STEADY-STATE POWER ANALYSIS

LEARNING GOALS

Instantaneous Power
For the special case of steady state sinusoidal signals

Average Power
Power absorbed or supplied during one cycle

Maximum Average Power Transfer
When the circuit is in sinusoidal steady state

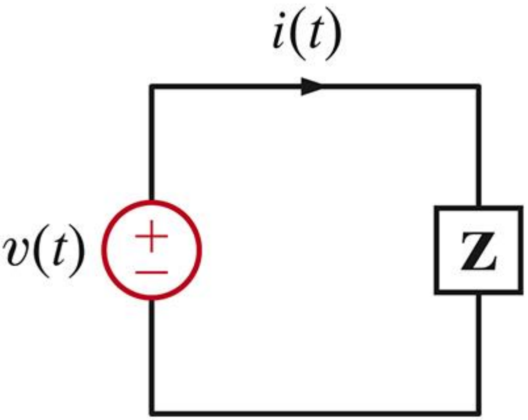
Effective or RMS Values
For the case of sinusoidal signals

Power Factor
A measure of the angle between current and voltage phasors

Power Factor Correction
How to improve power transfer to a load by “aligning” phasors

Single Phase Three-Wire Circuits
Typical distribution method for households and small loads

INSTANTANEOUS POWER



Instantaneous Power Supplied to Impedance

$$p(t) = v(t)i(t)$$

In steady State

$$v(t) = V_M \cos(\omega t + \theta_v)$$

$$i(t) = I_M \cos(\omega t + \theta_i)$$

$$p(t) = V_M I_M \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$\cos\phi_1 \cos\phi_2 = \frac{1}{2}[\cos(\phi_1 - \phi_2) + \cos(\phi_1 + \phi_2)]$$

$$p(t) = \frac{V_M I_M}{2}[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)]$$

constant

Twice the frequency

EXAMPLE 1

Assume: $v(t) = 4\cos(\omega t + 60°)$,

$$Z = 2\angle 30° \Omega$$

Find: $i(t), p(t)$

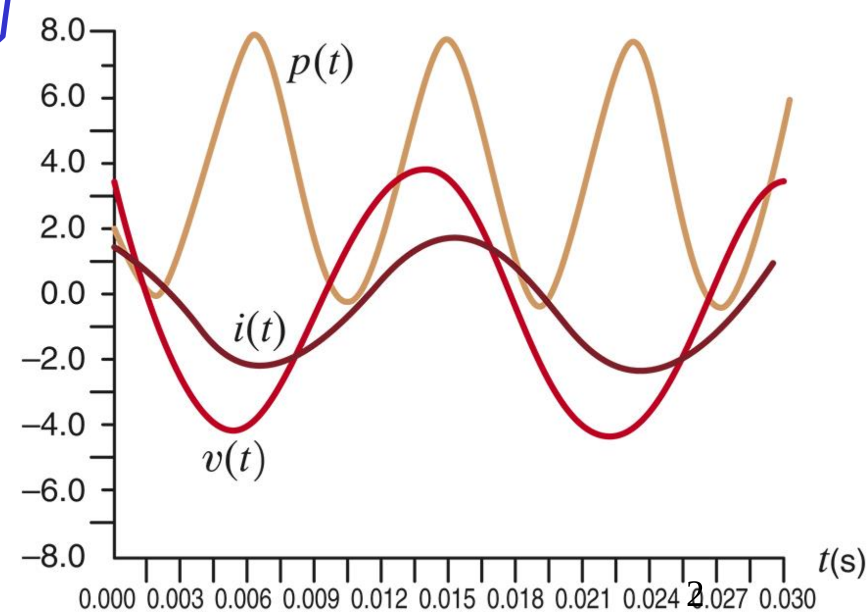
$$I = \frac{V}{Z} = \frac{4\angle 60°}{2\angle 30°} = 2\angle 30° (A)$$

$$i(t) = 2\cos(\omega t + 30°)(A)$$

$$V_M = 4, \theta_v = 60°$$

$$I_M = 2, \theta_i = 30°$$

$$p(t) = 4\cos 30° + 4\cos(2\omega t + 90°)$$



AVERAGE POWER

For sinusoidal (and other periodic signals) we compute averages over one period

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt \qquad T = \frac{2\pi}{\omega}$$

$$p(t) = \frac{V_M I_M}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right]$$

$$P = \frac{V_M I_M}{2}\cos(\theta_v - \theta_i)$$

It does not matter who leads

If voltage and current are in phase

$$\theta_v = \theta_i \Rightarrow P = \frac{1}{2}V_M I_M$$

Purely resistive

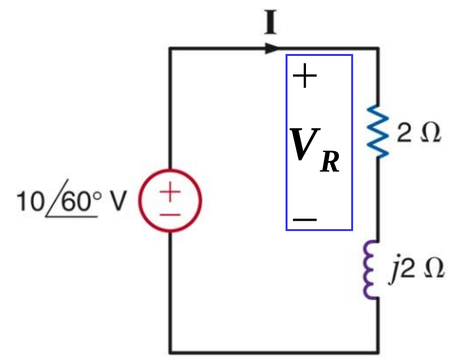
If voltage and current are in quadrature

$$\theta_v - \theta_i = \pm 90° \Rightarrow P = 0$$

Purely inductive or capacitive

EXAMPLE 2

Find the average power absorbed by impedance



$$I = \frac{10\angle 60°}{2 + j2} = \frac{10\angle 60°}{2\sqrt{2}\angle 45°} = 3.53\angle 15° (A)$$

$$V_M = 10, I_M = 3.53, \theta_v = 60°, \theta_i = 15°$$

$$P = \frac{1}{2}\left(35.3\cos(45°)\right) = 12.5W$$

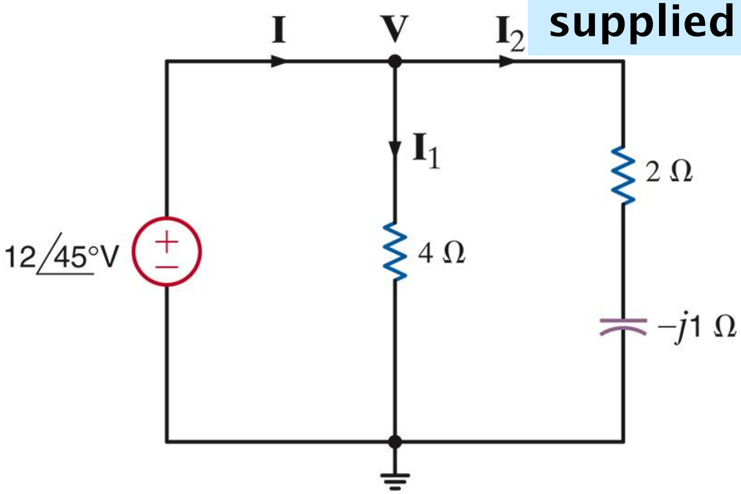
Since inductor does not absorb power one can use voltages and currents across the resistive part

$$V_R = \frac{2}{2 + j2}10\angle 60° = 7.06\angle 15° (V)$$

$$P = \frac{1}{2}7.06 \times 3.53 = 12.5W$$

EXAMPLE 3

Determine the average power absorbed by each resistor, the total average power absorbed and the average power supplied by the source



Inductors and capacitors do not absorb power in the average

$$P_{total} = 18 + 28.7W$$

$$P_{\text{supplied}} = P_{\text{absorbed}} \Rightarrow \quad P_{\text{supplied}} = 46.7W$$

Verification

$$I = I_1 + I_2 = 3\angle 45° + 5.36\angle 71.57°$$

$$I = 8.15\angle 62.10°(A)$$

$$P = \frac{V_M I_M}{2}\cos(\theta_v - \theta_i)$$

$$P_{\text{supplied}} = \frac{1}{2} 12 \times 8.15 \times \cos(45° - 62.10°)$$

$$P_{\text{supplied}} = 46.7W$$

If voltage and current are in phase

$$\theta_v = \theta_i \Rightarrow P = \frac{1}{2} V_M I_M = \frac{1}{2} R I_{1M}^2 = \frac{1}{2} \frac{V_M^2}{R}$$

$$I_1 = \frac{12\angle 45°}{4} = 3\angle 45°(A)$$

$$P_{4\Omega} = \frac{1}{2} 12 \times 3 = 18W$$

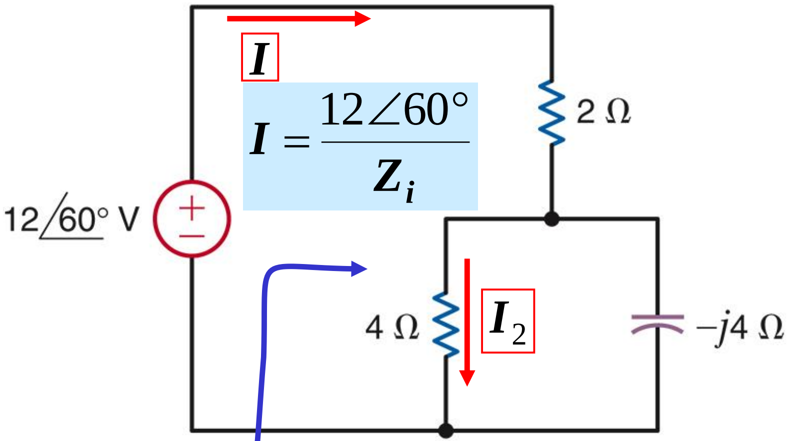
$$I_2 = \frac{12\angle 45°}{2 - j1} = \frac{12\angle 45°}{\sqrt{5}\angle -26.57°} = 5.36\angle 71.57°(A)$$

$$P_{2\Omega} = \frac{1}{2} \times 2 \times 5.36^2 = 28.7W$$

$$P_{2\Omega} = \frac{1}{2} \times 12 \times 5.36 \cos(45^0 - 71.57^0) = 28.7W$$

EXAMPLE 4

Find average power absorbed by each resistor



$$\boldsymbol{I} = \frac{12\angle 60^\circ}{\boldsymbol{Z_i}}$$

$$\boldsymbol{Z_i} = 2 + (4 \| -\boldsymbol{j}4)$$

$$\boldsymbol{Z_i} = 2 + \frac{4(-\boldsymbol{j}4)}{4 - \boldsymbol{j}4} = \frac{8 - \boldsymbol{j}8 - \boldsymbol{j}16}{4 - \boldsymbol{j}4} = \frac{25.3\angle -71.6^\circ}{4\sqrt{2}\angle -45^\circ}$$

$$\boldsymbol{Z_i} = 4.47\angle -26.6^\circ \Omega$$

$$\boldsymbol{I} = \frac{12\angle 60^\circ}{4.47\angle -26.6^\circ} = 2.68\angle 86.6^\circ (\boldsymbol{A})$$

$$\boldsymbol{P}_{2\Omega} = \frac{1}{2}\boldsymbol{R}\boldsymbol{I}_{\boldsymbol{M}}^2 = \frac{1}{2} \times 2 \times 2.68^2 = 7.20\boldsymbol{W}$$

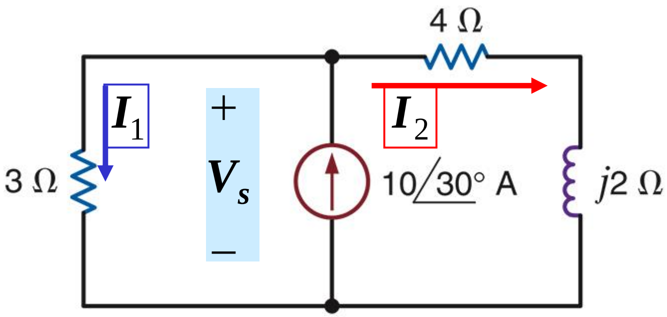
$$\boldsymbol{I}_2 = \frac{-\boldsymbol{j}4}{4 - \boldsymbol{j}4}\boldsymbol{I} = \frac{4\angle -90^\circ}{4\sqrt{2}\angle -45^\circ} \times 2.68\angle 86.6^\circ$$

$$\boldsymbol{I}_2 = 1.90\angle 41.6^\circ$$

$$\boldsymbol{P}_{4\Omega} = \frac{1}{2} \times 4 \times 1.90^2 = 7.22(\boldsymbol{W})$$

EXAMPLE 5

Find the AVERAGE power absorbed by each PASSIVE component and the total power supplied by the source



$$I_1 = \frac{4+j2}{3+4+j2} 10\angle 30°$$

$$I_1 = \frac{4.47\angle 26.57°}{7.28\angle 15.95°} 10\angle 30° = 6.14\angle 40.62° (A)$$

$$P_{3\Omega} = \frac{1}{2} R I_M^2 = \frac{1}{2} \times 3 \times 6.14^2 = 56.55 (W)$$

$$I_2 = 10\angle 30° - 6.14\angle 40.62°$$

$$I_2 = \frac{3}{3+4+j2} 10\angle 30° = \frac{30\angle 30°}{7.28\angle 15.95°}$$
$$= 4.12\angle 14.05° (A)$$

$$P_{4\Omega} = \frac{1}{2} \times 4 \times 4.12^2 = 33.95 (W)$$

$$P_{j2\Omega} = 0 (W)$$

Power supplied by source

Method 1. $P_{\text{supplied}} = P_{\text{absorbed}}$

$$P_{\text{supplied}} = P_{3\Omega} + P_{4\Omega} = 90.50W$$

Method 2: $P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$

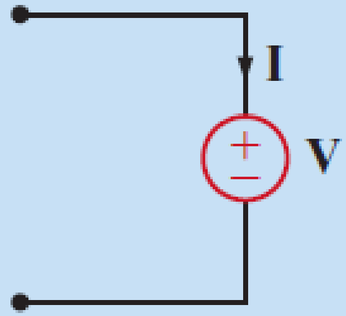
$$V_s = 3 I_1 = 18.42\angle 40.62°$$

$$P = \frac{1}{2} \times 18.42 \times 10 \times \cos(40.62° - 30°) = 90.5W$$

PASSIVE SIGN CONVENTION

- At this point an obvious question arises: how do we know whether the source is supplying power to the remainder of the network or absorbing it?
- If we employ the passive sign convention adopted in the earlier—that is, if the current reference direction enters the positive terminal of the source and the answer is positive—the source is absorbing power. If the answer is negative, the source is supplying power to the remainder of the circuit.

Under the following condition:

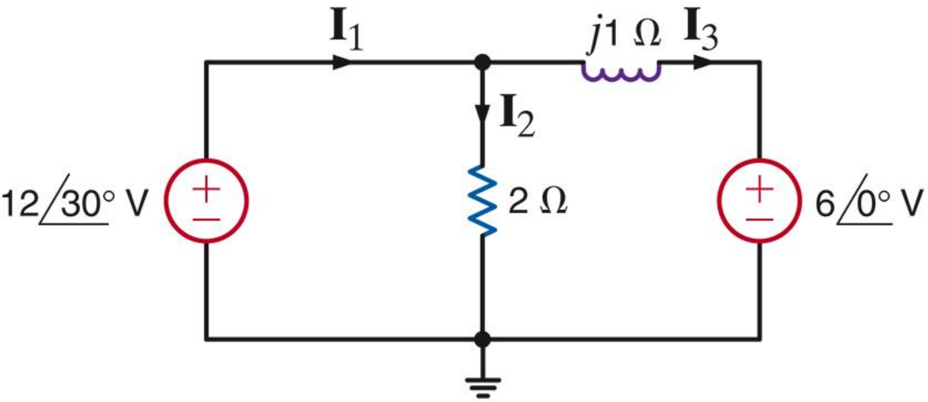


if $P = IV$ is positive, power is being absorbed.

If $P = IV$ is negative, power is being generated.

EXAMPLE 6

Determine average power absorbed or supplied by each element



$$I_3 = \frac{12\angle 30^\circ - 6\angle 0^\circ}{j1} = \frac{10.39 + j6 - 6}{j} = 6 - j4.39$$
$$= 7.43\angle -36.19^\circ (A)$$

$$P_{6\angle 0^\circ} = \frac{1}{2} \times 6 \times 7.43 \cos(0 + 36.19^\circ) = 18W$$

Passive sign convention

$$I_2 = \frac{12\angle 30^\circ}{2} = 6\angle 30^\circ (A)$$

$$I_1 = I_2 + I_3 = 5.20 + j3 + 6 - j4.39 = 11.2 - j1.39(A$$
$$= 11.28\angle -7.07^\circ$$

$$P_{2\Omega} = \frac{1}{2} R I_M^2 = \frac{1}{2} \times 2 \times 6^2 = 36(W)$$

$$P_{12\angle 30^\circ} = \frac{1}{2} \times 12 \times 11.28 \times \cos(30^\circ + 7.07^\circ)$$
$$= -54(W) \qquad = 36 + 18$$

$$P_{j1\Omega} = 0$$

To determine power absorbed/supplied by sources we need the currents I1, I2

Average Power

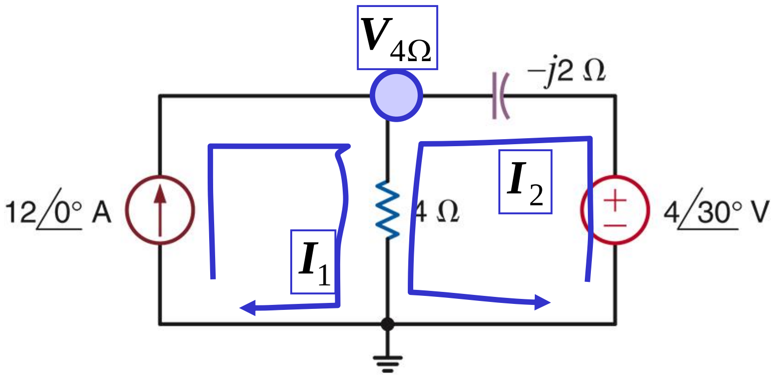
$$P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$$

For resistors

$$P = \frac{1}{2} R I_M^2 = \frac{1}{2} \frac{V_M^2}{R}$$

EXAMPLE 7

Determine average power absorbed/supplied by each element



$$P_{12\angle 0°} = -\frac{1}{2} \times 19.92 \times 12 \times \cos(-54.5° - 0°) = -69.4(W)$$

$$P_{4\angle 30°} = -\frac{1}{2} \times 4 \times (9.97)\cos(30° - 204°) = 19.8(W)$$

Check: Power supplied =power absorbed

Loop Equations

$$I_1 = 12\angle 0°$$

$$4\angle 30° = -j2I_2 + 4(I_2 + 12\angle 0°)$$

$$I_2 = \frac{4\angle 30° - 48\angle 0°}{4 - j2} = \frac{3.46 + j2 - 48}{4.47\angle -26.57°}$$

$$I_2 = \frac{44.58\angle 177.43°}{4.47\angle -26.57°} = 9.97\angle 204°(A)$$

$$V_{4\Omega} = 4(I_1 + I_2) = 4(12 + 9.97\angle 204°)(V)$$
$$= 4(12 - 9.108 - j4.055)(V) = 19.92\angle -54.5°(V)$$

$$P_{4\Omega} = \frac{1}{2}\frac{V_M^2}{R} = \frac{1}{2} \times \frac{19.92^2}{4} = 49.6W$$

$$P_{-2j\Omega} = 0(W)$$

Alternative Procedure

Node Equations

$$-12\angle 0° + \frac{V_{4\Omega}}{4} + \frac{V_{4\Omega} - 4\angle 30°}{-j2} = 0$$

$$I_2 = \frac{4\angle 30° - V_{4\Omega}}{-2j}$$

Average Power

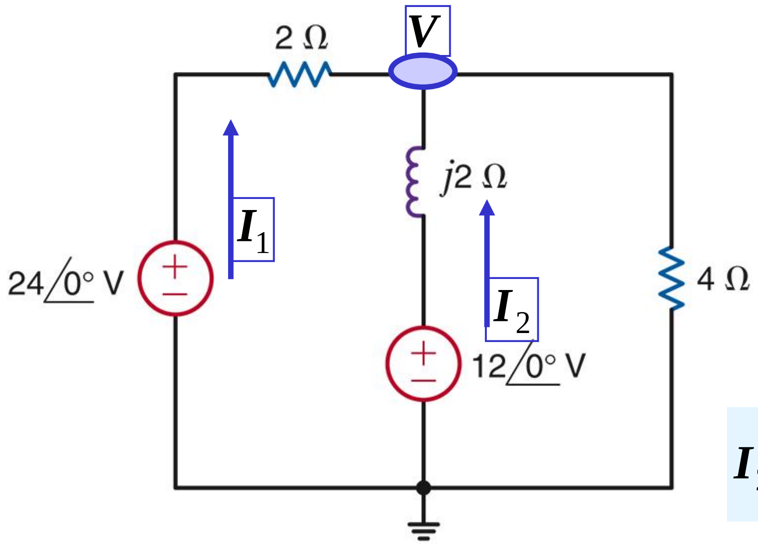
$$P = \frac{1}{2}V_M I_M \cos(\theta_v - \theta_i)$$

For resistors

$$P = \frac{1}{2}RI_M^2 = \frac{1}{2}\frac{V_M^2}{R}$$

EXAMPLE 8

Determine average power absorbed/supplied by each element



$$I_1 = \frac{24\angle 0° - V}{2} = \frac{24 - 14.77 - j1.85}{2} = 4.62 - j0.925$$

$$I_1 = 4.71\angle -11.32°(A)$$

$$I_2 = \frac{12\angle 0° - V}{j2} = \frac{12 - 14.77 + j1.85}{j2} \times \frac{-j}{-j}$$

$$I_2 = \frac{-1.85 + j2.77}{2} = -0.925 + j1.385(A) = 1.67\angle 123.73°(A)$$

Node Equation

$$\frac{V - 24\angle 0°}{2} + \frac{V - 12\angle 0°}{j2} + \frac{V}{4} = 0 \quad \times j4$$

$$2j(V - 24) + 2(V - 12) + jV = 0$$

$$(2 + 3j)V = 24 + j48$$

$$V = \frac{24 + j48}{2 + j3} \times \frac{2 - j3}{2 - j3} = \frac{192 + j24}{13}$$

$$= 14.88\angle 7.125°(V)$$

$$= 14.77 + j1.85(V)$$

$$P_{2\Omega} = \frac{1}{2} \times 2 \times 4.71^2 = 22.18(W)$$

$$P_{4\Omega} = \frac{1}{2} \times \frac{14.88^2}{4} = 27.67(W)$$

For resistors

$$P = \frac{1}{2} R I_M^2 = \frac{1}{2} \frac{V_M^2}{R}$$

$$P_{12\angle 0°} = -\frac{1}{2} \times 12 \times 1.67 \cos(0° - 123.73°) = 5.565(W)$$

$$P_{24\angle 0°} = -\frac{1}{2} \times 24 \times 4.71 \times \cos(0° + 11.32°) = -55.42(W)$$

Average Power

$$P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$$

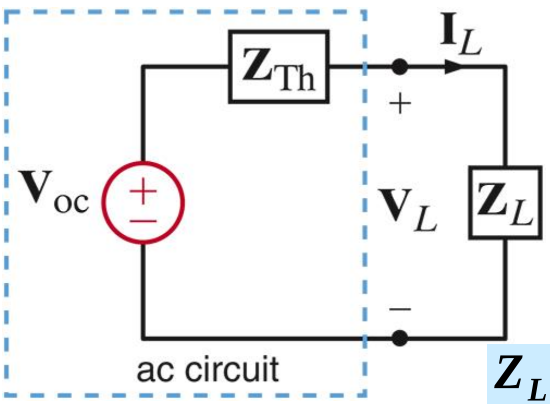
Check :

$$P_{\text{absorbed}} = 22.18 + 27.67 + 5.565(W)$$

$$P_{\text{supplied}} = 55.42(W)$$

MAXIMUM AVERAGE POWER TRANSFER

$$Z_{TH} = R_{TH} + jX_{TH}$$



$$Z_L = R_L + jX_L$$

$$P_L = \frac{1}{2} V_{LM} I_{LM} \cos(\theta_{V_L} - \theta_{I_L})$$

$$= \frac{1}{2} |V_L| |I_L| \cos(\theta_{V_L} - \theta_{I_L})$$

$$V_L = \frac{Z_L}{Z_L + Z_{TH}} V_{OC} \quad \Rightarrow |V_L| = \left| \frac{Z_L}{Z_L + Z_{TH}} \right| |V_{OC}|$$

$$I_L = \frac{V_L}{Z_L} \quad \Rightarrow \angle I_L = \angle V_L - \angle Z_L \quad \Rightarrow |I_L| = \frac{|V_L|}{|Z_L|}$$

$$\Rightarrow \theta_{V_L} - \theta_{I_L} = \angle Z_L$$

$$Z_L = R_L + jX_L \quad \Rightarrow \tan(\angle Z_L) = \frac{X_L}{R_L}$$

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}} \quad \therefore \cos(\theta_{V_L} - \theta_{I_L}) = \frac{R_L}{\sqrt{R_L^2 + X_L^2}}$$

$$P_L = \frac{1}{2} \frac{\cancel{|Z_L|} |V_{OC}|^2}{|Z_L + Z_{TH}|^2} \frac{R_L}{\cancel{\sqrt{R_L^2 + X_L^2}}}$$

$$Z_L + Z_{TH} = (R_L + R_{TH}) + j(X_L + X_{TH})$$

$$|Z_L + Z_{TH}|^2 = (R_L + R_{TH})^2 + (X_L + X_{TH})^2$$

$$P_L = \frac{1}{2} \frac{|V_{OC}|^2 R_L}{(R_L + R_{TH})^2 + (X_L + X_{TH})^2}$$

$$\left.\begin{array}{l} \dfrac{\partial P_L}{\partial X_L} = 0 \\ \dfrac{\partial P_L}{\partial R_L} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} X_L = -X_{TH} \\ R_L = R_{TH} \end{array}\right.$$

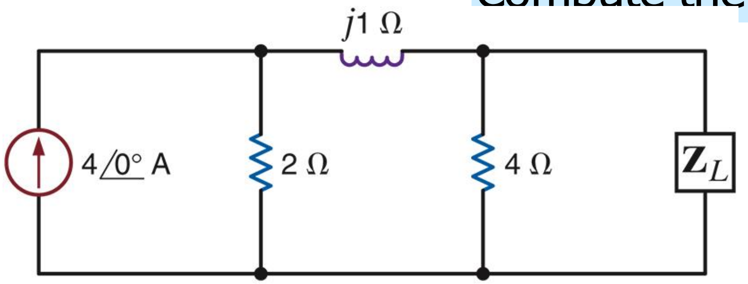
$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$

EXAMPLE 1

Find Z_L for maximum average power transfer.

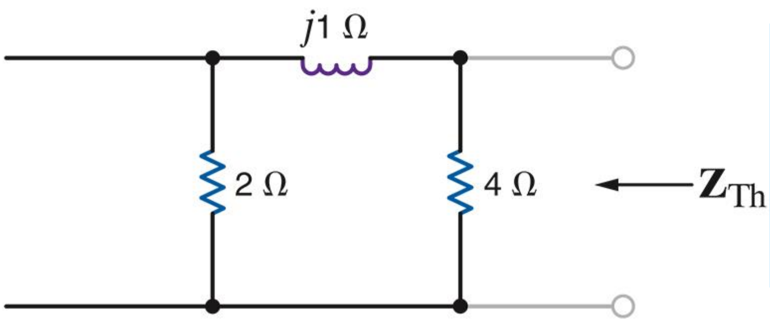
Compute the maximum average power supplied to the load



$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$

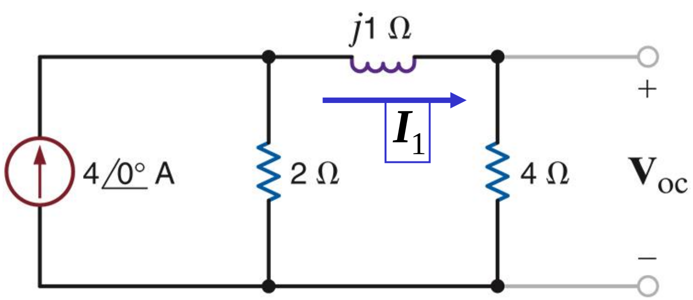
Remove the load and determine the Thevenin equivalent of remaining circuit



$$Z_{TH} = 4 \| (2+j1) = \frac{8+j4}{6+j1} = \frac{(8+j4)(6-j1)}{37} = \frac{52+j16}{37}\Omega$$

$$= \frac{8+j4}{6+j1} = \frac{8.94\angle 26.57^\circ}{6.08\angle 9.64^\circ} = 1.47\angle 16.93^\circ \Omega$$

$$Z_L^* = 1.47\angle -16.93^\circ = 1.41 - j0.43\Omega$$



$$V_{OC} = 4\times\frac{2}{6+j1}4\angle 0^\circ = \frac{32\angle 0^\circ}{6.08\angle 9.64^\circ} = 5.26\angle -9.64^\circ$$

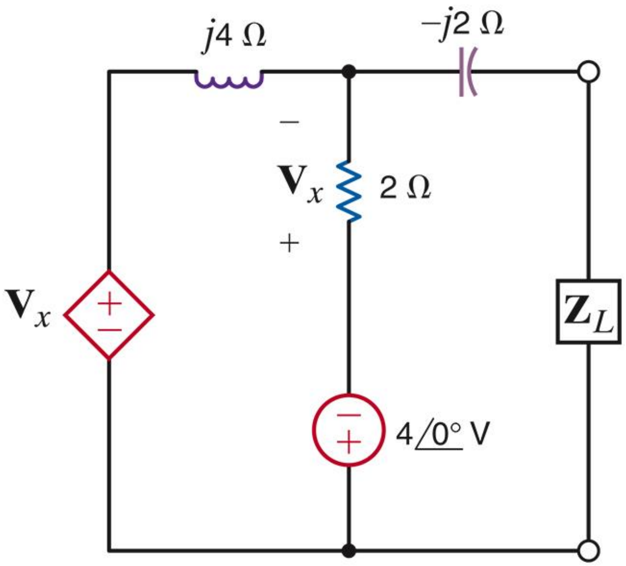
$$P_L^{\max} = \frac{1}{2}\times\frac{5.26^2}{4\times 1.41} = 2.45(W)$$

We are asked for the value of the power. We need the Thevenin voltage

EXAMPLE 2

Find Z_L for maximum average power transfer.

Compute the maximum average power supplied to the load

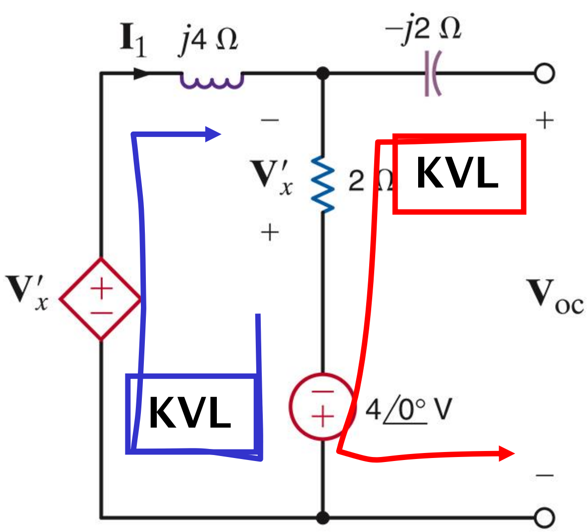


$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$

Circuit with dependent sources!

$$Z_{TH} = \frac{V_{OC}}{I_{SC}}$$



$$4\angle 0° = -V_x' + (2 + j4)I_1$$

$$V_X' = -2I_1$$

$$4\angle 0° = (4 + j4)I_1 = (4\sqrt{2}\angle 45°)I_1$$

$$I_1 = \frac{4\angle 0°}{4\sqrt{2}\angle 45°} = 0.707\angle -45°(A)$$

$$V_{OC} = 2I_1 - 4\angle 0° = 1 - j1 - 4 = -3 - j1 = \sqrt{10}\angle -161.5°$$

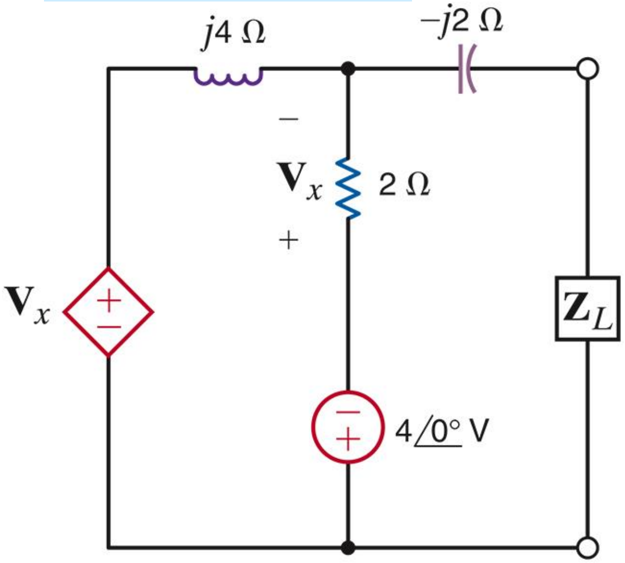
Next: the short circuit current …

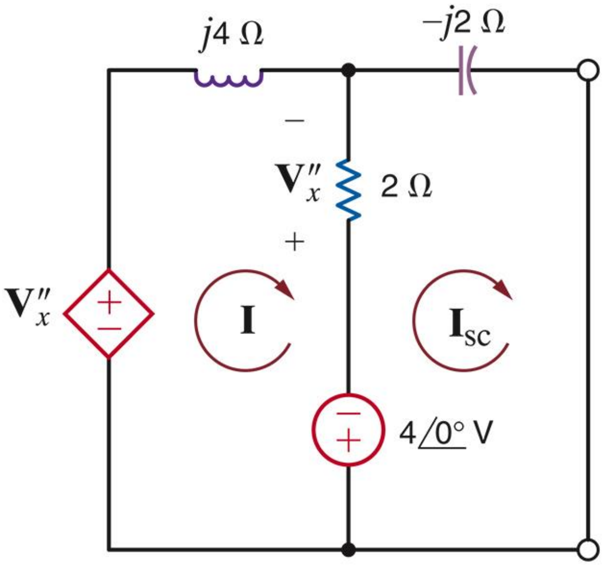
EXAMPLE 2 (continued)...

$$\therefore \boldsymbol{Z}_L^{opt} = \boldsymbol{Z}_{TH}^*$$

$$\boldsymbol{P}_L^{\max} = \frac{1}{2}\left(\frac{|\boldsymbol{V}_{OC}|^2}{4\boldsymbol{R}_{TH}}\right)$$

Original circuit





LOOP EQUATIONS FOR SHORT CIRCUIT CURRENT

$$-\boldsymbol{V}_x'' + \boldsymbol{j}4\boldsymbol{I} + 2(\boldsymbol{I} - \boldsymbol{I}_{SC}) - 4\angle° = 0$$

$$4\angle 0° + 2(\boldsymbol{I}_{SC} - \boldsymbol{I}) - \boldsymbol{j}2\boldsymbol{I}_{SC} = 0$$

CONTROLLING VARIABLE

$$\boldsymbol{V}_x'' = 2(\boldsymbol{I}_{SC} - \boldsymbol{I})$$

Substitute and rearrange

$$(4+\boldsymbol{j}4)\boldsymbol{I} - 4\boldsymbol{I}_{SC} = 4$$

$$-2\boldsymbol{I} + (2-\boldsymbol{j}2)\boldsymbol{I}_{SC} = -4 \Rightarrow \boldsymbol{I} = (1-\boldsymbol{j}1)\boldsymbol{I}_{SC} + 2$$

$$4(1+\boldsymbol{j})[(1-\boldsymbol{j})\boldsymbol{I}_{SC} + 2] - 4\boldsymbol{I}_{SC} = 4$$

$$\boldsymbol{I}_{SC} = -1 - \boldsymbol{j}2(\boldsymbol{A}) = \sqrt{5}\angle -116.57°$$

$$\boldsymbol{V}_{OC} = 2\boldsymbol{I}_1 - 4\angle 0° = 1 - \boldsymbol{j}1 - 4 = -3 - \boldsymbol{j}1 = \sqrt{10}\angle -161.57°$$

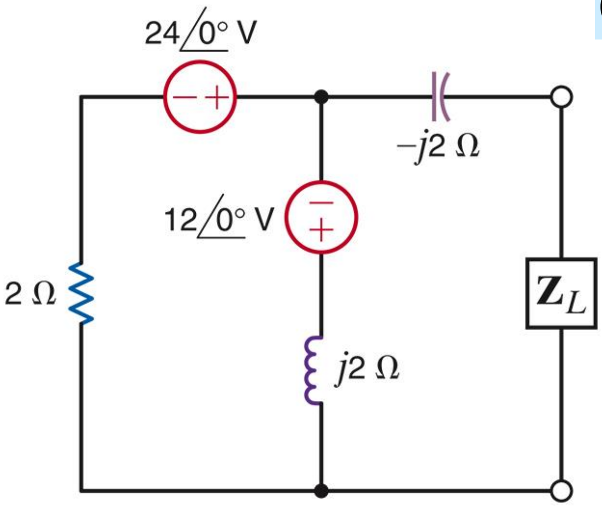
$$\boldsymbol{Z}_{TH} = \frac{\sqrt{10}\angle -161.57°}{\sqrt{5}\angle -116.57°} = \sqrt{2}\angle -45° = 1 - \boldsymbol{j}1\Omega \qquad \Rightarrow \boldsymbol{Z}_L^{opt} = 1 + \boldsymbol{j}1\Omega$$

$$\boldsymbol{P}_L^{\max} = \frac{1}{2}\times\frac{(\sqrt{10})^2}{4} = 1.25(\boldsymbol{W})$$

EXAMPLE 3

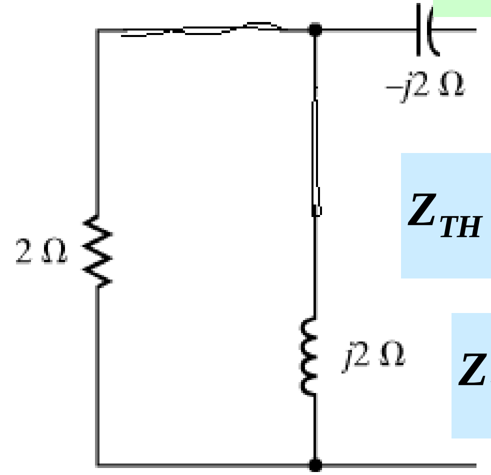
Find Z_L for maximum average power transfer.

Compute the maximum average power supplied to the load



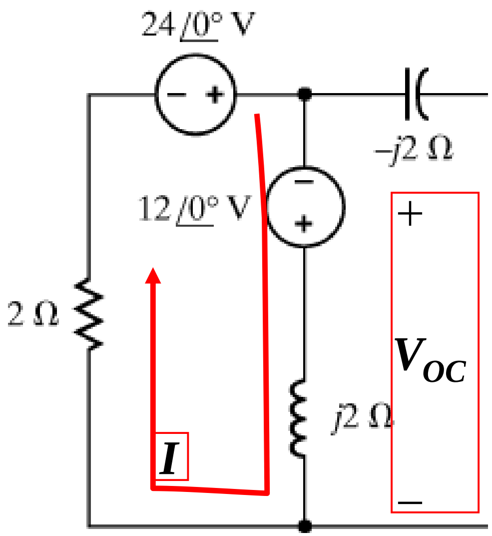
$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$



$$Z_{TH} = -j2 + (2 \| j2) = -j2 + \frac{4j}{2+j2}\Omega$$

$$Z_{TH} = \frac{4}{2+j2} = \frac{8-j8}{8} = 1 - j(\Omega)$$



$$V_{OC} = -12\angle 0° + j2I$$
$$= -12 + j2 \times 9(1-j)$$
$$= 6 + j18$$

$$Z_L^{opt} = 1 + j(\Omega)$$

$$V_{OC} = 18.974\angle 71.57°(V)$$

$$|V_{OC}|^2 = 6^2 + 18^2 = 360$$

$$P_L^{\max} = \frac{1}{2} \times \frac{360}{4} = 45(W)$$

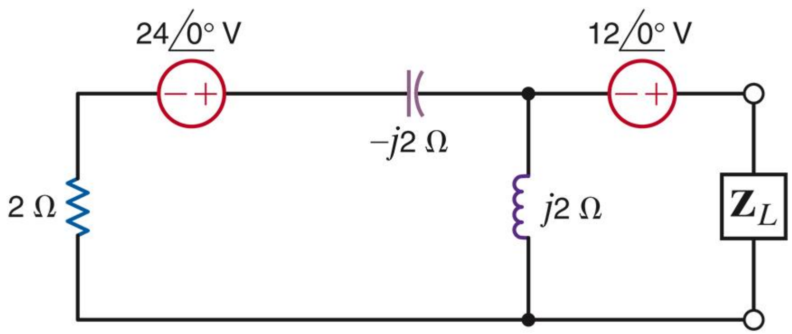
$$36\angle 0° = (2+j2)I$$

$$I = \frac{36(2-j2)}{8} = 9(1-j) = 12.73\angle -45°$$

EXAMPLE 4

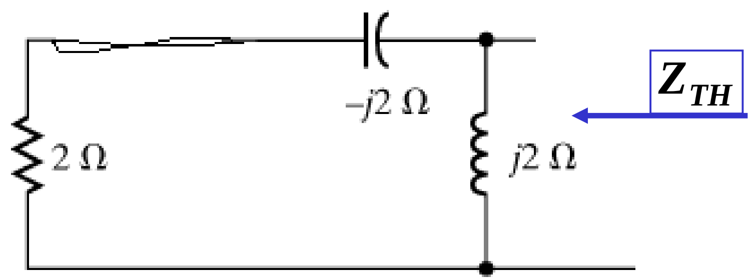
Find Z_L for maximum average power transfer.

Compute the maximum average power supplied to the load



$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$

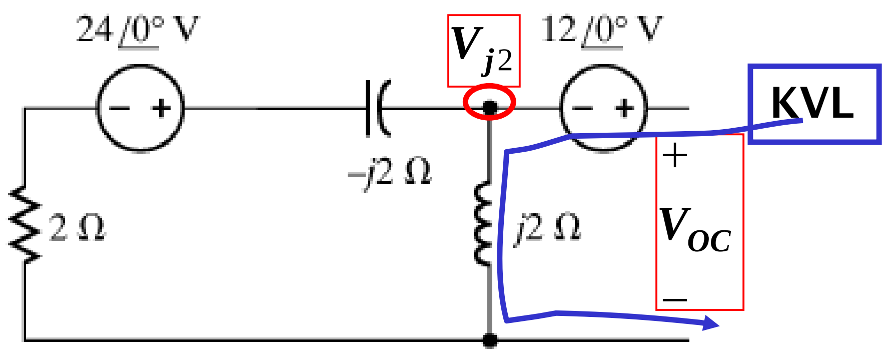


$$Z_{TH} = j2 \,||\, (2-j2) = \frac{j2(2-j2)}{2+j2-j2}$$

$$Z_{TH} = 2 + j2(\Omega)$$

$$Z_L^{opt} = 2 - j2(\Omega)$$

$$P_L^{\max} = \frac{1}{2} \times \frac{720}{4 \times 2} = 45(W)$$

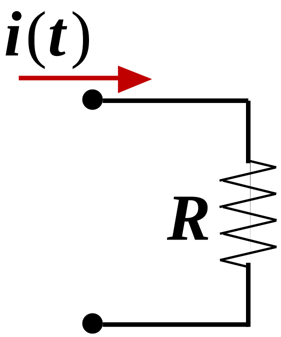


$$V_{j2} = \frac{j2}{j2-j2+2} 24\angle 0^\circ = 24\angle 90^\circ$$

$$V_{OC} = 12\angle 0^\circ + 24\angle 90^\circ = 12 + j24(V)$$

$$|V_{OC}|^2 = 12^2 + 24^2 = 720$$

EFFECTIVE OR RMS VALUES



Instantaneous power

$$p(t) = i^2(t)R$$

If the current is sinusoidal the average power is known to be $P_{av} = \frac{1}{2} I_M^2 R$

$$\therefore I_{eff}^2 = \frac{1}{2} I_M^2$$

For a sinusoidal signal

$$x(t) = X_M \cos(\omega t + \theta)$$

the effective value is

$$X_{eff} = \frac{X_M}{\sqrt{2}}$$

The effective value is the equivalent DC value that supplies the same average power

If current is periodic with period T

$$P_{av} = \frac{1}{T} \int_{t_0}^{t_0+T} p(t)dt = R\left(\frac{1}{T} \int_{t_0}^{t_0+T} i^2(t)dt \right)$$

If current is DC ($i(t) = I_{dc}$) then

$$P_{dc} = RI_{dc}^2$$

$$I_{eff} : P_{av} = P_{dc}$$

$$I_{eff}^2 = \frac{1}{T} \int_{t_0}^{t_0+T} i^2(t)dt$$

$$I_{eff} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} i^2(t)dt}$$

Definition is valid for ANY periodic signal with period T

For sinusoidal case $P_{av} = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$

$$P_{av} = V_{eff} I_{eff} \cos(\theta_v - \theta_i)$$

effective $\approx$ rms (root mean square)

EFFECTIVE OR RMS VALUES

EXAMPLE 9.7

We wish to compute the rms value of the waveform $i(t) = I_M \cos(\omega t - \theta)$, which has a period of $T = 2\pi/\omega$.

SOLUTION

Substituting these expressions into Eq. (9.23) yields

$$I_{\text{rms}} = \left[\frac{1}{T}\int_0^T I_M^2 \cos^2(\omega t - \theta)\, dt\right]^{1/2}$$

Using the trigonometric identity

$$\cos^2\phi = \frac{1}{2} + \frac{1}{2}\cos 2\phi$$

we find that the preceding equation can be expressed as

$$I_{\text{rms}} = I_M\left\{\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left[\frac{1}{2} + \frac{1}{2}\cos(2\omega t - 2\theta)\right] dt\right\}^{1/2}$$

Since we know that the average or mean value of a cosine wave is zero,

$$I_{\text{rms}} = I_M\left(\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{1}{2}\, dt\right)^{1/2}$$

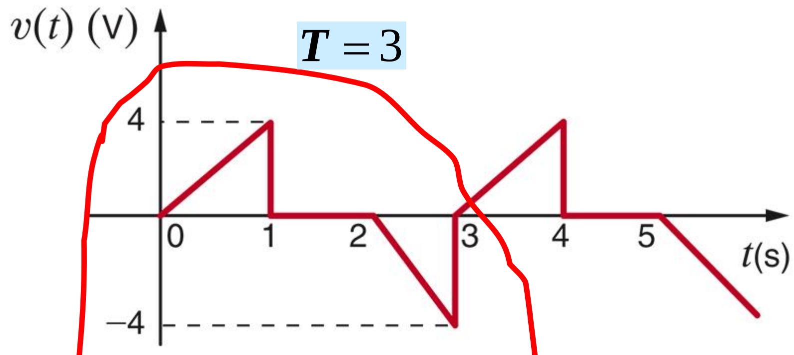
$$= I_M\left[\frac{\omega}{2\pi}\left(\frac{t}{2}\right)\Bigg|_0^{2\pi/\omega}\right]^{1/2} = \frac{I_M}{\sqrt{2}} \qquad 9.24$$

EXAMPLE 1

Compute the rms value of the voltage waveform

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)dt}$$

$T = 3$



One period

$$v(t) = \begin{cases} 4t & 0 < t \le 1 \\ 0 & 1 < t \le 2 \\ -4(t-2) & 2 < t \le 3 \end{cases}$$

The two integrals have the same value

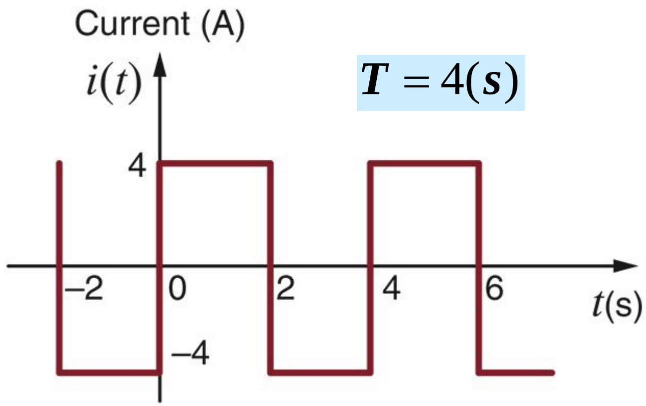
$$\int_0^T v^2(t)dt = \int_0^1 (4t)^2 dt + \int_2^3 (4(t-2))^2 dt$$

$$\int_0^3 v^2(t)dt = 2 \times \left[\frac{16}{3}t^3\right]_0^1 = \frac{32}{3}$$

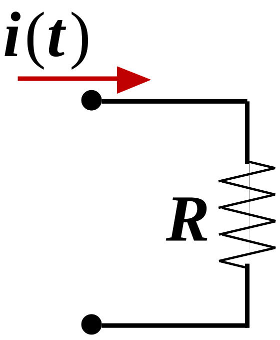
$$V_{rms} = \sqrt{\frac{1}{3} \times \frac{32}{3}} = 1.89(V)$$

EXAMPLE 2

Compute the rms value of the voltage waveform and use it to determine the average power supplied to the resistor



$\boldsymbol{T} = 4(\boldsymbol{s})$



$\boldsymbol{R} = 2\Omega$

$\boldsymbol{i}^2(\boldsymbol{t}) = 16;\ 0 \le \boldsymbol{t} < 4$

$$\boldsymbol{X_{rms}} = \sqrt{\frac{1}{\boldsymbol{T}} \int_{\boldsymbol{t}_0}^{\boldsymbol{t}_0+\boldsymbol{T}} \boldsymbol{x}^2(\boldsymbol{t})\boldsymbol{dt}}$$

The average power delivered to a 2-Ω resistor with this current is

$$P = I_{\mathrm{rms}}^2 R = (4)^2(2) = 32\ \mathrm{W}$$

The current waveform is periodic with a period of $T = 4$ s. The rms value is

$$I_{\mathrm{rms}} = \left\{\frac{1}{4}\left[\int_0^2 (4)^2\,dt + \int_2^4 (-4)^2\,dt\right]\right\}^{1/2}$$

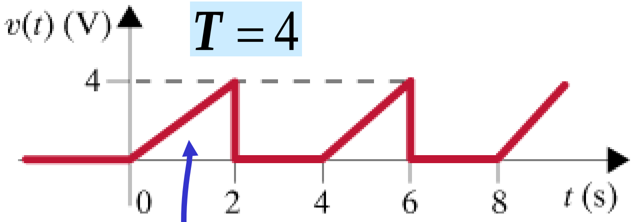
$$= \left[\frac{1}{4}\left(16t\Big|_0^2 + 16t\Big|_2^4\right)\right]^{1/2}$$

$$= 4\ \mathrm{A}$$

$\boldsymbol{I_{rms}} = 4(\boldsymbol{A})$

EXAMPLE 3

Compute rms value of the voltage waveform



$T = 4$

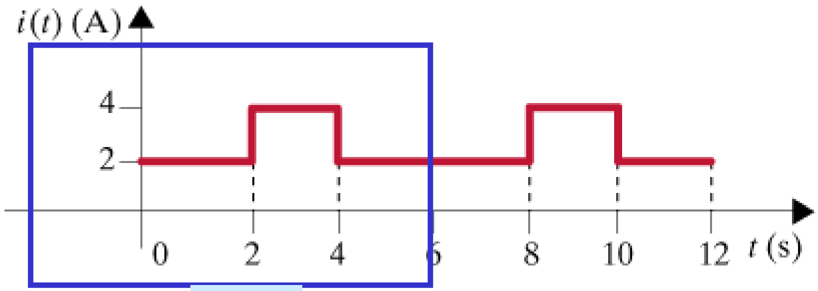
$v = 2t$

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)dt}$$

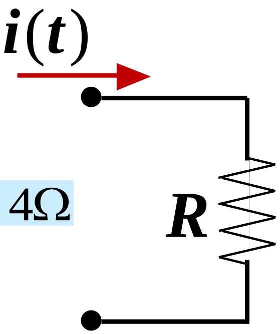
$$V_{rms} = \sqrt{\frac{1}{4}\int_0^2 (2t)^2 dt} \qquad = \left[\frac{1}{3}t^3\right]_0^2 = \frac{8}{3}(V)$$

EXAMPLE 4

Compute the rms value for the current waveforms and use them to determine average power supplied to the resistor



$T = 6$



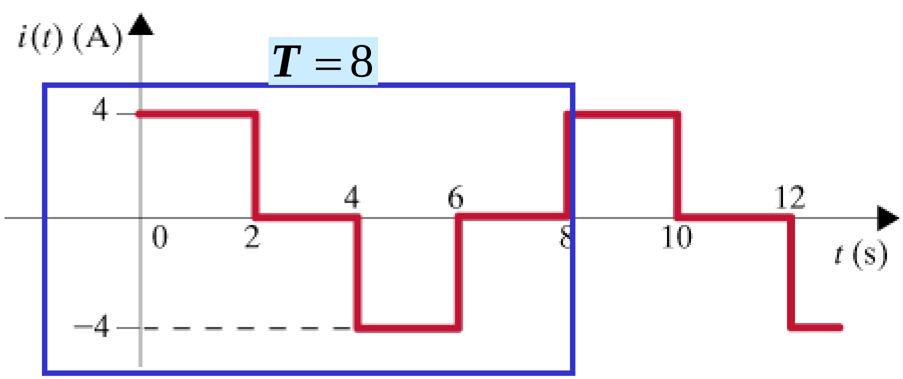
$R = 4\Omega$

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)dt}$$

$$P_{av} = I_{rms}^2 R$$

$$I_{rms}^2 = \frac{1}{6}\left[\int_0^2 4dt + \int_2^4 16dt + \int_4^6 4dt\right] = \frac{8+32+8}{6} = 8$$

$$P = 8 \times 4 = 32(W)$$

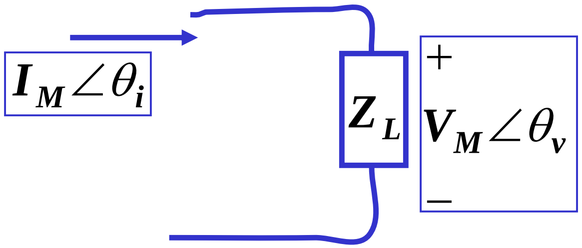


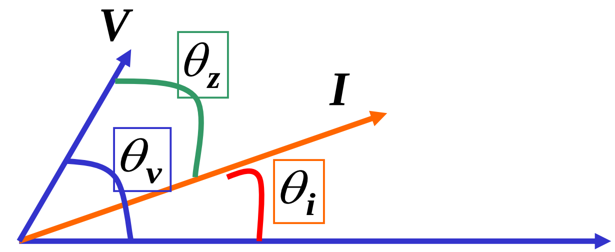
$T = 8$

$$I_{rms}^2 = \frac{1}{8}\left[\int_0^2 16dt + \int_4^6 16dt\right] = 8$$

$$P = 32(W)$$

THE POWER FACTOR





$$V = ZI \Rightarrow \angle V = \angle Z + \angle I$$

$$\theta_v = \theta_z + \theta_i$$

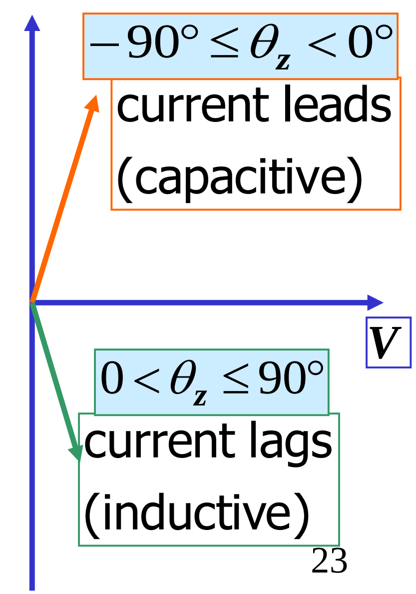
$$P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$P_{\text{apparent}} = V_{rms} I_{rms}$$

$$pf = \frac{P}{P_{\text{apparent}}} = \cos(\theta_v - \theta_i) = \cos\theta_z$$

$$P = V_{rms} \times I_{rms} \times pf$$

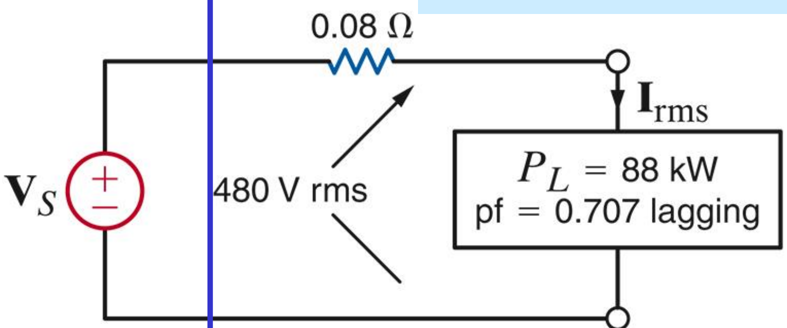
pf	θ_z	
0	$-90°$	pure capacitive
$0 < pf < 1$	$-90° < \theta_z < 0°$	leading or capacitive
1	$0°$	resistive
$0 < pf < 1$	$0° < \theta_z < 90°$	lagging or inductive
0	$90°$	pure inductive



EXAMPLE 1

Find the power supplied by the power company.
Determine how it changes if the power factor is changed to 0.9

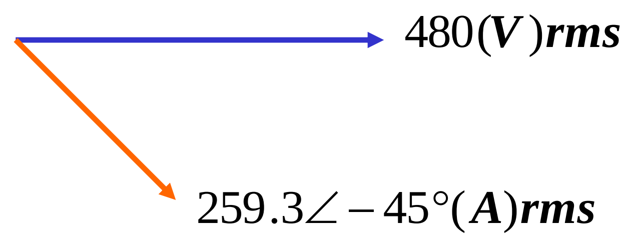
$$P = V_{rms} \times I_{rms} \times pf$$



Power company

$$\Rightarrow \cos\theta_z = 0.707 \Rightarrow \theta_z = -45°$$

Current lags the voltage



$$I_{rms} = \frac{88 \times 10^3 (W)}{480 \times 0.707} = 259.3(A)rms$$

$$P_{losses} = I_{rms}^2 R = 259.3^3 \times 0.08 = 5.378kW$$

$$P_S = P_{losses} + 88{,}000(W) = 93.378(kW)$$

$$V_{S_{rms}} = 0.08 I_{rms} + V_L$$
$$= 0.08 \times 259.3\angle -45° + 480$$

$$V_{S_{rms}} = 0.08 \times (183.4 - j183.4) + 480$$
$$= 494.7 - j14.7 = 495\angle -1.7°(V)$$

If pf=0.9

$$I_{rms} = \frac{88{,}000}{480 \times 0.9} = 203.7(A)rms$$

$$P_{losses} = I_{rms}^2 R = 3.32kW$$

If pf=0.9

$$I_{rms} = 203.7\angle -25.8°$$

$$V_S = 14.47 - j7.09 + 480 = 494\angle -0.82°$$

Losses can be reduced by 2kW!

Examine also the generated voltage

EXAMPLE 2

$$P_L = 100kW, V_L = 480(V)rms, pf = 0.707$$
$$R_{line} = 0.1\Omega$$

$$P = V_{rms} \times I_{rms} \times pf$$

Determine the power savings if the power factor can be increased to 0.94

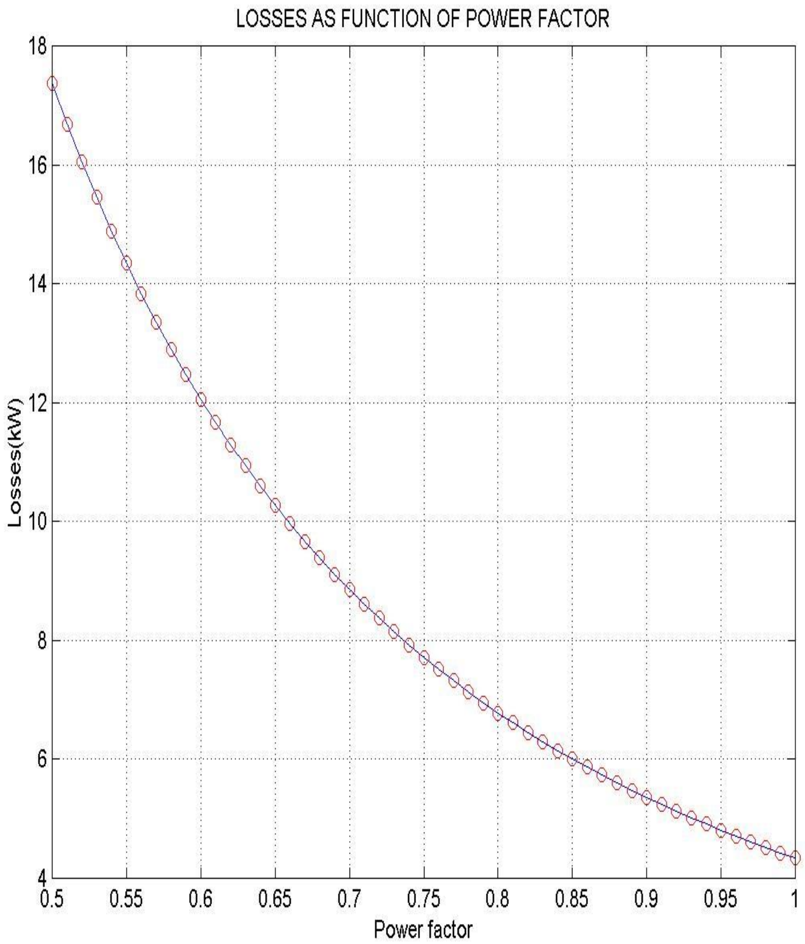
$$I_{rms} = \frac{P}{V_{rms} \times pf}$$

$$P_{losses} = I_{rms}^2 R_{line} = \frac{P^2 R_{line}}{V_{rms}^2} \times \frac{1}{pf^2}$$

$$P_{losses}(pf = 0.707) = \frac{10^{10} \times 0.1}{480^2} \times \frac{1}{0.707^2}(W)$$
$$= 2 \times 4.34kW$$

$$P_{losses}(pf = 0.94) = \frac{10^{10} \times 0.1}{480^2} \times \frac{1}{0.94^2}(W)$$
$$= 1.13 \times 4.34kW$$

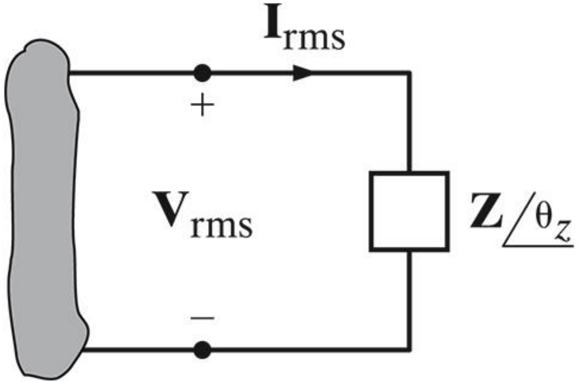
$$P_{saved} = 0.87 \times 4.34kW = 3.77kW$$



COMPLEX POWER

Definition of Complex Power

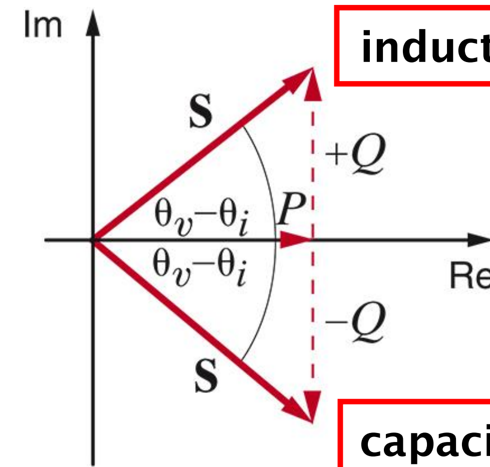
$$S = V_{rms} I^{*}_{rms}$$



$$S = V_{rms}\angle\theta_v{}^{\circ} \times [I_{rms}\angle\theta_i{}^{\circ}]^{*}$$
$$S = V_{rms} I_{rms}\angle\theta_v - \theta_i$$

The units of apparent and reactive power are Volt-Ampere

$$|S| = V_{rms} I_{ms}$$
$$P = |S| \times pf$$



inductive

capacitive

$$S = \underbrace{V_{rms} I_{rms}\cos(\theta_v - \theta_i)}_{P} + j\underbrace{V_{rms} I_{rms}\sin(\theta_v - \theta_i)}_{Q}$$

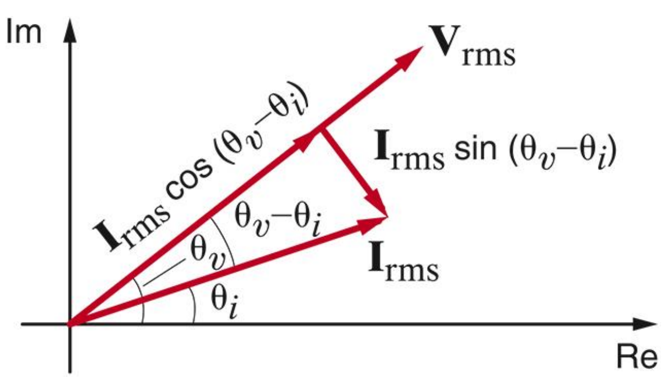
Active Power

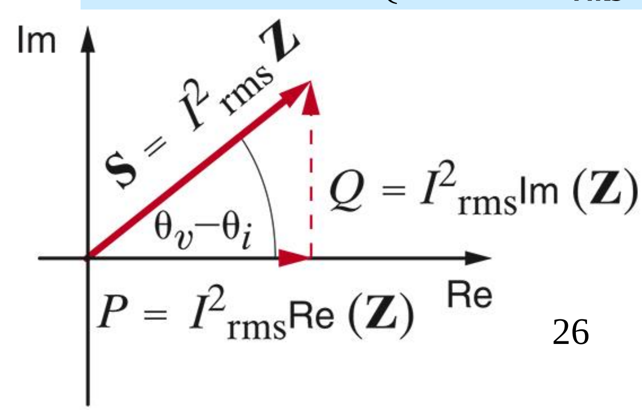
Reactive Power

Another useful form

$$V_{rms} = Z I_{rms} \Rightarrow S = (Z I_{rms}) I^{*}_{rms} = Z\,|I_{rms}|^{2}$$

$$Z = R + jX \Rightarrow \begin{cases} P = R\,|I_{rms}|^{2} \\ Q = X\,|I_{rms}|^{2} \end{cases}$$





ANALYSIS OF BASIC COMPONENTS

RESISTORS

For a resistor, $\theta_v - \theta_i = 0°$, $\cos(\theta_v - \theta_i) = 1$

$\therefore \sin(\theta_v - \theta_i) = 0$ $\quad \Rightarrow Q = 0$

INDUCTORS

For an inductor, $\theta_v - \theta_i = 90°$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(90°) = 0$$

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(90°) > 0$$

CAPACITORS

$$\theta_v - \theta_i = -90°$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(-90°) = 0$$

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(-90°) < 0$$

Supplies reactive power!!

Determining P or S

If $v(t)$ and $i(t)$ are known and we wish to find P given an impedance $\mathbf{Z}\angle\theta = R + jX$, two viable approaches are as follows:

Step 1. Determine $\mathbf{V}$ and $\mathbf{I}$ and then calculate

$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta \quad \text{or} \quad P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

Step 2. Use $\mathbf{I}$ to calculate the real part of $\mathbf{S}$; that is,

$$P = R_e(\mathbf{S}) = I^2 R$$

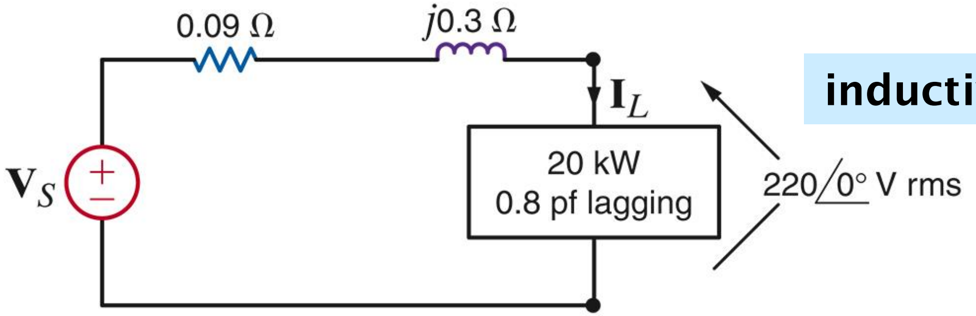
WARNING $P \neq \dfrac{V^2}{R}$ $\left(IF \quad X \neq 0\right)$

EXAMPLE 1

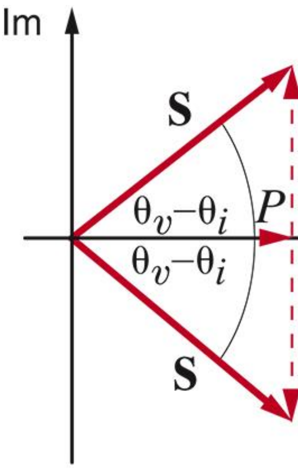
Given:

$P_L = 20kW,\ pf = 0.8\,lagging, V_L = 220\angle 0°\,rms, Z_L = 0.09 + j0.3\Omega,\ f = 60Hz$

Determine the voltage and power factor at the input to the line



inductive



inductive

capacitive

$P = \mathrm{Re}\{S\}$ $= |S|\cos(\theta_v - \theta_i) = |S| \times pf$

$$\therefore |S_L| = \frac{P}{pf} = 25kVA$$

$Q^2 = S_L{}^2 - P^2 \Rightarrow Q = 15kVA$ $\quad S_L = 20 + j15(kVA) = 25\angle 36.87°$

$S_L = V_L I_L^*$

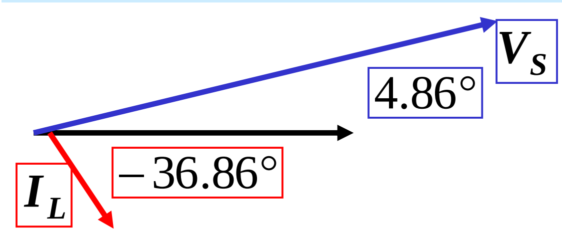
$$\Rightarrow I_L = \left[\frac{S_L}{V_L}\right]^* = \left[\frac{25{,}000\angle 36.87°}{220\angle 0°}\right]^* = 113.64\angle -36.86°(A)$$

$$I_L = \left[\frac{20{,}000 + j15{,}000}{220}\right]^* = 90.91 - j68.18(A)$$

$V_S = (0.09 + j0.3)I_L + 220\angle 0°$

$V_S = (0.09 + j0.3)(90.91 - j68.18) + 220(V)$

$V_S = 248.63 + j21.14 = 249.53\angle 4.86°$

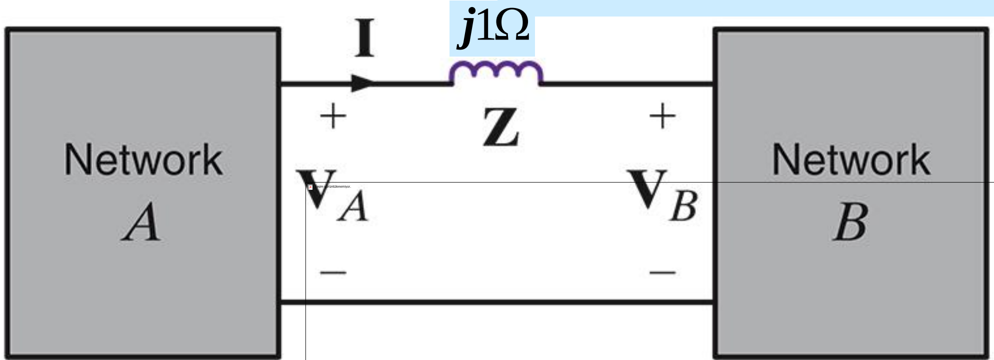


$pf_{\text{source}} = \cos(41.72°) = 0.746$

lagging

EXAMPLE 2

Compute the average power flow between networks
Determine which is the source



$\boldsymbol{V_A} = 120\angle 30°(V)rms$ $\quad$ $\boldsymbol{V_B} = 120\angle 0°(V)rms$

$$\boldsymbol{I} = \frac{\boldsymbol{V_A} - \boldsymbol{V_B}}{\boldsymbol{Z}} = \frac{120\angle 30° - 120\angle 0°}{j1}$$

$$\boldsymbol{I} = \frac{(103.92 + j60) - 120}{j} = 60 + j16.08(A)rms$$

$$\boldsymbol{I} = 62.12\angle 15°(A)rms$$

$$\boldsymbol{S_A} = \boldsymbol{V_A}(-\boldsymbol{I})^* = 120\angle 30° \times 62.12\angle -195° = 7{,}454\angle -165° VA_{rms}$$

Passive sign convention.
Power received by A

$$\boldsymbol{P_A} = 7{,}454\cos(165°) = -7{,}200(W)$$

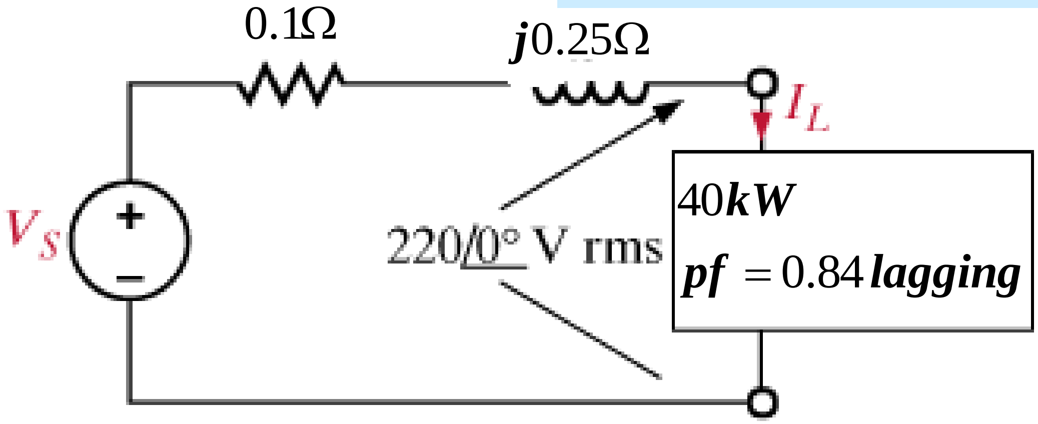
$$\boldsymbol{S_B} = \boldsymbol{V_B}(\boldsymbol{I})^* = 120\angle 0° \times 62.12\angle -15° = 7{,}454\angle -15° VA_{rms}$$

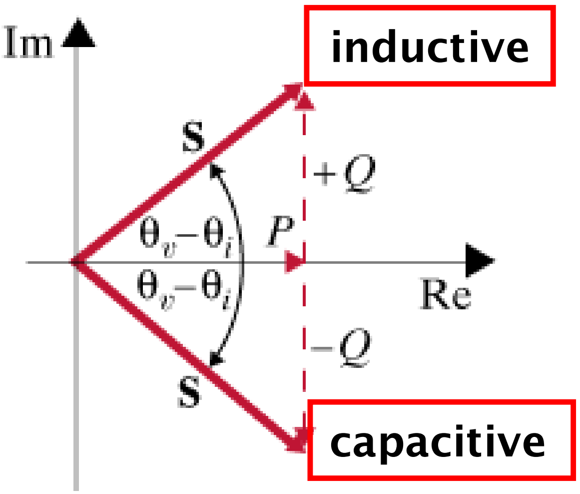
$$\boldsymbol{P_B} = 7{,}454\cos(-15°) = 7{,}200(W)$$

A supplies 7.2kW average power to B

EXAMPLE 3

Determine real and reactive power losses and real and reactive power supplied



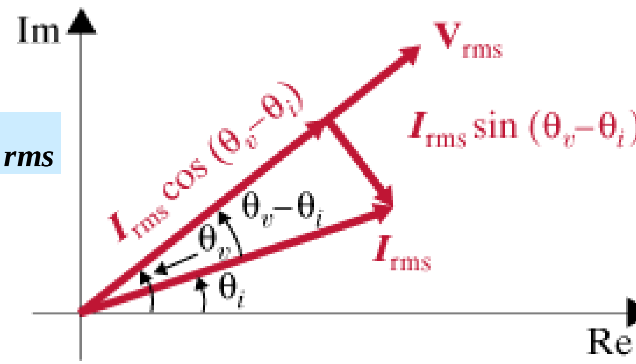


$P = \mathrm{Re}\{S\} = |S|\cos(\theta_v - \theta_i) = |S| \times pf$

$|S_L| = \dfrac{P}{pf} = \dfrac{40}{.84} = 47.62kVA$ $\quad |Q_L| = \sqrt{|S_L|^2 - P^2} = 25{,}839(VA)$

$S = VI^* \Rightarrow |I_L| = \dfrac{|S_L|}{|V_L|} = 216.45(A)_{rms}$ $\quad I_L = 216.45\angle -32.86°(A)_{rms}$

$pf = \cos(\theta_v - \theta_i) \Rightarrow \theta_v - \theta_i = 32.86°$



$S_{\text{losses}} = (Z_{line} I_L) I_L^* = Z_{line} |I_L|^2$

$S_{\text{losses}} = (0.1 + j0.25)(216.45)^2 = 4{,}685 + j11{,}713VA$

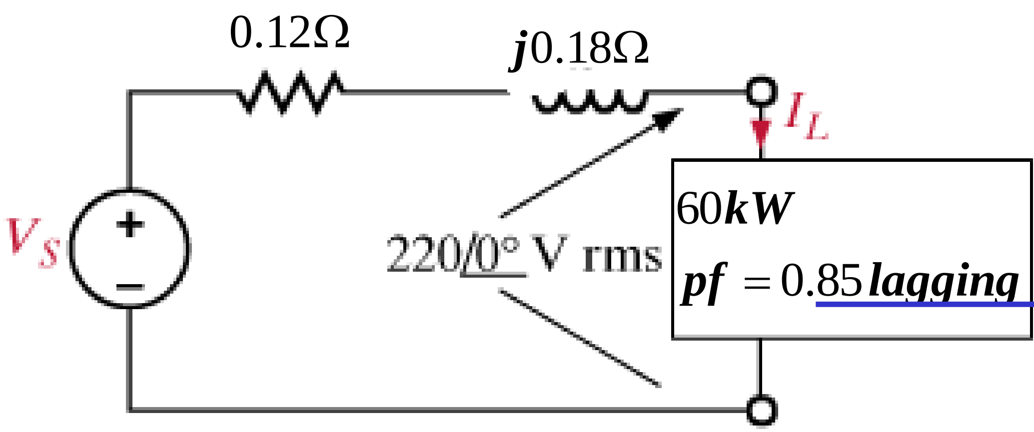
Balance of power

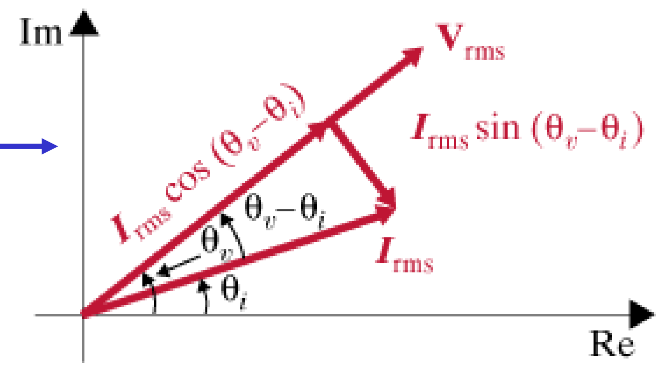
$S_{\text{supplied}} = S_{\text{losses}} + S_{\text{load}}$

$= 4.685 + j11.713 + 40 + j25.839 = 44.685 + j37.552kVA$

EXAMPLE 4

Determine line voltage and power factor at the supply end





$$P = \mathrm{Re}\{S\} = |S|\cos(\theta_v - \theta_i) = |V_L| \times |I_L| \times pf$$

$$S_L = V_L I_L^*$$

$$|I_L| = \frac{P}{|V_L| \times pf} = 320.86(A)_{rms}$$

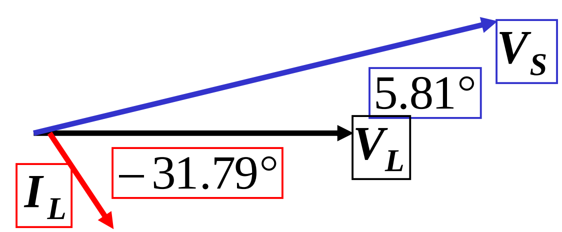
$$\theta_v - \theta_i = \cos^{-1}(pf) \Rightarrow \theta_v - \theta_i = 31.79°$$

$$I_L = 320.86\angle -31.79°(A)_{rms} = 272.72 - j169.03(A)_{rms}$$

$$V_S = Z_{\text{line}} I_L + V_L = (0.12 + j0.18)(272.72 - j169.03) + 220$$

$$V_S = 283.15 + j28.81(V)_{rms} = 284.61\angle 5.81°(V)_{rms}$$

The phasor diagram helps in visualizing the relationship between voltage and current

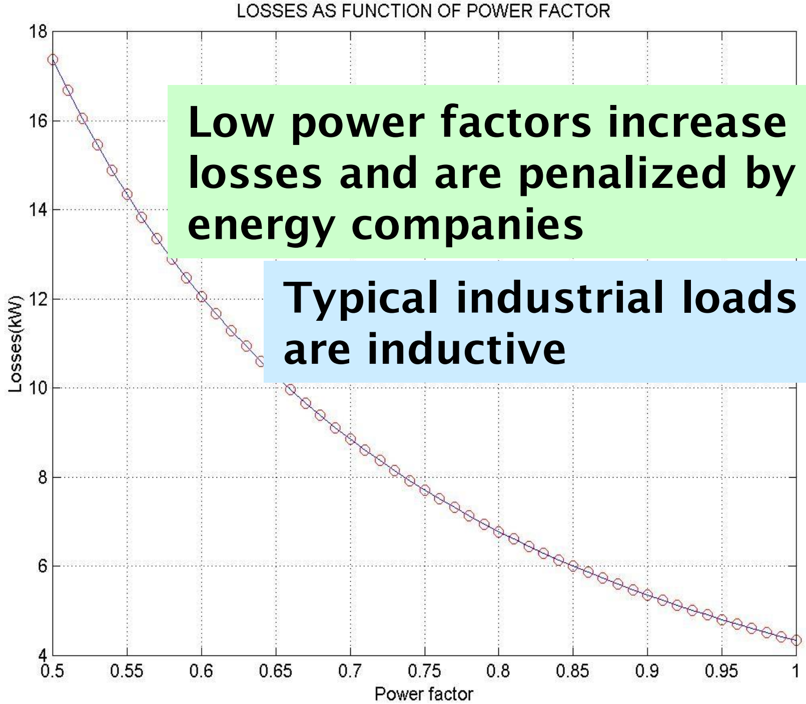


$$pf_{\text{source}} = \cos(37.6°) = 0.792$$

lagging

POWER FACTOR CORRECTION

$$\boldsymbol{V_L} = \frac{1}{j\omega \boldsymbol{C}} \boldsymbol{I}_{\text{capacitor}}$$



Low power factors increase losses and are penalized by energy companies

Typical industrial loads are inductive

Without capacitor :

$$\boldsymbol{S}_{\text{old}} = \boldsymbol{P}_{\text{old}} + j\boldsymbol{Q}_{\text{old}} = |\boldsymbol{S}_{\text{old}}| \angle \theta_{\text{old}}$$

$$pf_{\text{old}} = \cos(\theta_{\text{old}})$$

With capacitor

$$\boldsymbol{S}_{\text{new}} = \boldsymbol{S}_{\text{old}} + \boldsymbol{S}_{\text{capacitor}}$$

$$= \boldsymbol{P}_{\text{old}} + j\boldsymbol{Q}_{\text{old}} - j\boldsymbol{Q}_{\text{capacitor}}$$

$$= |\boldsymbol{S}_{\text{new}}| \angle \theta_{\text{new}}$$

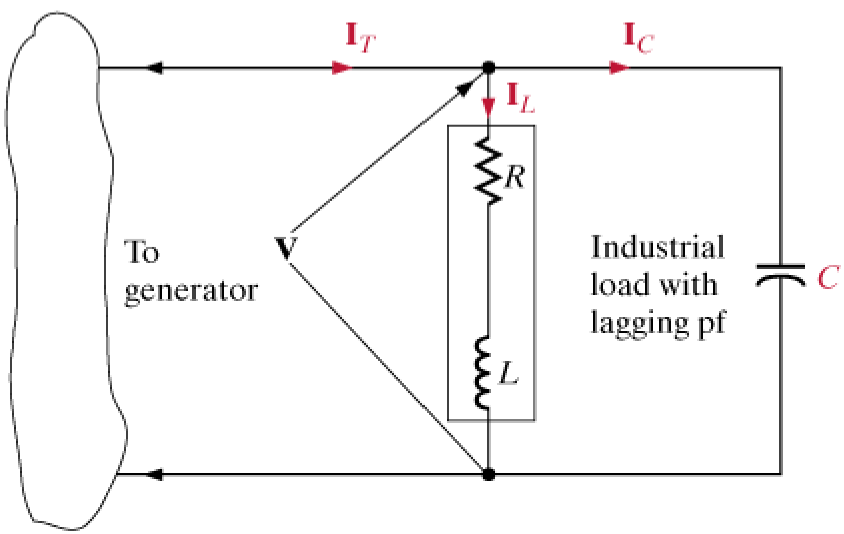
$$pf_{\text{new}} = \cos(\theta_{\text{new}})$$

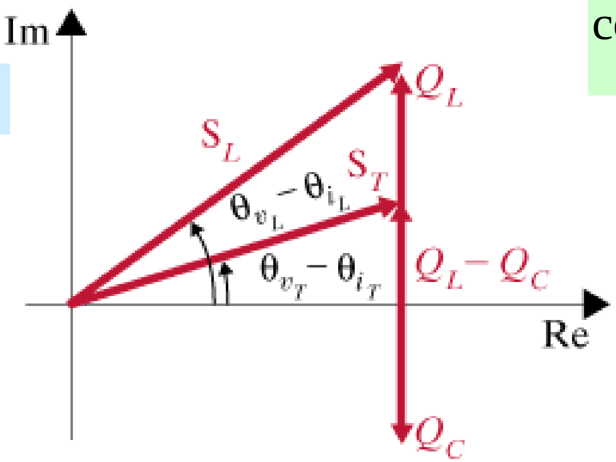
$$\boldsymbol{Q}_{\text{capacitor}} = |\boldsymbol{V_L}||\boldsymbol{I}_{\text{capacitor}}| = |\boldsymbol{V_L}|^2 \omega \boldsymbol{C}$$

$$\tan\theta_{\text{new}} = \frac{\boldsymbol{Q}_{\text{old}} - \boldsymbol{Q}_{\text{capacitor}}}{\boldsymbol{P}_{\text{old}}}$$

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}}$$

Simple approach to power factor correction





EXAMPLE 1 **Economic Impact of Power Factor Correction**

It is common for an industrial facility operating at a poor power factor to be charged more by the electric utility providing electrical service.

Operating Conditions 277 V rms and requires 500 kW at a power factor of 0.75 lagging

energy charge of 2¢ per kWh and a demand charge of \$3.50 per kW per month if the power factor is between 0.9 lagging and unity and \$5 per kVA per month if the power factor is less than 0.9 lagging.

Current Monthly Utility Bi monthly energy charge is $500 \times 24 \times 30 \times \$0.02 = \$7{,}200$.

$$\mathbf{S}_{\text{old}} = \frac{500}{0.75}\underline{/\cos^{-1}(0.75)} = 666.67\underline{/41.4^\circ} = 500 + j441 \text{ kVA}$$

monthly demand charge is $666.67 \times \$5 = \$3{,}333.35$.

Correcting to pf=0.9

$$\mathbf{S}_{\text{new}} = \frac{500}{0.9}\underline{/\cos^{-1}(0.9)} = 555.6\underline{/25.84^\circ} = 500 + j242.2 \text{ kVA}$$

New demand charge $500 \times \$3.50 = \$1{,}750$ per month

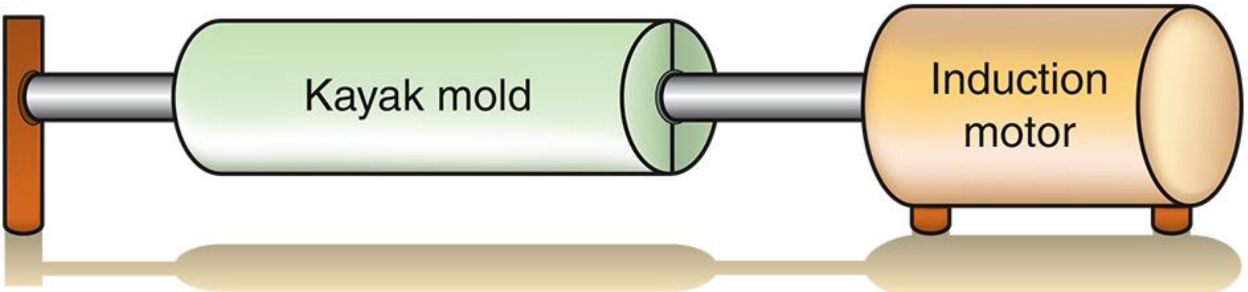
Additional energy charge due to capacitor bank is negligible

Monthly savings are approx \$1853 per month! A reasonable capacitor bank should Itself in about a year

EXAMPLE 2

$f = 60\,Hz.$

Determine the capacitor required to increase the power factor to 0.95 lagging



Roto-molding process

$50\,kW, V_L = 220\angle 0°_{rms}$

$pf = 0.8\,lagging$

$P = \mathrm{Re}\{S\} = |S|\cos(\theta_v - \theta_i) = |S| \times pf$

$|S_{old}| = \dfrac{P}{pf} = \dfrac{50}{.80} = 62.5\,kVA$ $\quad |Q_{old}| = \sqrt{|S_{old}|^2 - P^2} = 37.5(kVA).$

$\cos\theta_{new} = 0.95 \Rightarrow \tan\theta_{new} = \dfrac{\sqrt{1 - pf_{new}^2}}{pf_{new}} = 0.329$ $= \dfrac{Q_{new}}{P} \Rightarrow Q_{new} = 0.329 \times P = 16.43\,kVA$

$\therefore Q_{capacitor} = Q_{old} - Q_{new} = 37.5 - 16.43 = 21.07\,kVA$

$Q_{\text{capacitor}} = |V_L||I_{\text{capacitor}}|$
$= |V_L|^2\,\omega C$

$\therefore C = \dfrac{Q_{capacitor}}{\omega|V_L|^2} = \dfrac{21.07 \times 10^3}{(220)^2 \times (2\pi \times 60)} = 0.001156\,(F) = 1156\,\mu F$

EXAMPLE 3

Determine the capacitor necessary to increase the power factor to 0.94

$$P_L = 100kW, V_L = 480(V)rms, pf = 0.707$$

$$R_{line} = 0.1\Omega, f = 60Hz$$

$$P = \mathrm{Re}\{S\} = |S|\cos(\theta_v - \theta_i) = |S| \times pf$$

$$|S_{old}| = \frac{P}{pf} = \frac{100}{.707} = 141.44kVA \quad |Q_{old}| = \sqrt{|S_{old}|^2 - P^2} = 100.02(kVA).$$

$$\cos\theta_{new} = 0.94 \Rightarrow \tan\theta_{new} = \frac{\sqrt{1 - pf_{new}^2}}{pf_{new}} = 0.363 \quad = \frac{Q_{new}}{P} \Rightarrow Q_{new} = 0.363 \times P = 36.3kVA$$

$$\therefore Q_{capacitor} = Q_{old} - Q_{new} = 100.02 - 36.3 = 63.72kVA$$

$$Q_{\text{capacitor}} = |V_L||I_{\text{capacitor}}| = |V_L|^2 \omega C$$

$$\therefore C = \frac{Q_{capacitor}}{\omega |V_L|^2} = \frac{63.72 \times 10^3}{(480)^2 \times (2\pi \times 60)} = 0.000733(F) = 733\mu F$$

HOMEWORK PROBLEMS

9.1 The voltage and current at the input of a network are given by the expressions

$$v(t) = 6 \cos \omega t \text{ V}$$
$$i(t) = 4 \sin \omega t \text{ A}$$

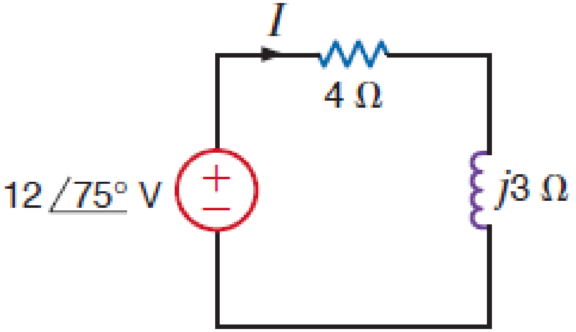
Determine the average power absorbed by the network.

9.2 The voltage and current at the input of a circuit are given by the expressions

$$v(t) = 170 \cos(\omega t + 30°) \text{ V}$$
$$i(t) = 5 \cos(\omega t + 45°) \text{ A}$$

Determine the average power absorbed by the circuit.

9.3 Determine the equations for the current and the instantaneous power in the network in Fig. P9.3.



HOMEWORK PROBLEMS

9.4 Given $v_s(t) = 100 \cos 100t$ volts, find the average power supplied by the source and the current $i_2(t)$ in the network in Fig. P9.4.

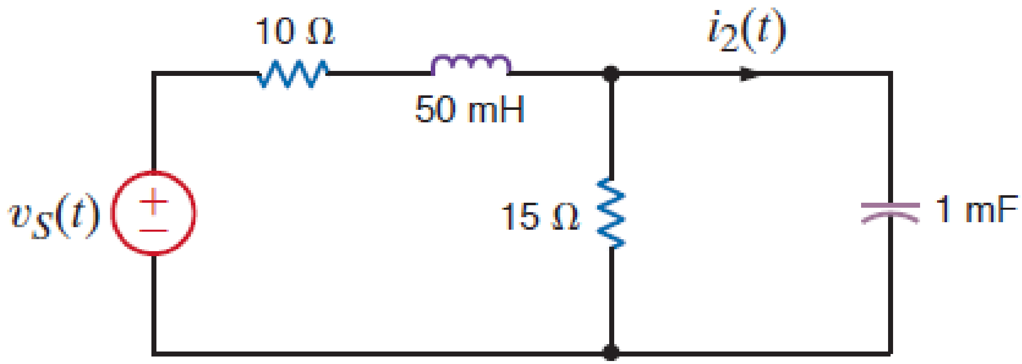
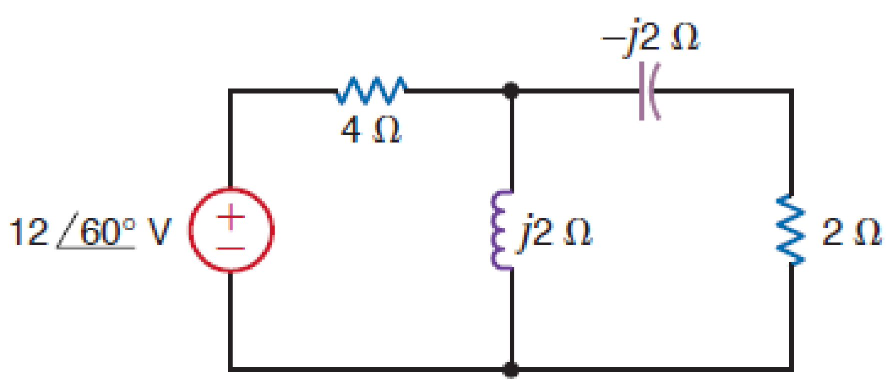


Figure P9.4

9.5 Determine the instantaneous power supplid by the source in the circuit in Fig. P9.5.



HOMEWORK PROBLEMS

9.10 If $i_g(t) = 0.5 \cos 2000t$ A, find the average power absorbed by each element in the circuit in Fig. P9.10.

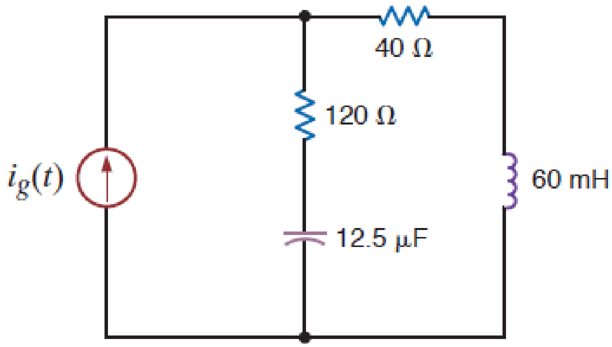


Figure P9.10

9.11 Find the average power absorbed by the network in Fig. P9.11.

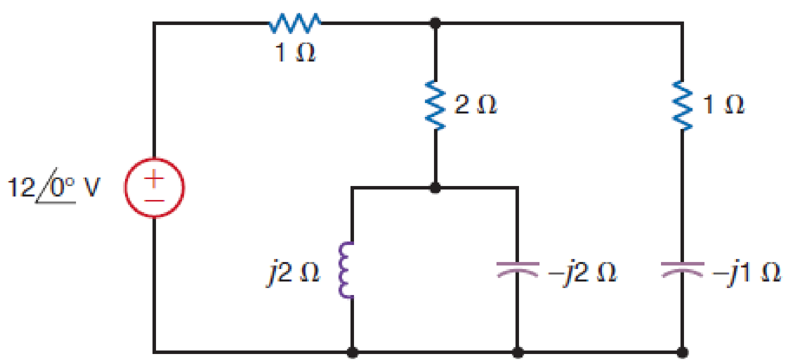


Figure P9.11

9.17 Find the average power absorbed by the network shown in Fig. P9.17.

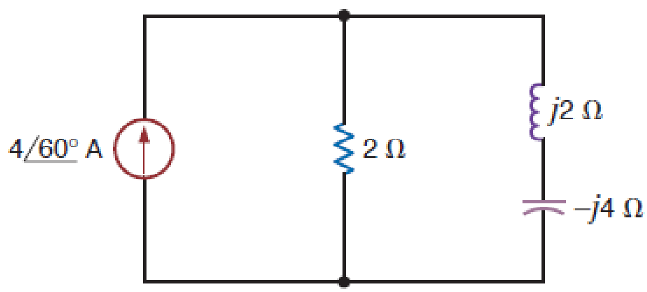


Figure P9.17

9.18 Determine the average power supplied by each source in the network shown in Fig. P9.18.

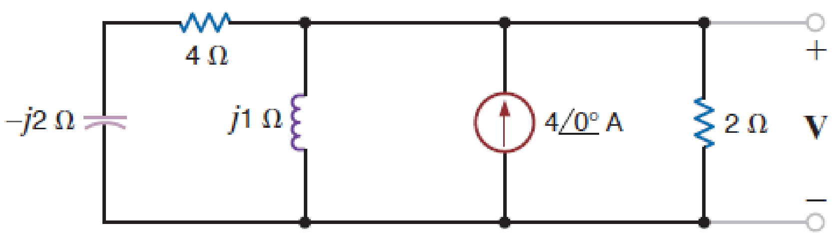


Figure P9.18

9.22 Calculate the average power absorbed by the 1-Ω resistor in the network shown in Fig. P9.22.

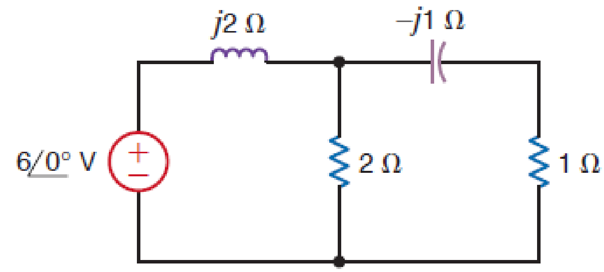


Figure P9.22

9.23 Determine the average power absorbed by the 4-Ω resistor in the network shown in Fig. P9.23.

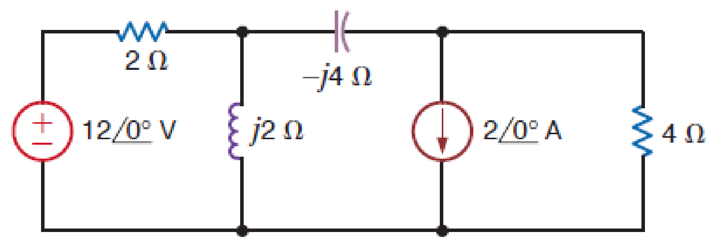


Figure P9.23

HOMEWORK PROBLEMS

9.33 Find the value of $\mathbf{Z}_L$ in Fig. P9.33 for maximum average power transfer to the load.

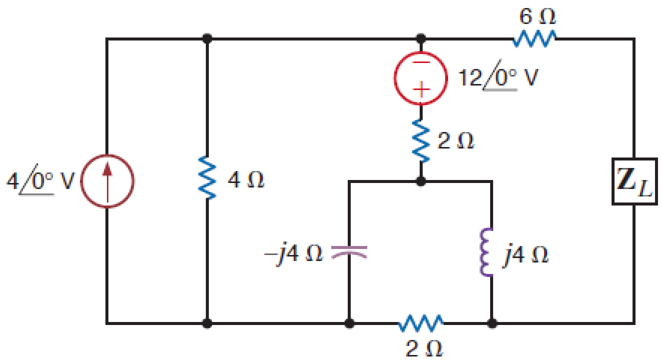


Figure P9.33

9.34 Find the value of $\mathbf{Z}_L$ in Fig. P9.34 for maximum average power transfer to the load.

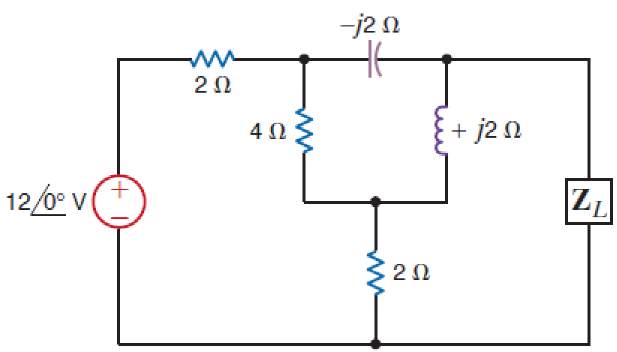


Figure P9.34

Figure P9.36

9.37 Determine the impedance $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power transferred to $\mathbf{Z}_L$ for the circuit shown in Fig. P9.37.

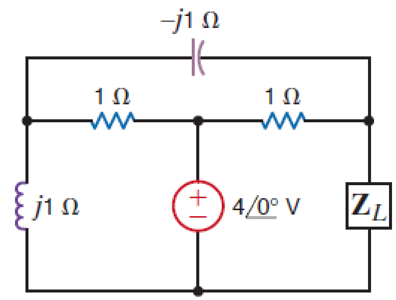


Figure P9.37

9.38 In the network in Fig. P9.38, find $\mathbf{Z}_L$ for maximum average power transfer and the maximum average power transferred.

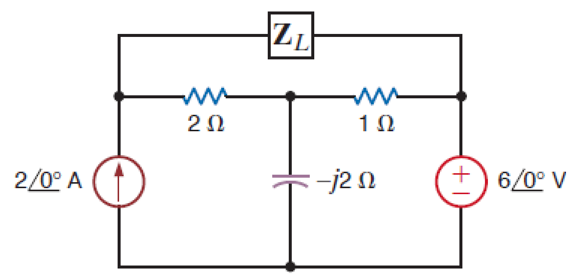


Figure P9.38

HOMEWORK PROBLEMS

9.49 Calculate the rms value of the waveform in Fig. P9.49.

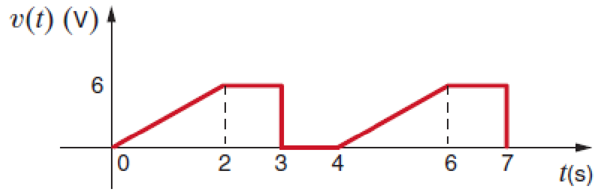


Figure P9.49

9.50 Calculate the rms value of the waveform shown in Fig. P9.50.

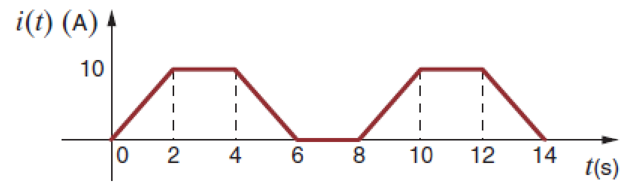


Figure P9.50

9.51 The current waveform shown in Fig. P9.51 is applied to a 4-Ω resistor. Calculate the average power dissipated in the resistor.

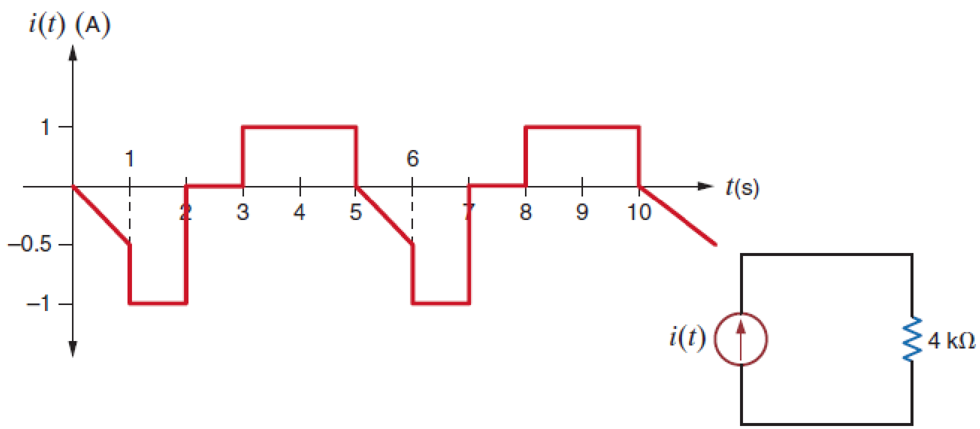


Figure P9.51

HOMEWORK PROBLEMS

9.62 The power supply in Fig. P9.59 generates 50 kW the line impedance is 0.095 Ω. If the load consumes 43 kW and the voltmeter leads 220 V rms, determine the ammeter leading and the power factor of the inductive load.

9.63 A plant consumes 100 kW of power at 0.9 pf lagging. If the load current is 200 A rms, find the load voltage.

9.64 A plant consumes 20 kW of power from a 240-V rms line. If the load power factor is 0.9, what is the angle by which the load voltage leads the load current? What is the load current phasor if the line voltage has a phasor of $240 \underline{/0^\circ}$ V rms?

9.65 A plant draws 250 A rms from a 240-V rms line to supply a load with 50 kW. What is the power factor of the load?

9.66 The power company must generate 100 kW to supply an industrial load with 94 kW through a transmission line with 0.09-Ω resistance. If the load power factor is 0.83 lagging, find the load voltage.

9.67 A transmission line with impedance of $0.08 + j0.25\ \Omega$ is used to deliver power to a load. The load is inductive, and the load voltage is $220 \underline{/0^\circ}$ V rms at 60 Hz. If the load requires 12 kW and the real power loss in the line is 560 W, determine the power factor angle of the load.

9.68 The power company supplies 80 kW to an industrial load. The load draws 220 A rms from the transmission line. If the load voltage is 440 V rms and the load power factor is 0.8 lagging, find the losses in the transmission line.

9.69 The power company supplies 40 kW to an industrial load. The load draws 200 A rms from the transmission line. If the load voltage is 240 V rms and the load power factor is 0.8 lagging, find the losses in the transmission line.

9.70 An industrial load that consumes 40 kW is supplied by the power company, through a transmission line with 0.1-Ω resistance, with 44 kW. If the voltage at the load is 240 V rms, find the power factor at the load.

9.71 A transmission line with impedance $0.1 + j0.2\ \Omega$ is used to deliver power to a load. The load is capacitive and the load voltage is $240 \underline{/0^\circ}$ V rms at 60 Hz. If the load requires 15 kW and the real power loss in the line is 660 W, determine the input voltage to the line.

9.72 An industrial load operates at 30 kW, 0.8 pf lagging. The load voltage is $240 \underline{/0^\circ}$ V rms. The real and reactive power losses in the transmission-line feeder are 1.8 kW and 2.4 kvar, respectively. Find the impedance of the transmission line and the input voltage to the line.

HOMEWORK PROBLEMS

9.73 Find the real and reactive power absorbed by each element in the circuit in Fig. P9.73.

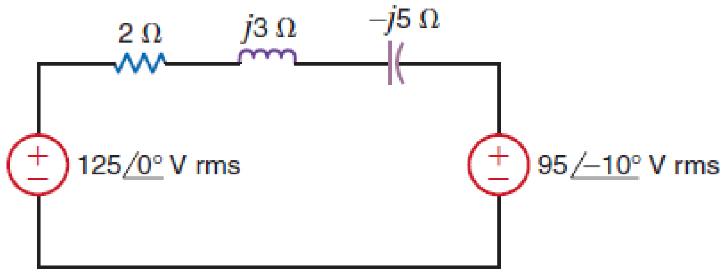
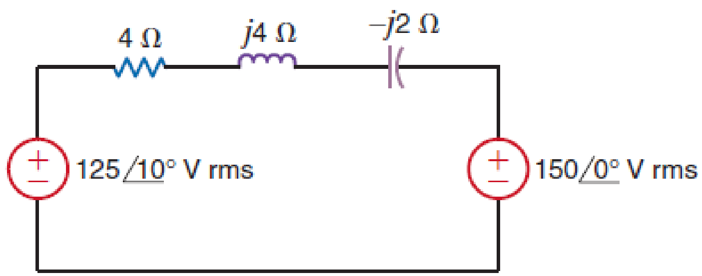


Figure P9.73

9.74 Calculate the real and reactive power absorbed by every element (including the sources) in the circuits in Fig. P9.74.



9.77 In the circuit shown in Fig. P9.77, calculate $\mathbf{V}_S$, the complex power supplied by the source, and the power factor of the source.

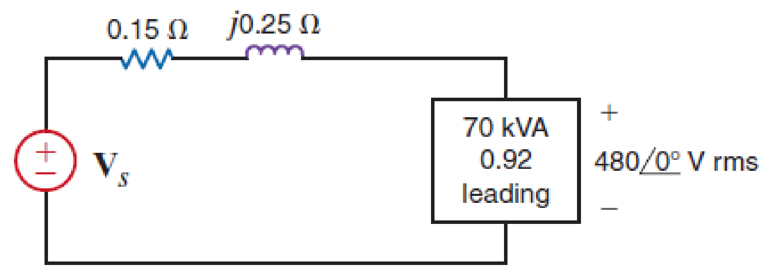


Figure P9.77

9.78 In the circuit shown in Fig. P9.78, calculate $\mathbf{V}_S$, the complex power supplied by the source, and the power factor of the source.

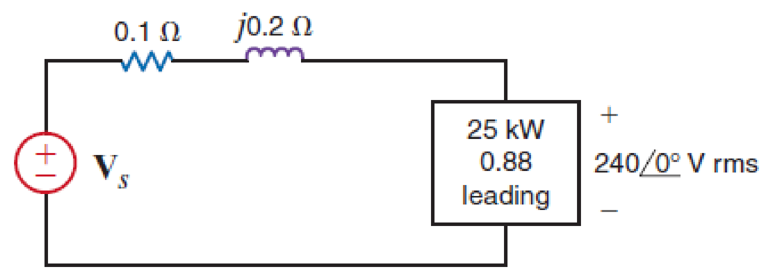


Figure P9.78

HOMEWORK PROBLEMS

9.82 Given the network in Fig. P9.82, determine the input voltage $\mathbf{V}_S$.

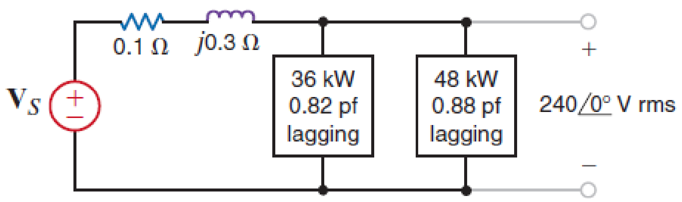


Figure P9.82

9.83 Given the network in Fig. P9.83, determine the input voltage $\mathbf{V}_S$.

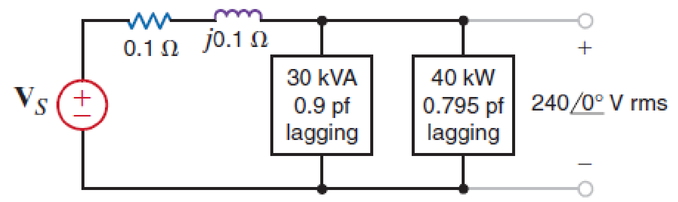


Figure P9.83

9.84 Given the circuit in Fig. P9.84, find the complex power supplied by the source and the source power factor. If $f = 60$ Hz, find $v_s(t)$.

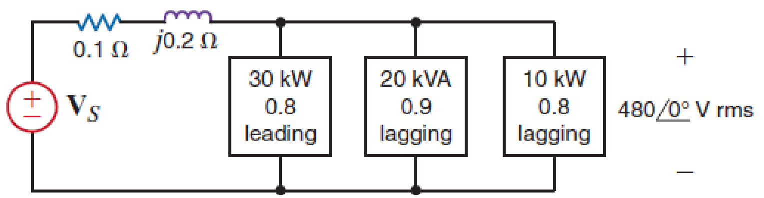


Figure P9.84

9.85 Given the network in Fig. P9.85, compute the input source voltage and the input power factor.

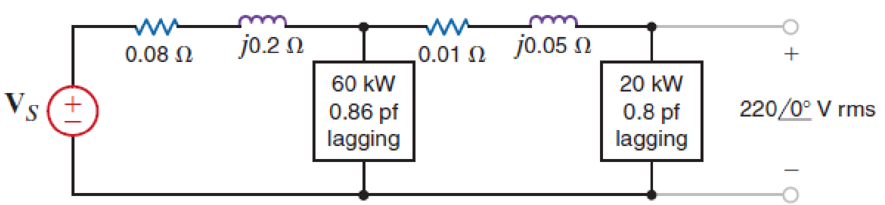


Figure P9.85

HOMEWORK PROBLEMS

9.93 A plant consumes 60 kW at a power factor of 0.75 lagging from a 240-V rms 60-Hz line. Determine the value of the capacitor that when placed in parallel with the load will change the load power factor to 0.9 lagging.

9.94 A bank of induction motors consumes 36 kW at 0.78 pf lagging from a 60-Hz 240 $\underline{/0^\circ}$-V rms line. If 200 μF of capacitors are placed in parallel with the load, what is the new power factor of the total load?

9.95 Calculate the value of capacitance in Fig. P9.95 that must be connected in parallel with the load to correct the source power factor with the load to correct the source power factor to 0.94 lagging. The frequncy $f = 60$ Hz.

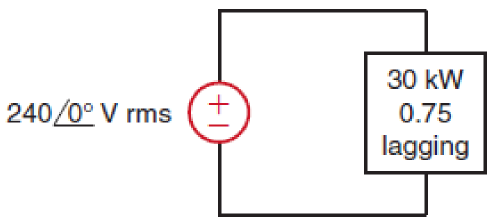


Figure P9.95

9.98 Determine the value of capacitance in Fig. P9.98 that must be connected in parallel with the load so that the power factor of the combined load and capacitor is unity. Calculate the complex power supplied by the source after the power factor has been corrected to unity. The frequency $f = 60$ Hz

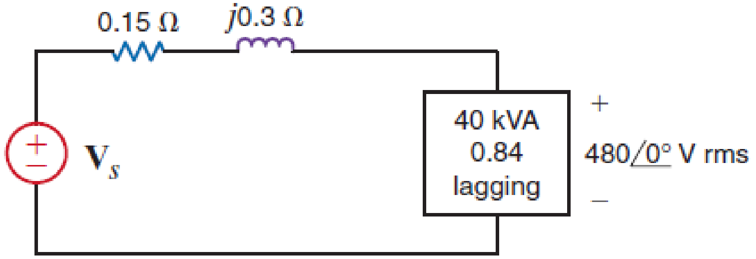


Figure P9.98

9.99 A 5-kW load operates at 60 Hz, 240-V rms and has a power factor of 0.866 lagging. We wish to create a power factor of at least 0.975 lagging using a single capacitor. Can this requirement be met using a single capacitor from Table 6.1?