.50 \\ j1.25 & -j6.25 & j2.50 \\ j2.50 & j2.50 & -j5.00 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

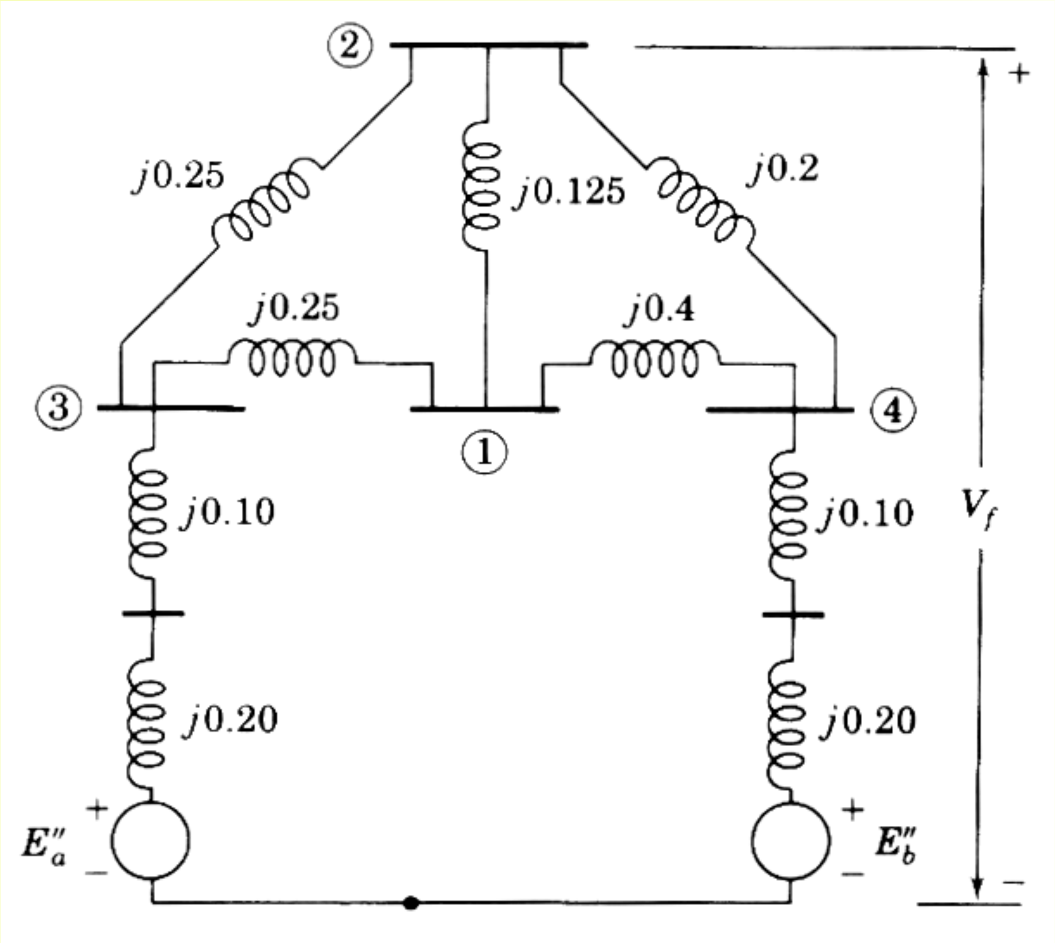
$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \quad \text{pu}$$

$$V_1(F) = V_1(0) - Z_{13} I_3(F) = 1.0 - (j0.12)(-j2.0) = 0.76 \quad \text{pu}$$
$$V_2(F) = V_2(0) - Z_{23} I_3(F) = 1.0 - (j0.16)(-j2.0) = 0.68 \quad \text{pu}$$
$$V_3(F) = V_3(0) - Z_{33} I_3(F) = 1.0 - (j0.34)(-j2.0) = 0.32 \quad \text{pu}$$

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \quad \text{pu}$$

Example 2

- A 3-phase fault occurs at bus 2



[Zbus]

$$j\begin{bmatrix} .2436 & .1938 & .1544 & .1456 \\ & .2295 & .1494 & .1506 \\ & & .1954 & .1046 \\ & & & .1954 \end{bmatrix}$$

$$V_f = 1.0\angle 0 \; p.u.$$

Example 2-Solution

- Fault current

$$I_f = V_f / Z_{22} = 1.0 / j.2295 = -j4.3573\ p.u.$$

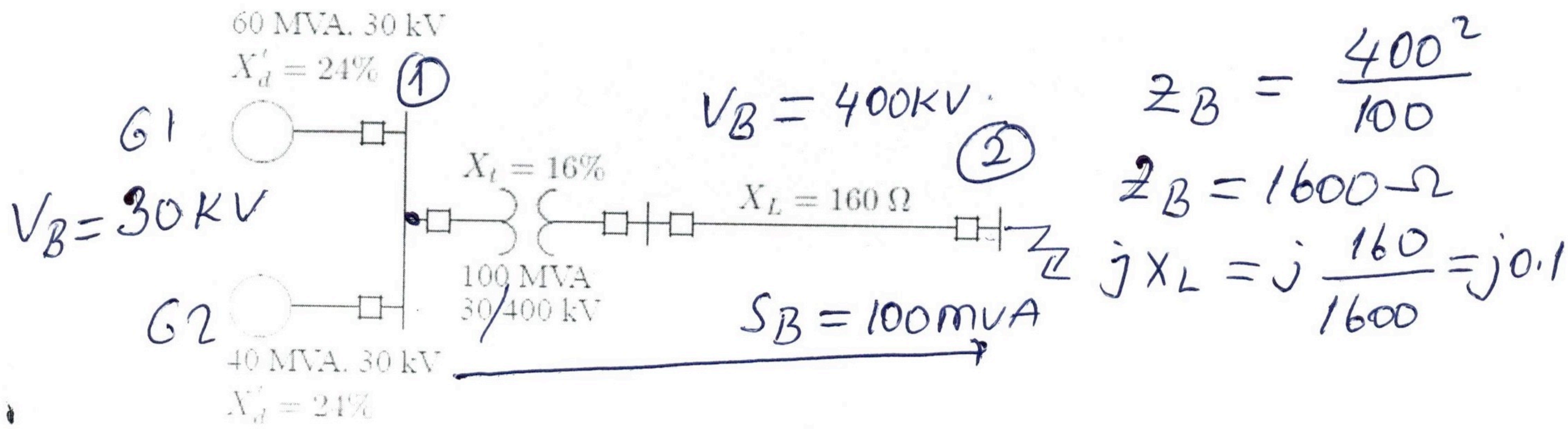
- Voltages during fault

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 - j0.1938 / j0.2295 \\ 0 \\ 1 - j0.1494 / j0.2295 \\ 1 - j0.1506 / j0.2295 \end{bmatrix} = \begin{bmatrix} 0.1556 \\ 0 \\ 0.3490 \\ 0.3438 \end{bmatrix}$$

- Fault currents contributed to bus 2
  - From bus 1 $I_{f1} = V_1 / Z_{b1} = 0.1556 / j0.125 = -j1.2448\ p.u.$
  - From bus 3 $I_{f3} = V_3 / Z_{b3} = 0.3490 / j0.25 = -j1.3960\ p.u.$
  - From bus 4 $I_{f4} = V_4 / Z_{b4} = 0.3438 / j0.20 = -j1.7190\ p.u.$

Example 3

3. A three-phase fault occurs at the receiving end of the transmission line. By ignoring the pre-fault loading conditions, find the fault current using Z-bus method. **(Choose V_B=30 kV of the generator voltage and S_B = 100 MVA) (25 pts)**



Example 3-Solution

$X_{G1} = j0.24\left(\frac{100}{60}\right) = j0.4\,pu$

$X_{G2} = j0.24\left(\frac{100}{40}\right) = j0.6\,pu$

$X_T = j0.16\,pu$

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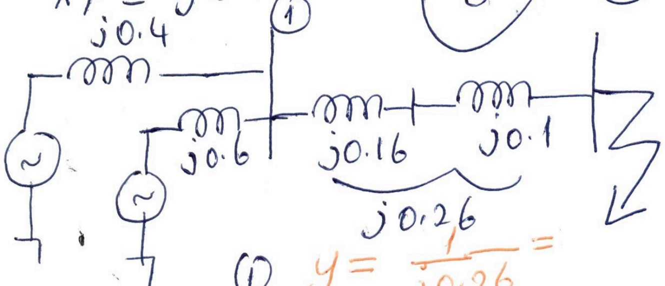
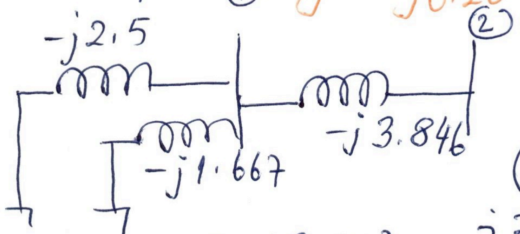


$y = \frac{1}{j0.26} =$



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$$Y_{BUS} = \begin{bmatrix} -j8.013 & j3.846 \\ j3.846 & -j3.846 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1}$$

$$|Y_{BUS}| = -30.818 + 14.792 = -16.026$$

5

$$Z_{BUS} = \frac{1}{-16.026}\begin{bmatrix} -j3.846 & j3.846 \\ -j3.846 & -j8.013 \end{bmatrix}$$

$$Z_{BUS} = \begin{bmatrix} j0.24 & j0.24 \\ j0.24 & j0.5 \end{bmatrix}$$

5

$Z_{22} = j0.5\,pu$

$$I_f^{3\phi} = \frac{1.0}{j0.5} = -j2.0\,pu$$

5

$$I_B = \frac{100 \times 10^6}{\sqrt{3}\,(400 \times 10^3)} = 144.34\,A$$

$$I_f^{3\phi} = (-j2.0pu)(144.34) = -j288.68\,A$$

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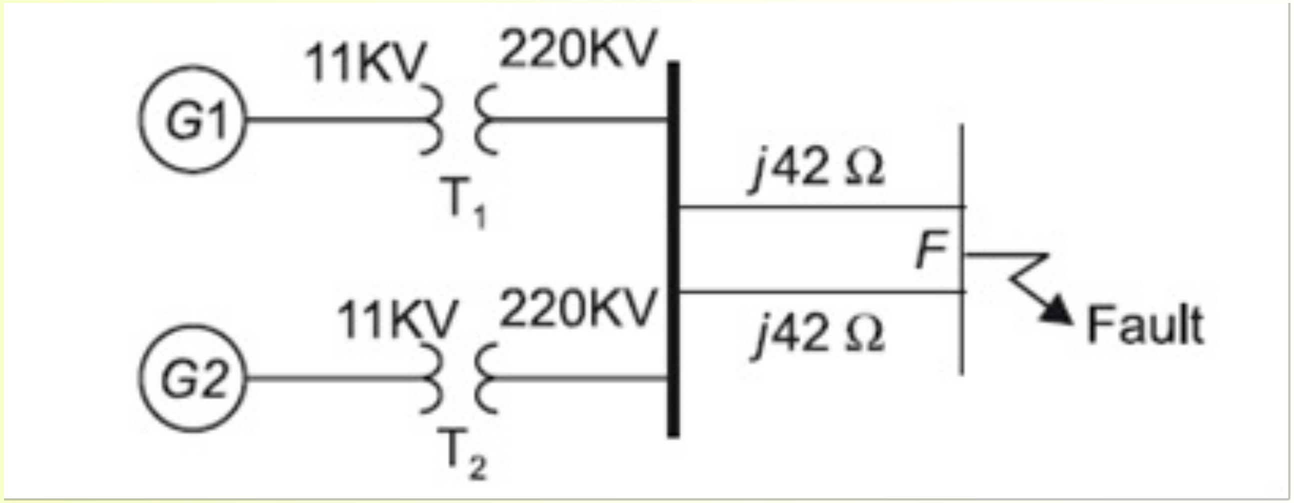
Example 4

The figure given below shows a generating station feeding a 220 kV system. Determine total fault current and fault current supplied by each generator for a three-phase fault at the receiving end of the line.



$G1$: 11 KV, 100 MVA, $x'_{g1} = j0.15$

$G2$: 11 KV, 75 MVA, $x'_{g2} = j0.125$

$T1$: 100 MVA, $x_{T1} = j0.10$, 11/220 KV

$T2$: 75 MVA, $x_{T2} = j0.08$, 11/220 KV

Example 4 Solution

Let $S_B = 100\,MVA$, $V_B = 11\,kV$

$X'_{g1} = j0.15\,pu$,

$X'_{g2} = j0.125\left(\frac{100}{75}\right) = j0.167\,pu$

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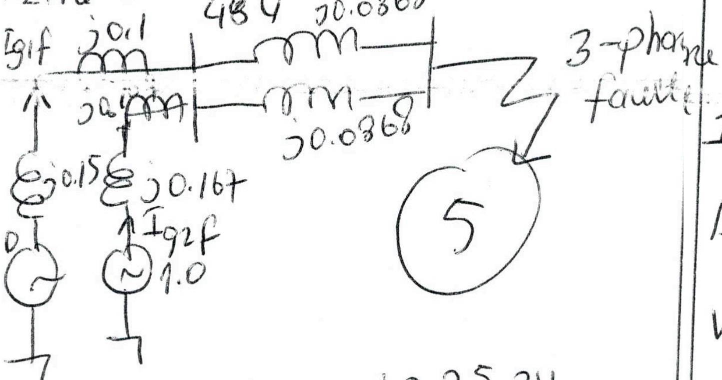
$X_{T1} = j0.10\,pu$

$X_{T2} = j0.08\left(\frac{100}{75}\right) = j0.107\,pu$

$Z_B = \frac{V_B^2}{S_B} = \frac{220^2}{100} = 484\,\Omega$

$X_{Line} = \frac{j42}{484} = j0.0868\,pu$



$X_1 = j0.1 + j0.15 = j0.25\,pu$

$y_1 = \frac{1}{X_1} = -j4\,pu$

$X_2 = j0.167 + j0.107 = j0.274\,pu$

$y_2 = \frac{1}{X_2} = -j3.65\,pu$

$X_3 = j0.0868 \,//\, j0.0868$

$= j0.0434\,pu$

$y_3 = \frac{1}{X_3} = -j23.041\,pu$

$Y_{Bus} = \begin{bmatrix} -j30.69 & j23.041 \\ j23.041 & -j23.041 \end{bmatrix}$ 5

$Z_{Bus} = Y_{Bus}^{-1} = \begin{bmatrix} j0.131 & j0.131 \\ j0.131 & j0.174 \end{bmatrix}$

The fault current:

$I_f = \frac{V_f}{Z_{22}} = \frac{1.0}{j0.174} = -j5.75\,pu$ 5

$I_B = \frac{100\times10^6}{\sqrt{3}\times220\times10^3} = 262.43\,A$.

$|I_f| = 5.75 \times 262.43 = 1508.98\,A$.

$V'_1 = 1 - \frac{Z_{12}}{Z_{22}} = 1 - \frac{j0.131}{j0.174} = 0.247$ 5

$V'_2 = 0$

$I_{g1f} = \frac{1-0.247}{j0.25} = -j3.012\,pu$

$I_{g2f} = \frac{1-0.247}{j0.274} = -j2.748\,pu$ 5

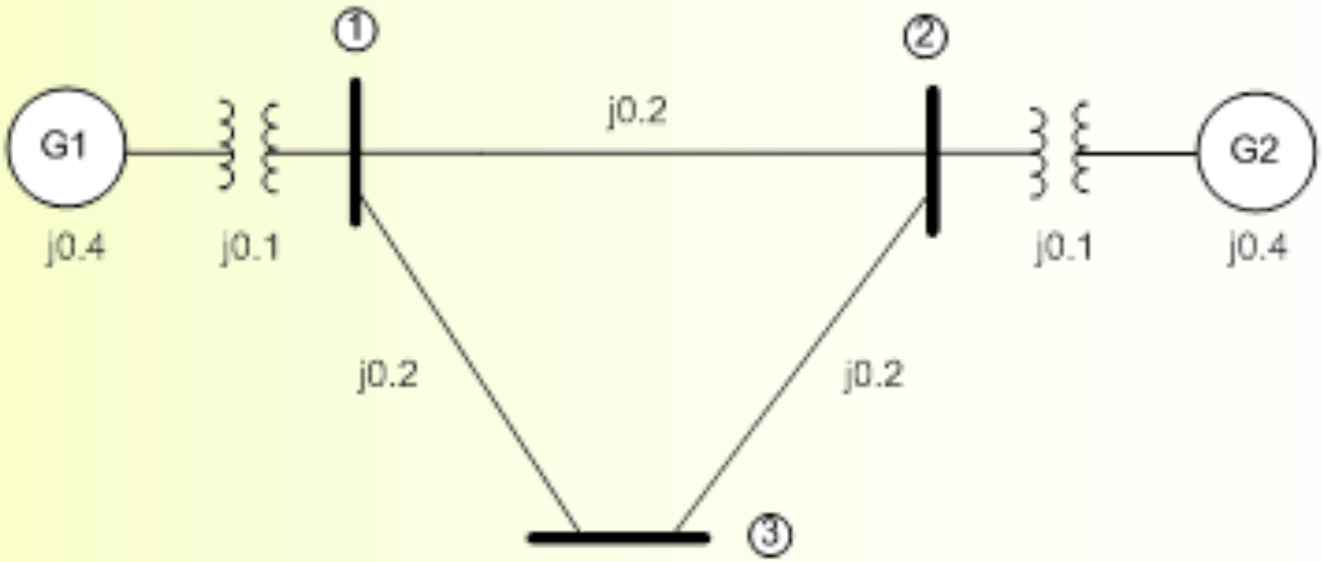
$I_B = \frac{100\times10^6}{\sqrt{3}\times11\times10^3} = 5248.6\,A$

$|I_{g2f}| = 14.423\,kA$

$|I_{g1f}| = 3.012 \times 5248 = 15.8\,kA$

Example 5

- For the three-bus system given below, a three-phase fault occurs at bus # 3. By ignoring the pre-fault loading conditions and using the Z-bus (bus impedance) method, compute the followings:
  - Fault current **(15)**
  - Bus voltages during the fault **(10)**
  - Fault currents of lines and generators**(10)**



Example 5-Solution

$$Y_B = \begin{bmatrix} -j12 & j5 & j5 \\ j5 & -j12 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

$$Z_B = Y_B^{-1} = \begin{bmatrix} j0.2793 & j0.2206 & j0.25 \\ j0.2206 & j0.2793 & j0.25 \\ j0.25 & j0.25 & j0.35 \end{bmatrix}$$

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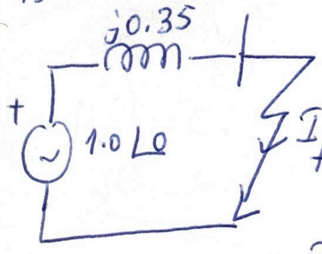
a) Fault current



$$I_f = \frac{1.0}{j0.35} = -j2.857 \text{ pu}$$

7

b) $V_1' = 1 - \frac{Z_{13}}{Z_{33}} = 1 - \frac{j0.25}{j0.35}$

$V_1' = 0.2857$ pu

$V_2' = 1 - \frac{Z_{22}}{Z_{33}} = 1 - \frac{j0.25}{j0.35}$

$V_2' = 0.2857$ pu

8

$V_3' = 0$ pu

c) Lines and generators fault currents:

Ignoring pre-fault loading condition:

$$I_{G1} = \frac{1.0 - 0.2857}{j0.5} = -j1.4286 \text{ pu}$$

$$I_{G2} = \frac{1.0 - 0.2857}{+j0.5} = -j1.4286 \text{ pu}$$

$$I_{12} = \frac{0.2857 - 0.2857}{j0.2} = 0 \text{ pu}$$

7

$$I_{13} = \frac{0.2857 - 0}{j0.2} = -j1.4285 \text{ pu}$$

$$I_{23} = \frac{0.2857 - 0}{j0.2} = -j1.4285 \text{ pu}$$

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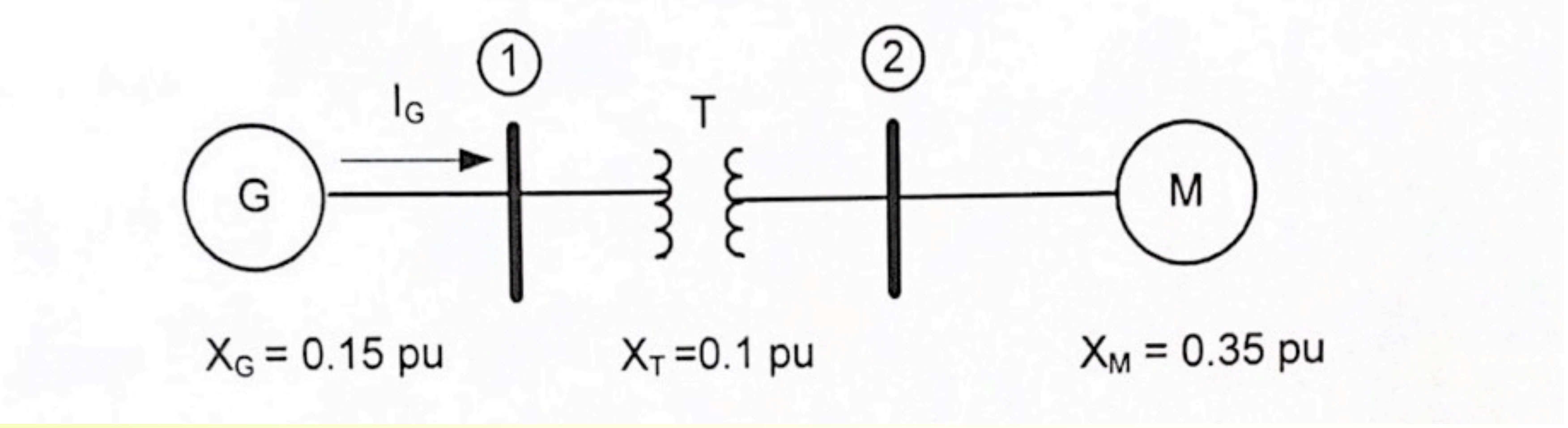
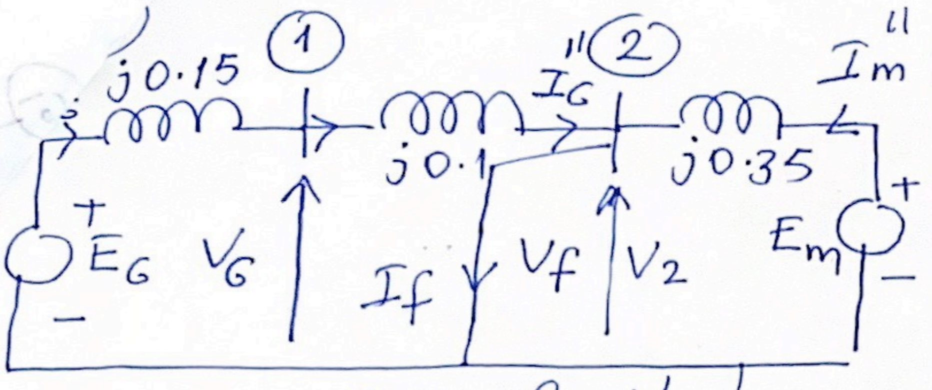
Example 6

The two-bus system below provides power to the motor when the voltage of the bus 1 is $V_1 = 0.9\angle 0^0$ pu and the generator current is $I_G = 1.0\angle 36.87^0$ pu. A three-phase fault occurs at bus 2. **Using the Z-bus matrix (bus impedance matrix)** find the fault current, the contribution of the generator and the motor to the fault current and bus voltages during the fault **(35 p)**



Example 6-Solution



pre-fault

Pre-fault load conditions:

$V_G = V_1 = 0.9\angle 0^\circ$

$I_G = 1.0\angle 36.87^\circ \text{ pu}$

The voltage at bus 2 before the fault:

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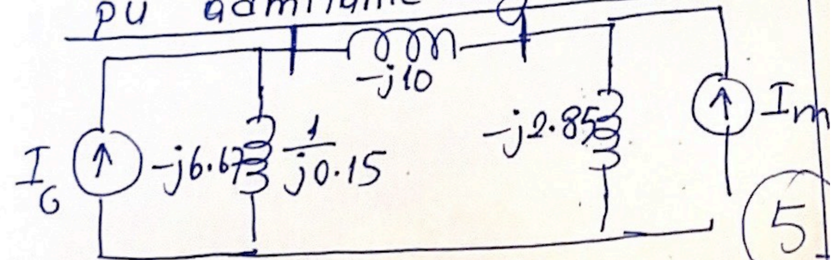
$V_2 = V_f = V_G - jX_T I_G$

$V_f = 0.9\angle 0^\circ - (j0.1)(1.0\angle 36.87^\circ)$

$\boxed{V_f = 0.963\angle -4.76^\circ \text{ pu}}$

pu admitance diagram



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Bus admitance matrix:

$$Y_{Bus} = \begin{bmatrix} -j16.67 & j10 \\ j10 & -j12.85 \end{bmatrix}$$

$$Z_{Bus} = [Y_{Bus}]^{-1} = \begin{bmatrix} j0.112 & j0.087 \\ j0.087 & j0.146 \end{bmatrix}$$

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Example 6-Solution

The fault current

$$I_f'' = \frac{V_f}{Z_{22}} = \frac{0.963\angle{-4.76^\circ}}{j0.146}$$

$$\boxed{I_f'' = 6.596\angle{-94.76^\circ}\ pu}$$

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Bus voltage during fault:

$$\boxed{V_2' = 0}$$

$$V_1' = V_1 - I_f'' Z_{12}$$

$$V_1' = 0.9\angle{0^\circ} - (6.596\angle{-94.76^\circ}).(j0.087)$$

$$\boxed{V_1' = 0.331\angle{8.25^\circ}\ pu}$$

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The contribution of the generator and the motor to the fault current:

$$I_G'' = \frac{V_1' - V_2'}{Z_{1-2}}$$

$$I_G'' = \frac{0.331\angle{8.25} - 0}{j0.1}$$

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$$\boxed{I_G'' = 3.31\angle{-81.75^\circ}\ pu}$$

5

$$I_f'' = I_G'' + I_m''$$

$$I_m'' = I_f'' - I_G''$$

$$I_m'' = 6.596\angle{-94.76^\circ} - 3.31\angle{-81.75^\circ}$$

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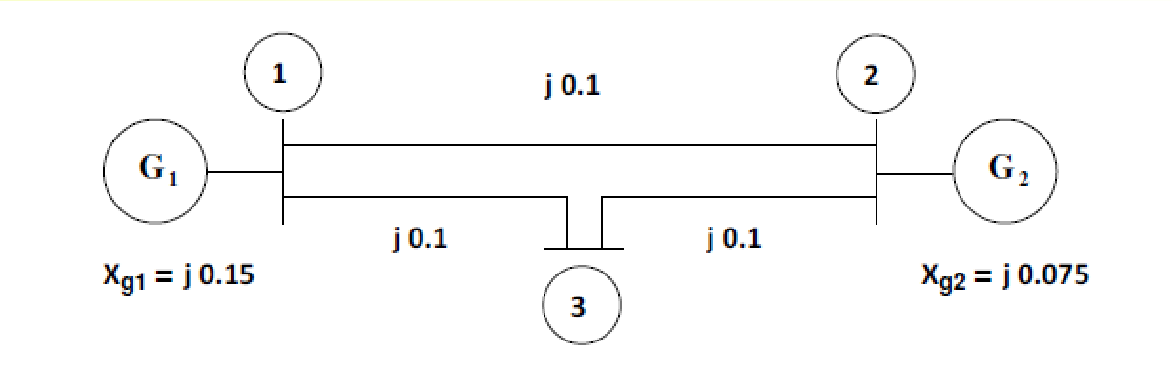
$$I_m'' = -1.022 - j3.298$$

$$I_m'' = 3.45\angle{-113.72^\circ}\ pu$$

Example 7



A three-bus power system is shown above. The p.u. impedances are on a base of 50 MVA and 12 kV. A three-phase symmetrical short circuit occurs at bus 3 with no fault impedance. Using Z-bus matrix method and ignoring pre-fault loading conditions:

a) Compute the short-circuit current (20)
b) Bus voltages during fault (5)
c) Contributions of generators to the short-circuit current (5)

Example 7-Solution

3)

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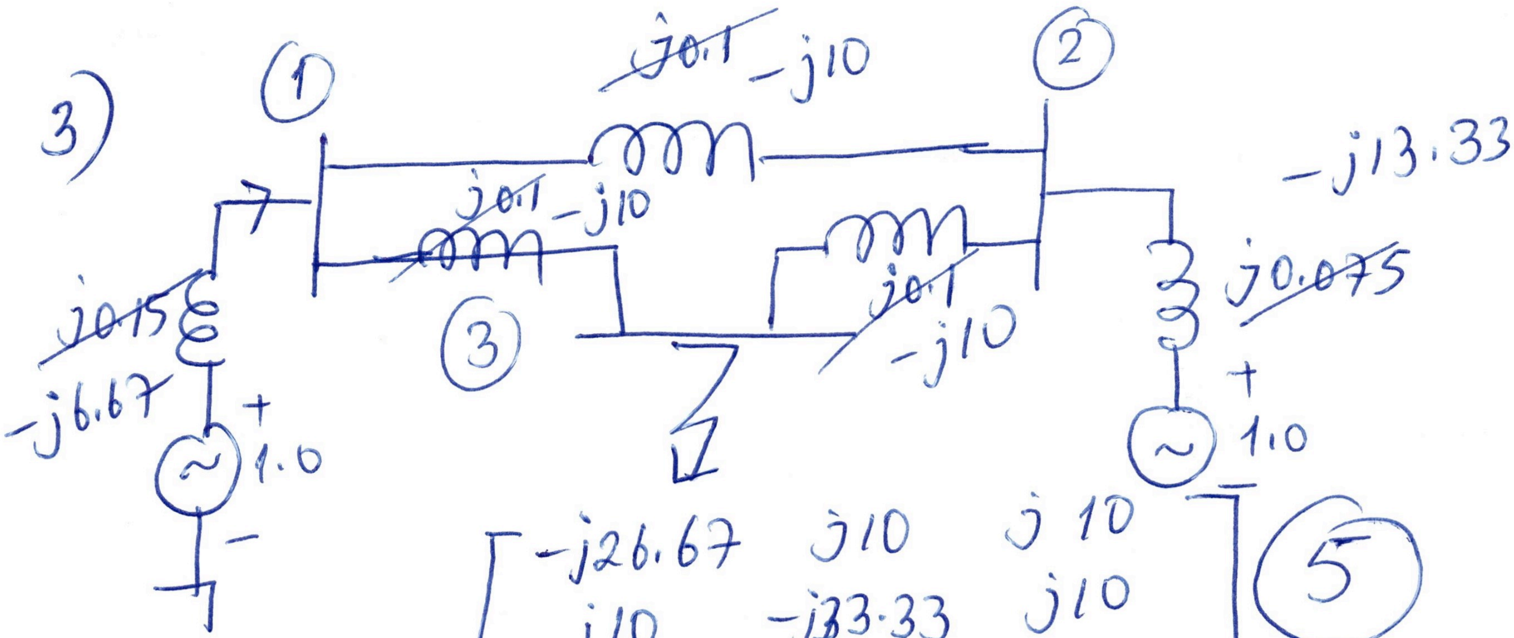

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a) $Y_{bus} = \begin{bmatrix} -j26.67 & j10 & j10 \\ j10 & -j33.33 & j10 \\ j10 & j10 & -j20 \end{bmatrix}$

$$Z_{bus} = [Y_{bus}]^{-1} = j\begin{bmatrix} 0.07286 & 0.03857 & 0.05571 \\ 0.03857 & 0.05571 & 0.04714 \\ 0.05571 & 0.04714 & 0.10143 \end{bmatrix}$$

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Example 7-Solution (cont'nd)

short circuit at bus 3 :

$$I_{3f} = \frac{1}{j0.10143} = -j9.86\ pu$$

Base current $$I_B = \frac{50\times10^6}{\sqrt{3}\,(12\times10^3)} = 2405.6\ A$$ 10

Fault current $I_{3f} = (-j9.86)(2405.6) = -j23.72\ kA$

b) $$V_1 = 1.0 - \frac{Z_{13}}{Z_{33}} = 1.0 - \frac{j0.05571}{j0.10143} = 0.451\ pu$$ 5

$$V_2 = 1.0 - \frac{Z_{23}}{Z_{33}} = 1.0 - \frac{j0.04714}{j0.10143} = 0.535\ pu$$

$V_3 = 0\ pu$

c) $$I_{G1} = \frac{1-0.451}{j0.15} = -j3.66\ pu\ \text{~~or~~}$$

$I_{G1} = -(j3.66)(2405.6) = -j8.804\ kA$ 5

$$I_{G2} = \frac{1-0.535}{j0.075} = -j6.2\ pu$$

$I_{G2} = (-j6.2)(2405.6) = -j14.91\ kA$