EEE 472 POWER SYSTEM ANALYSIS II
Introduction and Symmetrical Fault Analysis
(Balanced 3-Phase Faults)

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Course Content

1	06 March	Introduction to Faults on Power Systems and Symmetrical Faults
2	13 March	Symmetrical Fault Analysis using Direct Method and Thevenin Theorem
3	20 March	Bus Admittance and Bus Impedance Matrices
4	27 March	Symmetrical Fault Analysis using Bus Impedance Matrice
5	03 April	Introduction to Symmetrical Components
6	10 April	Sequence Impedances and Sequence Networks
7	17 April	Single Line to Ground Fault
8	24 April	Midterm I
9	30 April	Double Line to Ground Fault
10	01 May	Official Holiday
10	08 May	Line to Line Fault
11	15 May	Introduction to Load Flow
12	22 May	Load Flow Analysis: Gauss and Gauss Siedel Methods
13	29 May	Load Flow Analysis: Newton-Raphson Method
14	05 June	Midterm II
15	12 June	Power System Stability

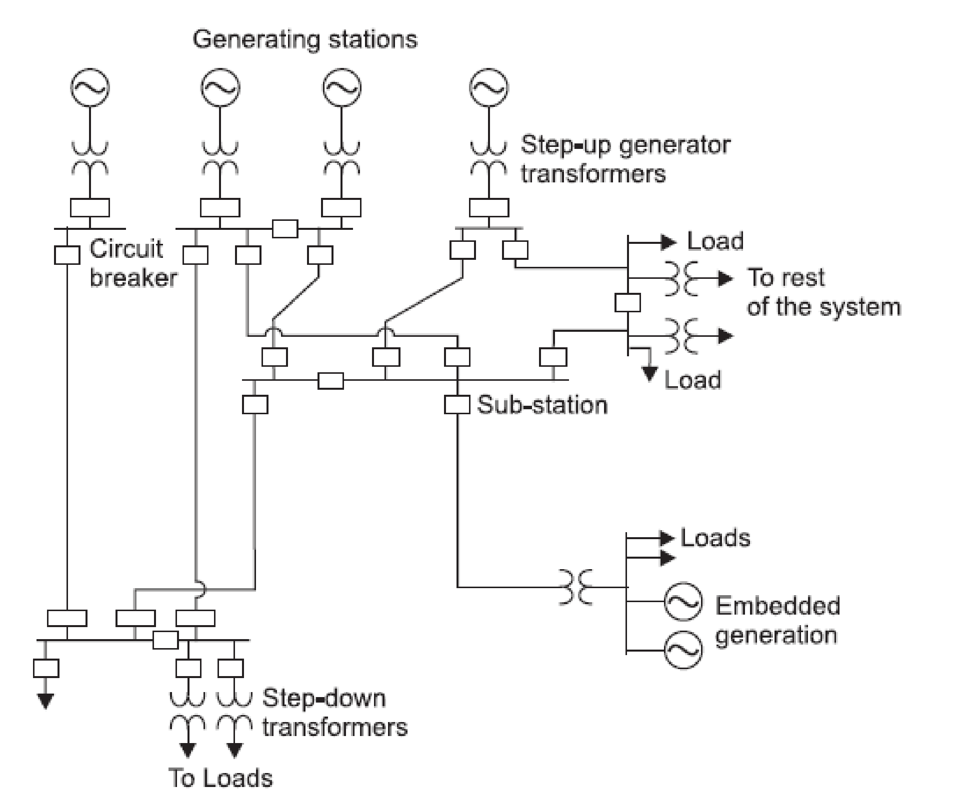
Reference Books

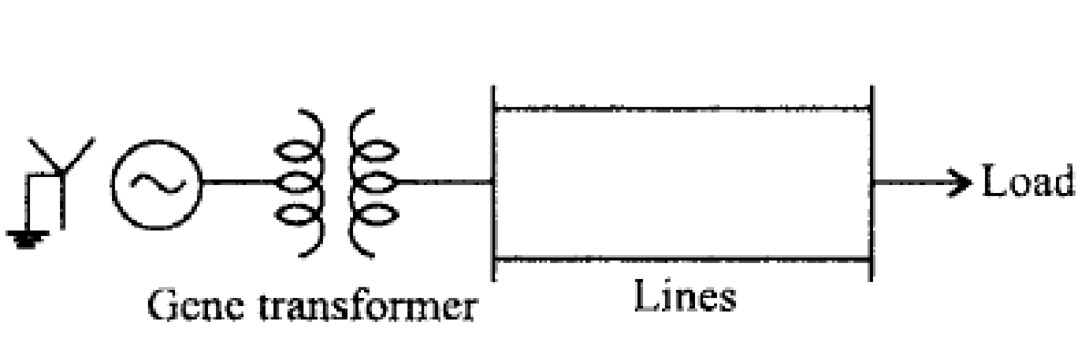
- H. Saadat, Power System Analysis, PSA Publishing 2010.
- J. H. Graigner and W. D. Stevenson, Power System Analysis, McGraw-Hill, 1994.
- A. R. Bergen and V. Vittal, Power System Analysis, Prentice Hall, 2000.
- J. D. Glover, T. J. Overbye and M. S. Sarma, Power System Analysis and Design, CENGAGE Learning, 2016.
- D. Das, Electrical Power Systems by, 2006, New Age International Limited Publishers 2006
- P. S. R. Murty, Power System Analysis BS Publications 2007.
- J.D. Glover, M. S. Sarma & T. J. Overbye, Çeviri Editörü: M. S. Dincer, Güç Sistemlerinin Analizi ve Tasarımı (5th baskıdan çeviri), Nobel Akademik Yayıncılık 2017

Evaluation

Final Grade = Midterm #1*0.3 + Midterm #2*0.3 + Final*0.4

One (single) Line Diagram of Power Systems





What is an Electrical Fault?

- The electrical power system is growing in size and complexity in all sectors such as generation, transmission, distribution, and load systems.
- Types of faults like short circuit conditions in the power system network result in severe economic losses and reduce the reliability of the electrical system.
- An electrical fault is an abnormal condition, caused by equipment failures such as transformers and rotating machines, human errors, and environmental conditions.
- These faults cause interruption to electric flows, equipment damages, and even cause the death of humans, birds, and animals.

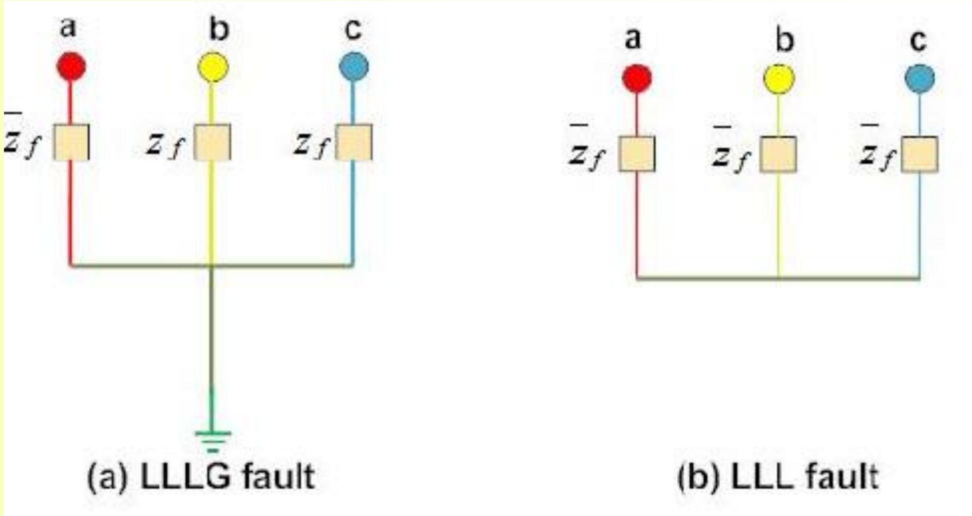
What is an Electrical Fault?

- **An electrical fault** is the deviation of voltages and currents from nominal values or states.
- Under normal operating conditions, power system equipment or lines carry normal voltages and currents which results in safer operation of the system.
- But when a fault occurs, it causes excessively high currents to flow which causes damage to equipment and devices.
- Fault detection and analysis are necessary to select or design suitable switchgear equipment, electromechanical relays, circuit breakers, and other protection devices.

Types of Faults in Electrical Power Systems

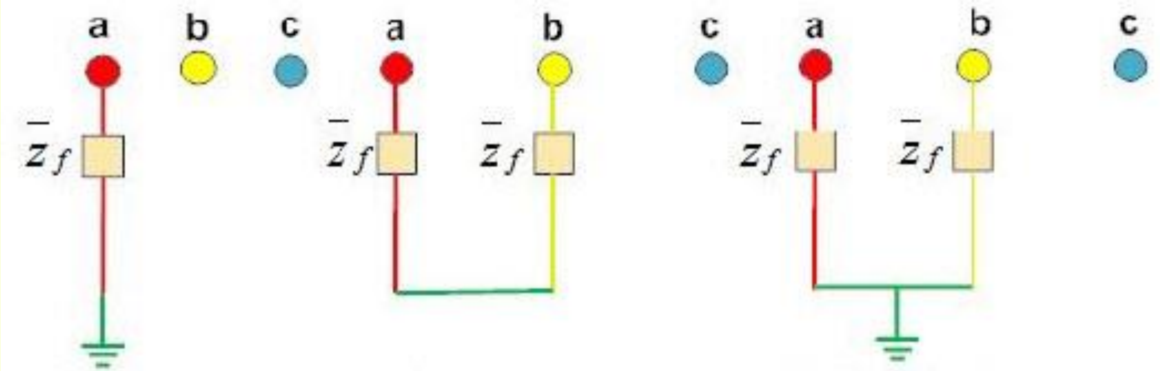
- In the electrical power system, the faults are mainly two types like
  - **open circuit faults**
  - **short circuit faults.**
- Further, these types of faults can be classified into
  - **Symmetrical (Balanced)**
  - **Unsymmetrical (unbalanced)**.

Symmetrical Faults



- These are very severe faults and occur infrequently in the power systems. These are also called balanced faults and are of two types namely line to line to ground **(L-L-L-G)** and line to line **(L-L-L).**
- Only 2-5 percent of system faults are symmetrical faults. If these faults occur, the system remains balanced but results in severe damage to the electrical power system equipment.
- It is the harsh kind of fault that holds the largest current.
- This current is used to determine the rating of the Circuit Breaker (CB)

Unsymmetrical Faults



(a) LG fault (b) LL fault (c) LLG fault

- These are very common and less severe than symmetrical faults.
- There are mainly three types namely **line to ground (L-G), line to line (L-L), and double line to ground (LL-G) faults.**
- The line to ground fault (L-G) is the most common fault and 65-70 percent of faults are of this type. It causes the conductor to make contact with the earth or ground.
- 15 to 20 percent of faults are double line to ground and causes the two conductors to make contact with the ground.
- The line to line faults occurs when two conductors make contact with each other mainly while swinging of lines due to winds and 5- 10 percent of the faults are of this type.
- These are also called unbalanced faults since their occurrence causes unbalance in the system. The unbalance of the system means that that impedance values are different in each phase causing unbalance current to flow in the phases.

Causes of Faults

- **Weather Conditions:** It includes lighting strikes, heavy rains, heavy winds, salt deposition on overhead lines and conductors, snow and ice accumulation on transmission lines, etc. These environmental conditions interrupt the power supply and also damage electrical installations.
- **Equipment Failures:** Various electrical equipment like generators, motors, transformers, reactors, switching devices, etc causes short circuit faults due to malfunctioning, aging, insulation failure of cables, and winding. These failures result in high current to flow through the devices or equipment which further damages it.
- **Human Errors:** Electrical faults are also caused due to human errors such as selecting improper rating of equipment or devices, forgetting metallic or electrical conducting parts after servicing or maintenance, switching the circuit while it is under servicing, etc.
- Other causes: insulation breakdown lightning ionizing air, animals or plants coming in contact with the wires, salt spray or pollution on insulators.

Effects of Faults

- **Over Current Flow:** When the fault occurs it creates a very low impedance path for the current flow. This results in a very high current being drawn from the supply, causing the tripping of relays, damaging insulation and components of the equipment.
- **Danger to Operating Personnel:** Fault occurrence can also cause shocks to individuals. The severity of the shock depends on the current and voltage at the fault location and even may lead to death.
- **Loss of Equipment:** Heavy current due to short circuit faults results in the components being burnt completely which leads to improper working of equipment or device. Sometimes heavy fire causes complete burnout of the equipment.
- **Disturbs Interconnected Power Network:** Faults not only affect the location at which they occur but also disturb the active interconnected circuits to the faulted line.
- **Electrical Fires:** Short circuit causes flashovers and sparks due to the ionization of air between two conducting paths which further leads to fire as we often observe in news such as building and shopping complex fires.

Fault Analysis

- Fault currents cause equipment damage due to both thermal and mechanical processes.
- Goal of fault analysis is to determine the magnitudes of the currents present during the fault:
  - need to determine the maximum current to ensure devices can survive the fault,
  - need to determine the maximum current the circuit breakers (CBs) need to interrupt to correctly size the CBs.

Fault Analysis

- **Analysis types**
  - power flow - evaluate normal operating conditions
  - fault analysis - evaluate abnormal operating conditions
- **Fault types:**
  - balanced faults: three-phase faults
  - unbalanced faults

    - single-line to ground faults

    - double-line to ground faults

    - line-to-line faults

Fault Analysis

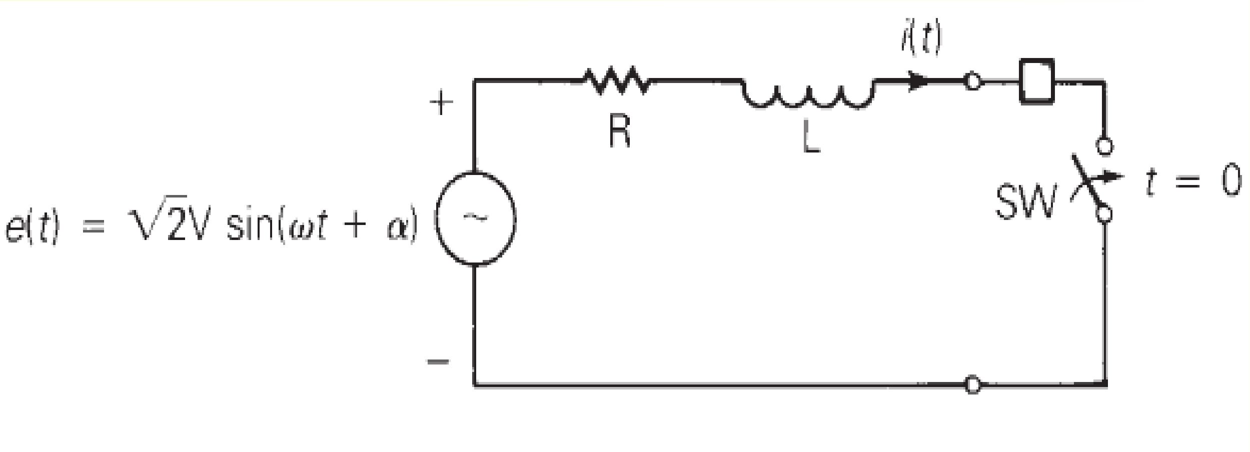
- **Results used for:**
  - specifying ratings for circuit breakers and fuses
  - protective relay settings
  - specifying the impedance of transformers and generators
- **Magnitude of fault currents depend on:**
  - the impedance of the network
  - the internal impedances of the generators
  - the resistance of the fault (arc and grounding resistances)

Fault Analysis

- **Network impedances are governed by**
  - transmission line impedances
  - transformer connections and impedances
  - grounding connections and resistances
- **Generator behavior is divided into three periods**
  - sub-transient period, lasting for the first few cycles
  - transient period, covering a relatively longer time
  - steady-state period

RL Circuit Transient Analysis

- To understand fault analysis we need to review the behavior of an *RL* circuit



$$L\frac{di(t)}{dt}+Ri(t)=\sqrt{2}V\sin(\omega t+\alpha) \quad t\geq 0$$

$$i(t)=i_{ac}(t)+i_{dc}(t)$$

$$=\frac{\sqrt{2}V}{Z}\left[\sin(\omega t+\alpha-\theta)-\sin(\alpha-\theta)e^{-t/T}\right]$$

RL Circuit Transient Analysis

$$i_{ac}(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t + \alpha - \theta) \text{ A}$$

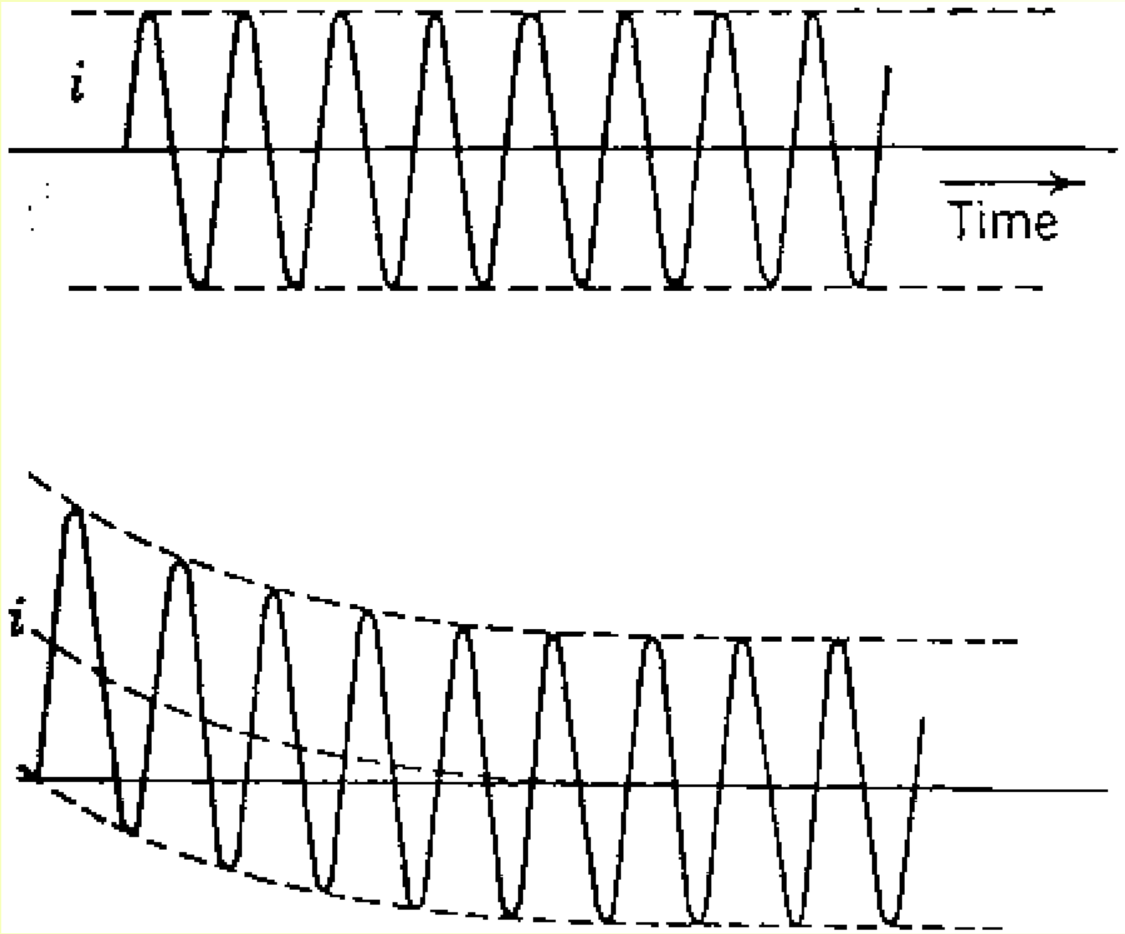
$$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\sin(\alpha - \theta)e^{-t/\tau} \text{ A}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{R^2 + X^2} \ \Omega$$

$$\theta = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{X}{R}$$

$$T = \frac{L}{R} = \frac{X}{\omega R} = \frac{X}{2\pi f R} \text{ s}$$

RL Circuit Transient Analysis: Current Waveform



$(\alpha - \theta = \mathbf{0})$

No DC offset

$(\alpha - \theta = -\pi/\mathbf{2})$

Maximum DC offset

$$i(t) = i_{ac}(t) + i_{dc}(t)$$

$$= \frac{\sqrt{2}V}{Z}\left[\sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta)e^{-t/T}\right]$$

RMS for Fault Current

- We are primarily interested in the largest fault current, we choose $\alpha = \theta - \pi/2$

$$i(t) = \sqrt{2} I_{ac} \left[\sin(\omega t - \pi/2) + e^{-t/T} \right] \text{ A}$$

$$I_{ac} = \frac{V}{Z} \text{ A}$$

$$\begin{aligned} I_{rms}(t) &= \sqrt{[I_{ac}]^2 + [I_{dc}(t)]^2} \\ &= \sqrt{[I_{ac}]^2 + \left[\sqrt{2} I_{ac} e^{-t/T} \right]^2} \\ &= I_{ac} \sqrt{1 + 2e^{-2t/T}} \text{ A} \end{aligned}$$

RMS for Fault Current

- **It is conveient to use** $T = X/(2\pi fR)$ **and** $t = \tau/f$

$$I_{rms}(\tau) = K(\tau)\mathrm{I}_{ac} \text{ A}$$

τ is the cycle or period.

$$K(\tau) = \sqrt{1 + 2e^{-4\pi\tau/(X/R)}} \text{ p.u.}$$

Comments/Observation:

- The rms asymmetrical fault current equals the rms ac fault current times an "asymmetry factor," $K(\tau)$
- For $\tau = 0$ $\quad I_{rms} = \sqrt{3}\mathrm{I}_{ac}$ A
- For large τ $\quad I_{rms} = \mathrm{I}_{ac}$ A

Summary

Component	Instantaneous Current (A)	rms Current (A)
Symmetrical (AC)	$i_{ac}(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t + \alpha - \theta)$ A	$I_{ac} = \frac{V}{Z}$ A
DC offset	$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\sin(\alpha - \theta)e^{-t/\tau}$ A	
Asymmetrical (total)	$i(t) = i_{ac}(t) + i_{dc}(t)$	$I_{rms}(t) = \sqrt{I_{ac}^2 + i_{dc}(t)^2}$ with maximum DC offset $I_{rms}(\tau) = K(\tau)I_{ac}$

Example 1

A bolted short circuit occurs in the series R–L circuit of Figure 3.2 with $V = 20\ \text{kV}$, $X = 8\ \Omega$, $R = 0.8\ \Omega$, and with maximum DC offset. The circuit breaker opens 3 cycles after fault inception. Determine (a) the rms AC fault current, (b) the rms "momentary" current at $\tau = 0.5$ cycle, which passes through the breaker before it opens, and (c) the rms asymmetrical fault current that the breaker interrupts.

$$I_{ac} = \frac{20\times10^3}{\sqrt{8^2+0.8^2}} = \frac{20\times10^3}{8.040} = 2.488\ \text{kA}$$

$$\left(X/R\right) = 8/0.8 = 10$$

$$\tau = 0.5\ \text{cycle}$$

$$K\left(0.5\ \text{cycle}\right) = \sqrt{1+2e^{-4\pi(0.5)/10}} = 1.438$$

$$I_{momentary} = K\left(0.5\ \text{cycle}\right) I_{ac} = (1.438)(2.488) = 3.576\ \text{kA}$$

$$\left(X/R\right) = 10 \quad \tau = 3\ \text{cycle}$$

$$K\left(3\ \text{cycle}\right) = \sqrt{1+2e^{-4\pi(3)/10}} = 1.023$$

$$I_{rms}\left(3\ \text{cycles}\right) = (1.023)(2.488) = 2.544\ \text{kA}$$

Example 2

In the circuit of Figure 8.1, let $R = 0.125\ \Omega$, $L = 10$ mH, and the source voltage be given by $v(t) = 151\sin(377t + \alpha)$. Determine the current response after closing the switch for the following cases.

(a) No dc offset.

(b) For maximum dc offset.

$$i(t) = i_{ac}(t) + i_{dc}(t)$$

$$= \frac{\sqrt{2}V}{Z}\left[\sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta)e^{-t/T}\right]$$

$$Z = 0.125 + j(377)(0.01) = 0.125 + j3.77 = 3.772\angle 88.1^\circ$$

$$I_m = \frac{151}{3.772} = 40 \quad \text{A}$$

and

$$\tau = \frac{L}{R} = 0.08 \quad \text{sec}$$

From (8.2) the response is

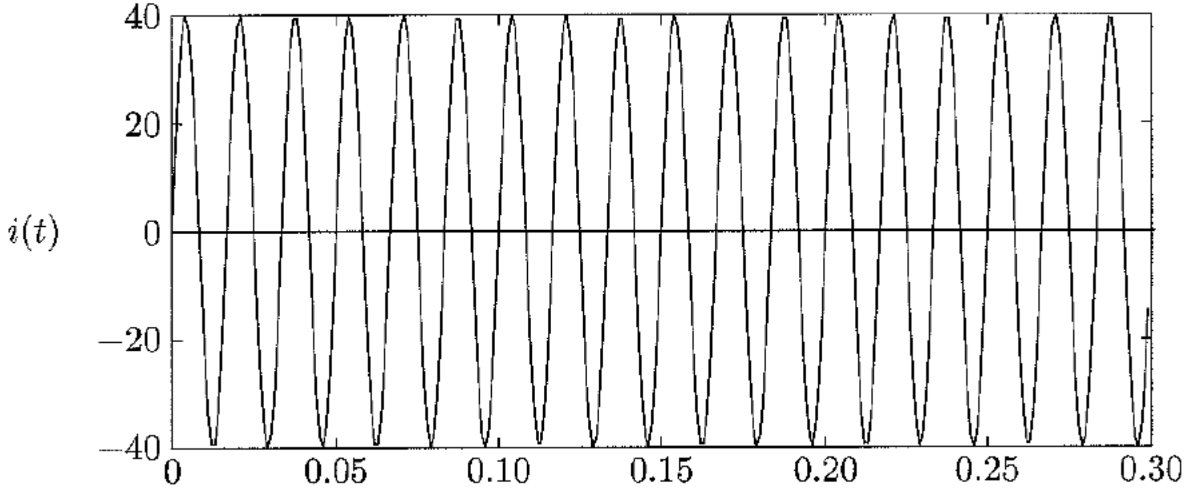
$$i(t) = 40\sin(\omega t + \alpha - 88.1^\circ) - 40e^{-t/0.08}\sin(\alpha - 88.1^\circ)$$

The response has no dc offset if switch is closed when $\alpha = 88.1^\circ$, and it has the maximum dc offset when $\alpha = 88.1^\circ - 90^\circ = -1.9^\circ$. The following commands produce the responses shown in Figures 8.2(a) and 8.2(b).

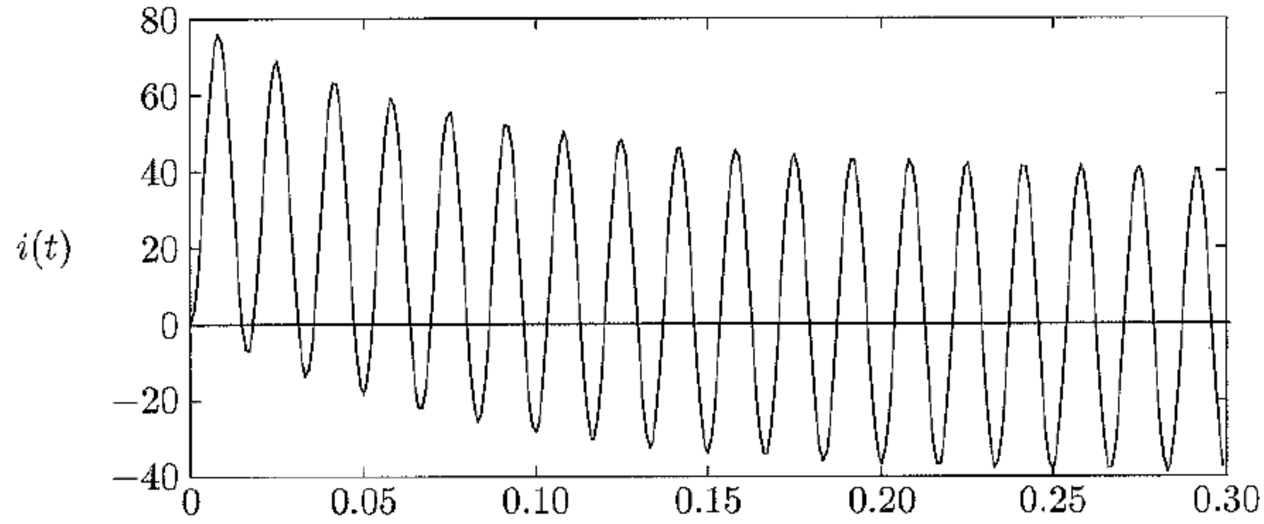
Example 2: Matlab M-File (Cont'd)

```
alf1 = 88.1*pi/180;
alf2 = -1.9*pi/180;
gamma = 88.1*pi/180;
t = 0:.001:.3;
i1 = 40*sin(377*t+alf1-gamma)-40*exp(-t/.08).*sin(alf1-gamma);
i2 = 40*sin(377*t+alf2-gamma)-40*exp(-t/.08).*sin(alf2-gamma);
subplot(2,1,1), plot(t, i1)
xlabel('t, sec'), ylabel('i(t)')
subplot(2,1,2), plot(t, i2)
xlabel('t, sec'), ylabel('i(t)')
subplot(111)
```

Example 2: Current Waveform (Cont'd)



(a)

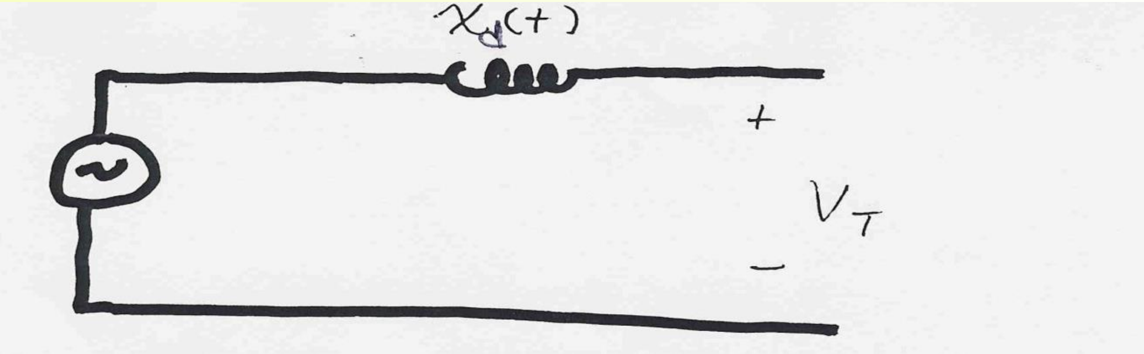


(b)

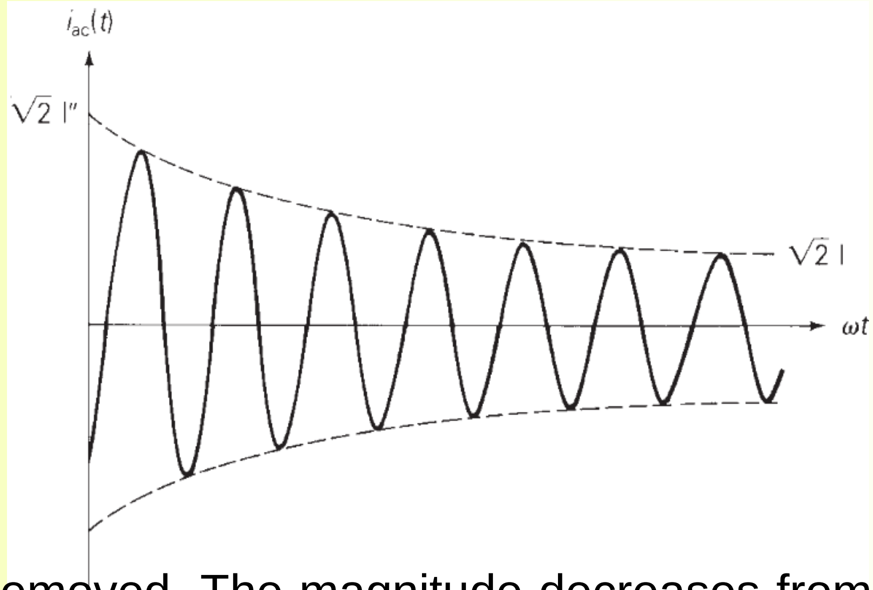
Generator Modeling During Faults

- During a fault the only devices that can contribute fault current are those with energy storage.
- Thus the models of generators (and other rotating machines) are very important since they contribute the bulk of the fault current.
- Generators can be approximated as a constant voltage behind a time-varying reactance:

E_g



Short Circuit Current of an Unloaded Generator



- DC offset is removed. The magnitude decreases from a high intial value to a lower steady-state value.
- A physical explanation for this phenomenon is that the magnetic flux caused by the short-circuit armature currents (or by the resultant armature MMF) is initially forced to flow through high reluctance paths that do not link the field winding or damper circuits of the machine.
- This is a result of the theorem of constant flux linkages, which states that the flux linking a closed winding cannot change instantaneously. The armature inductance, which is inversely proportional to reluctance, is therefore initially low. As the flux then moves toward the lower reluctance paths, the armature inductance increases.

Generator Modeling (cont'd)

The time varying reactance is typically approximated using three different values, each valid for a different time period:

X_d'' = direct-axis subtransient reactance

X_d' = direct-axis transient reactance

X_d = direct-axis synchronous reactance

Can then estimate currents using circuit theory:

For example, could calculate steady-state current that would occur after a three-phase short-circuit if no circuit breakers interrupt current.

$$X_d'' < X_d' < X_d$$

Generator Behavior During Fault

- **Sub-transient period,** $X_G = X''_d$
  - determine the interrupting capacity of HV circuit breakers
  - determine the operation timing of the protective relay system for high-voltage networks
- **Transient period,** $X_G = X'_d$
  - determine the interrupting capacity of MV circuit breakers
  - determine the operation timing of the protective relay system for medium-voltage networks
  - transient stability studies

Generator 3-Phase Fault Current

- The instantaneous AC fault current can be written as

$$i_{ac}(t) = \sqrt{2}E_g \left[\left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/T_d''} + \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/T_d'} + \frac{1}{X_d} \right] \sin\left(\omega t + \alpha - \frac{\pi}{2} \right)$$

- Note that at t = 0 , when the fault occurs, the rms value

$$I_{ac}(0) = \frac{E_g}{X_d''} = I''$$

$T_d'' =$ direct−axis subtransient time constant (≈ 0.035sec)

$T_d' =$ direct−axis transient time constant (≈ 1sec)

- The rms ac fault current then equals the rms transient fault current

$$I' = \frac{E_g}{X_d'}$$

Generator 3-Phase Fault Current

- The instantaneous AC fault current can be written as

$$i_{ac}(t) = \sqrt{2}E_g\left[\left(\frac{1}{X_d''}-\frac{1}{X_d'}\right)e^{-t/T_d''}+\left(\frac{1}{X_d'}-\frac{1}{X_d}\right)e^{-t/T_d'}+\frac{1}{X_d}\right]\sin\left(\omega t+\alpha-\frac{\pi}{2}\right)$$

- Note that at t = 0 , when the fault occurs, the rms value

$$I_{ac}(0) = \frac{E_g}{X_d''} = I''$$

$T_d'' =$ direct−axis subtransient time constant (≈ 0.035sec)

$T_d' =$ direct−axis transient time constant (≈ 1sec)

- The rms ac fault current then equals the rms transient fault current

$$I' = \frac{E_g}{X_d'}$$

Generator 3-Phase Fault Current (cont'd)

- When t is much larger than T_d', the rms AC fault current approaches its steady-state value, given by

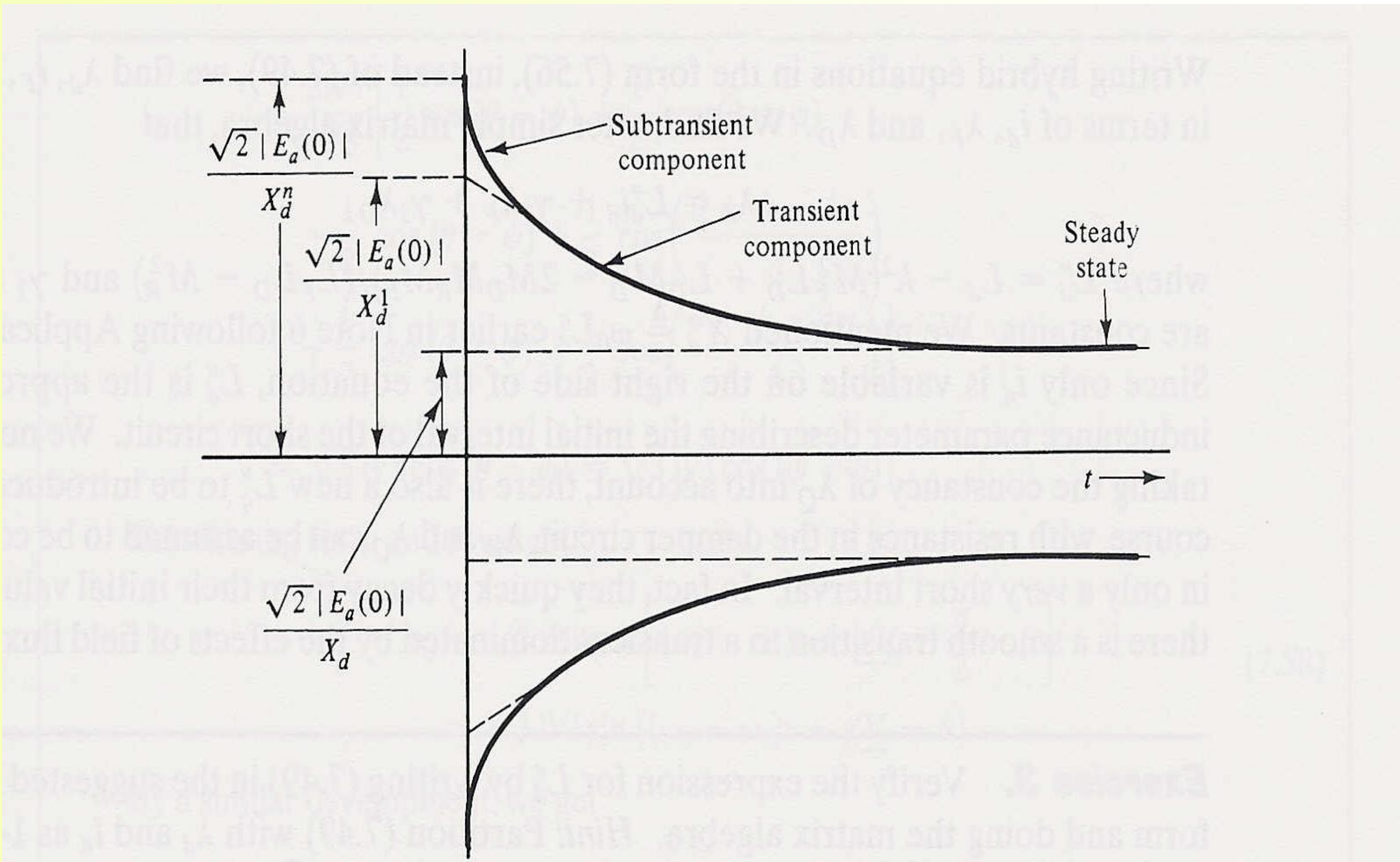
$$I_{ac}(\infty) = \frac{E_g}{X_d} = I$$

The maximum DC offset is

$$I_{\mathrm{DC}}(t) = \frac{\sqrt{2}\,E_g}{X_d''} e^{-t/T_A} = \sqrt{2} I'' e^{-t/T_A}$$

where T_A is the armature time constant (≈ 0.2 seconds)

Generator Short Circuit Currents



Generator Short Circuit Currents

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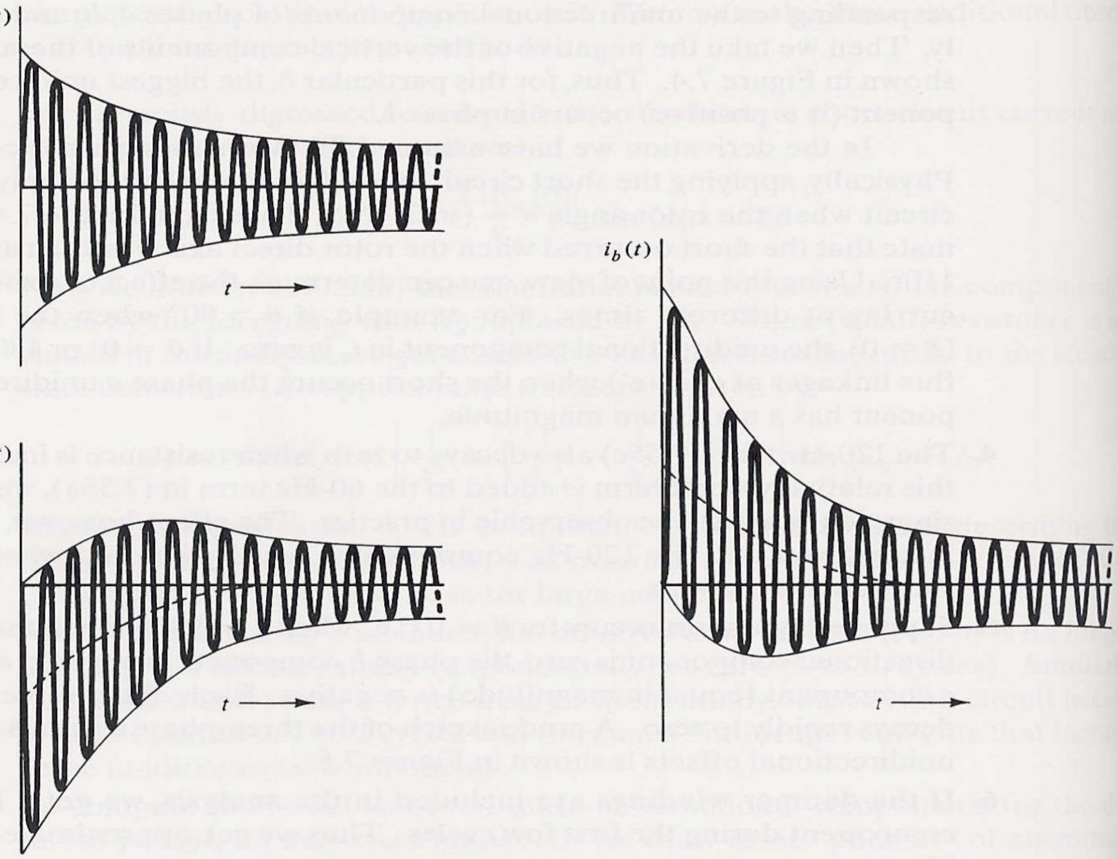


Figure 7.5 Short-circuit currents.

Generator Short Circuit Currents (Summary)

Component	Instantaneous Current (A)	rms Current (A)
Symmetrical (AC)	$i_{ac}(t)=\sqrt{2}\left[\begin{array}{l}\left(\frac{1}{X''_d}-\frac{1}{X'_d}\right)e^{-t/T''_d}\\+\left(\frac{1}{X'_d}-\frac{1}{X_d}\right)e^{-t/T'_d}+\frac{1}{X_d}\end{array}\right]\sin\left(\omega t+\alpha-\frac{\pi}{2}\right)$	$i_{ac}(t)=\sqrt{2}\left[\begin{array}{l}\left(\frac{1}{X''_d}-\frac{1}{X'_d}\right)e^{-t/T''_d}\\+\left(\frac{1}{X'_d}-\frac{1}{X_d}\right)e^{-t/T'_d}+\frac{1}{X_d}\end{array}\right]\sin\left(\omega t+\alpha-\frac{\pi}{2}\right)$
Subtransient		$I''=E_g/X''_d$
Transient		$I'=E_g/X'_d$
Steady-state		$I=E_g/X_d$
Maximum DC offset	$i_{dc}(t)=\sqrt{2}I''e^{-t/T_A}$	
Asymmetrical (total)	$i(t)=i_{ac}(t)+i_{dc}(t)$	$I_{rms}(t)=\sqrt{I_{ac}^2+i_{dc}(t)^2}$ with maximum DC offset $I_{rms}(t)=\sqrt{I_{ac}(t)^2+\left[\sqrt{2}I''e^{-t/T_A}\right]^2}$

Example 3

A 500 MVA , 20 kV , 60 Hz synchronous generator with reactances $X''_d = 0.15$, $X'_d = 0.24$, $X_d = 1.1$ per unit and time constants $T''_d = 0.035$ s , $T'_d = 2.0$ s , $T_A = 0.20$ s is connected to a circuit breaker. The generator is operating at 5% above rated voltage and at no-load when a bolted three-phase short circuit occurs on the load side of the breaker. The breaker interrupts the fault 3 cycles after fault inception. Determine (a) the subtransient fault current in per-unit and kA rms; (b) maximum DC offset as a function of time; and (c) rms asymmetrical fault current, which the breaker interrupts, assuming maximum DC offset.

Solution.

a. The no-load voltage before the fault occurs is $E_g = 1.05$ p.u.. From (3.14), the subtransient fault current that occurs in each of the three phases is

$$I'' = \frac{1.05}{0.15} = 7.0 \text{ p.u.}$$

Example 3 (Cont'd)

$$I_{base} = \frac{S_{rated}}{\sqrt{3}V_{rated}} = \frac{500 \text{ MVA}}{\sqrt{3}20 \text{ kV}} = 14.43 \text{ kA}$$

The rms subtransient fault current in kA is the per-unit value multiplied by the base current:

$$I'' = (7.0)(14.43 \text{ kA}) = 101.0 \text{ kA}$$

b. From (3.17), the maximum DC offset that may occur in any one phase is

$$i_{dc\max}(t) = \sqrt{2}(101.0 \text{ kA})e^{-t/0.20} = 142.9e^{-5t} \text{ kA}$$

c. From (3.13), the rms ac fault current at $t = 3 \text{ cycles} = 0.05 \text{ s}$ is

$$I_{ac}(0.05 \text{ s}) = 1.05\left[\left(\frac{1}{0.15} - \frac{1}{0.24}\right)e^{-0.05/0.035} + \left(\frac{1}{0.24} - \frac{1}{1.1}\right)e^{-0.05/0.20} + \frac{1}{1.1}\right]$$
$$= 4.920 \text{ p.u.}$$
$$= (4.920)(14.43 \text{ kA}) = 71.01 \text{ kA}$$

Modifying (3.10) to account for the time-varying symmetrical component of fault current, we obtain

$$I_{rms}(0.05 \text{ s}) = \sqrt{\left[I_{ac}(0.05)\right]^2 + \left[\sqrt{2}I''e^{-t/T_A}\right]^2}$$
$$= I_{ac}(0.05)\sqrt{1 + 2\left[\frac{I''}{I_{ac}(0.05)}\right]^2 e^{-2t/T_A}}$$
$$= (71.1 \text{ kA})\sqrt{1 + 2\left[\frac{101}{71.01}\right]^2 e^{-2(0.05)/0.20}}$$
$$= (71.1 \text{ kA})(1.8585) = 132 \text{ kA}$$

Three-Phase Short Circuit Analysis

To simplify analysis of fault currents in networks we'll make several simplifications:

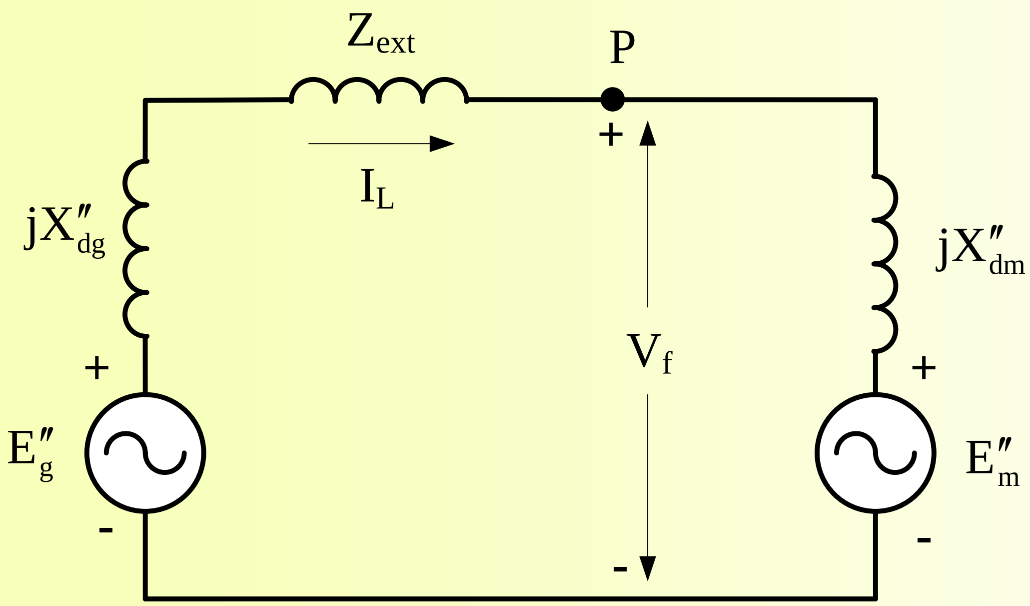
- Transmission lines are represented by their series reactance
- Transformers are represented by their leakage reactances
- Synchronous machines are modeled as a constant voltage behind direct-axis subtransient reactance
- Induction motors are ignored or treated as synchronous machines
- Other (nonspinning) loads are ignored

Fault Analysis Solution Techniques

- Circuit models used during the fault allow the network to be represented as a linear circuit
- There are two main methods for solving for fault currents:
  - ✓ **Direct method:** Use prefault conditions to solve for the internal machine voltages; then apply fault and solve directly.
  - ✓ **Superposition:** Fault is represented by two opposing voltage sources; solve system by superposition:
    - first voltage just represents the prefault operating point
    - second system only has a single voltage source and Thevenin theorem is used.

Direct Method: Prefault Condition

- Prefault condition
  - A generator is supplying a motor load
  - Using the generator terminal voltage V_t or the prefault voltage at the point of fault, P, V_f, and prefault load current we compute the internal voltages of the generator and motor is determined



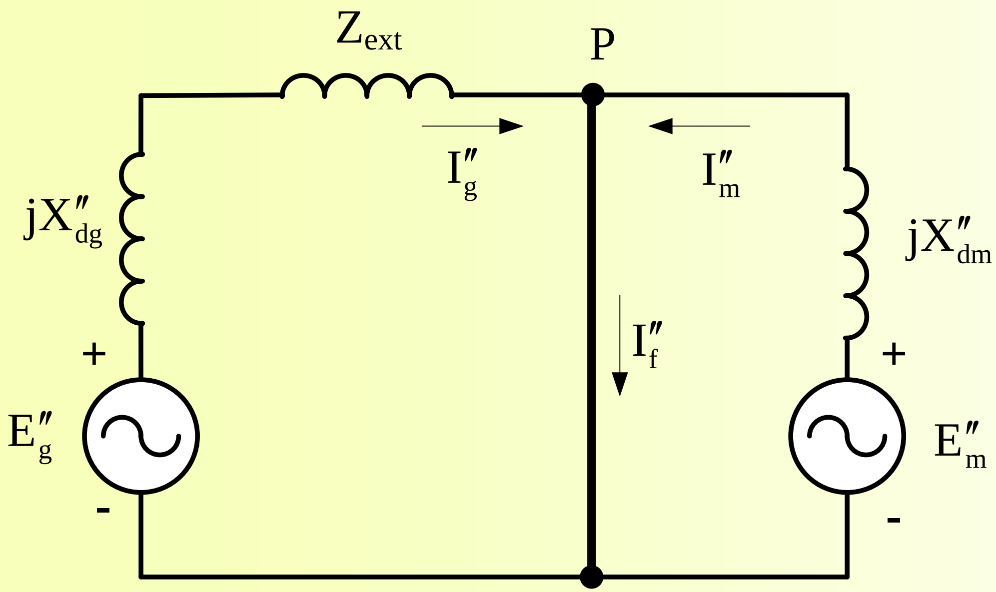
$$E''_g = V_t + jX''_{dg} I_L$$

$$= V_f + (Z_{ext} + jX''_{dg}) I_L$$

$$E''_m = V_f - jX''_{dm} I_L$$

Direct Method: Underfault Condition

- Underfault



Internal voltage

$$E''_g = V_f + (Z_{ext} + jX''_{dg})I_L$$

$$E''_m = V_f - jX''_{dm}I_L$$

Currents

$$I''_g = \frac{E''_g}{Z_{ext} + jX''_{dg}} = \frac{V_f}{Z_{ext} + jX''_{dg}} + I_L$$

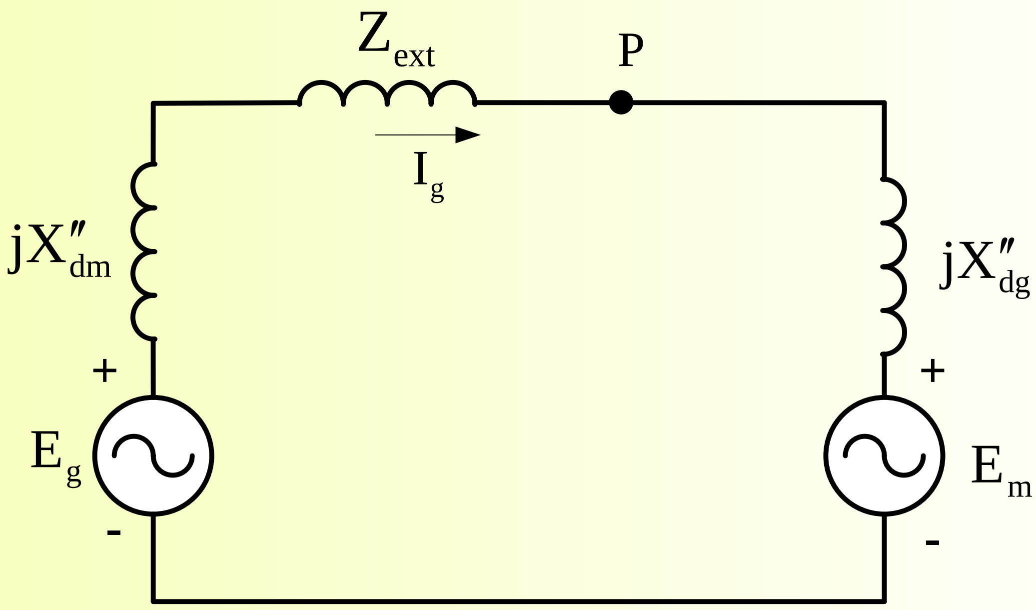
$$I''_m = \frac{E''_m}{jX''_{dm}} = \frac{V_f}{jX''_{dm}} - I_L$$

Fault currents

$$I''_f = I''_m + I''_g = \underbrace{\frac{V_f}{Z_{ext} + jX''_{dg}}}_{I''_{gf}} + \underbrace{\frac{V_f}{jX''_{dm}}}_{I''_{mf}}$$

Example 4: Direct Method

- A synchronous generator and motor are rated 30,000kVA, 13.2 kV with subtransient reactances of 20% connected thru a reactance of 10%. The motor is drawing 20,000kW at 0.8 p.f. leading @12.8kV. A 3-phase fault occurs @motor terminals. Find fault currents



Example 4: Direct Method (cont'd)

- Prefault condition
  - Line current

$$V_f = \frac{12.8}{13.2} = 0.97\angle 0\, pu$$

  - Current in p.u.

$$I_L = \frac{20{,}000\angle 36.9}{0.8\times\sqrt{3}\times 12.8} = 1128\angle 36.9\, A$$

$$I_{base} = \frac{30{,}000}{13.2\sqrt{3}} = 1312\, A \quad \Rightarrow \quad I_L = \frac{1128\angle 36.9}{1312} = 0.86\angle 36.9\, pu$$

$$= 0.69 + j0.52\, pu$$

- During fault
  - Generator

$$V_t = 0.97 + j0.1(0.69 + j0.52) = 0.918 + j0.069\, pu$$

$$E_g'' = 0.918 + j0.069 + j0.2(0.69 + j0.52) = 0.814 + j0.207\, pu$$

Example 4: Direct Method (cont'd)

- During fault
  - Generator: $I_g'' = (0.814 + j0.207) / j0.3 = 0.69 - j2.71 pu$
  - Motor: $V_f = 0.97\angle 0 pu$

$$E_m'' = 0.97 - j0.2(0.69 + j0.52) = 1.074 - j0.138 pu$$

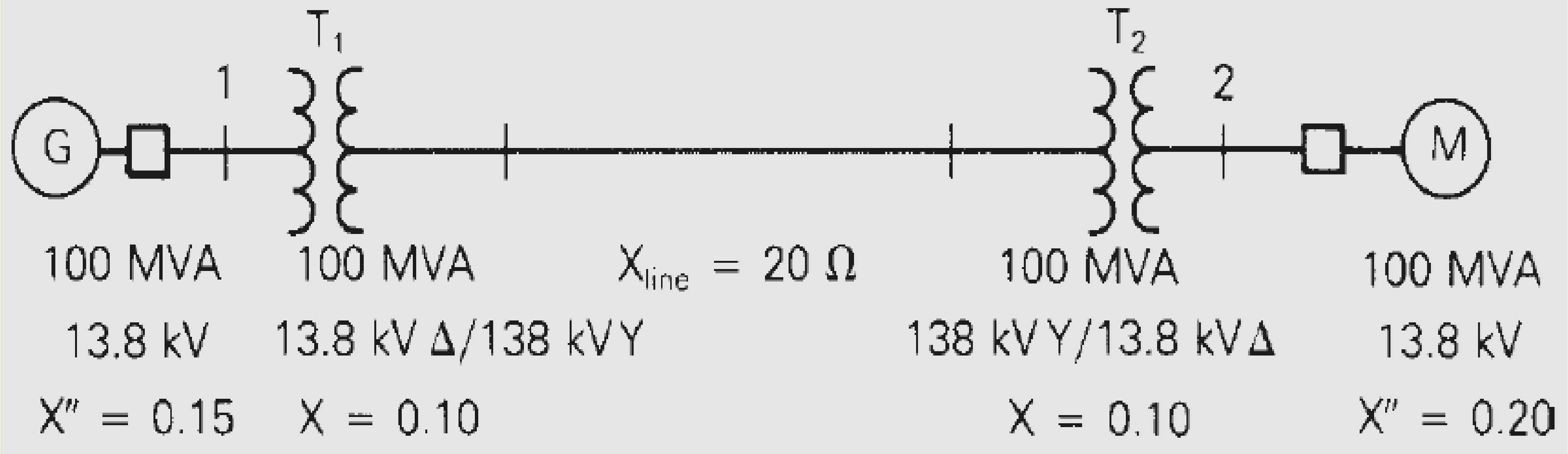
$$I_m'' = (1.074 - j0.138) / j0.2 = -0.69 - j5.37$$

- The fault

$$I_f'' = I_g'' + I_m'' = 0.69 - j2.71 - 0.69 - j5.37 = -j8.08 pu$$

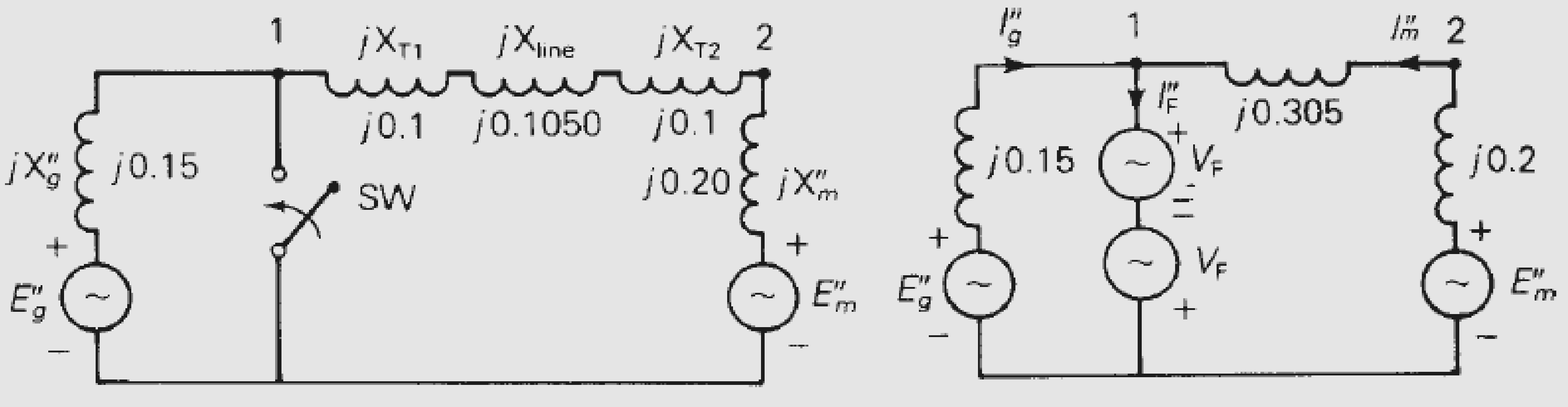
$$I_f'' = (-j8.08 pu)(1312 A) = -j10601 A$$

Superposition Approach



- Consider a three-phase short circuit at bus 1

Superposition Approach (cont'd)

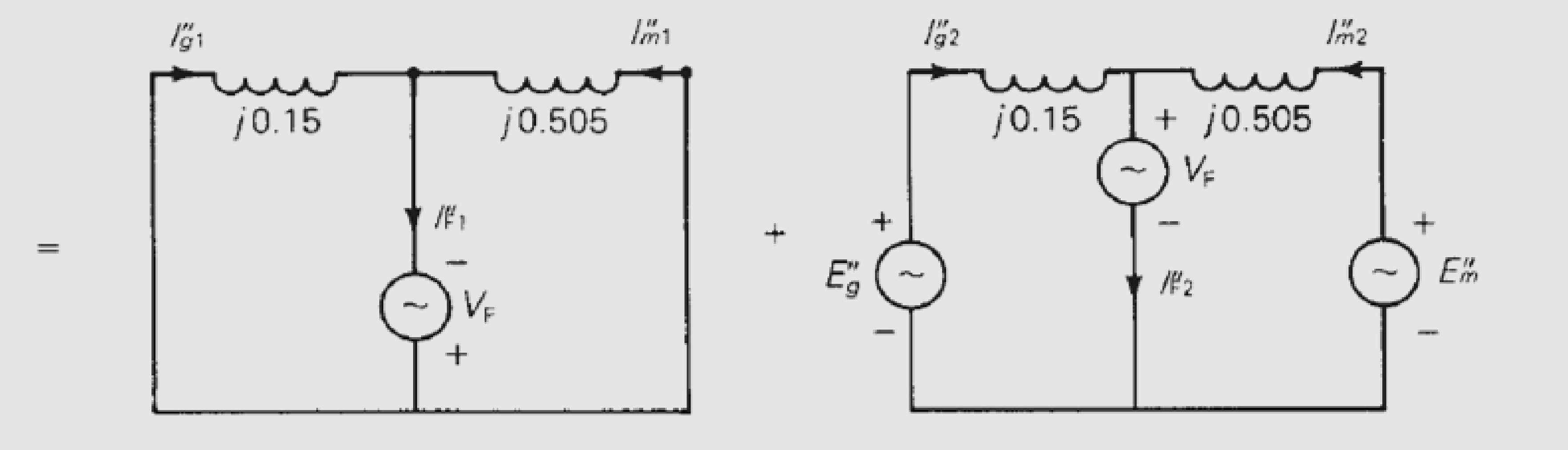


(a) Three-phase short circuit

(b) Short circuit represented by two opposing voltage sources

- The voltages Eg and Em are the prefault internal voltages behind the subtransient reactances of the machines, and the closing of switch SW represents the fault.
- For purposes of calculating the subtransient fault current, Eg and Em are assumed to be constant-voltage sources.
- The fault is represented by two opposing voltage Sources with equal phasor values V_F.

Superposition Approach (cont'd)

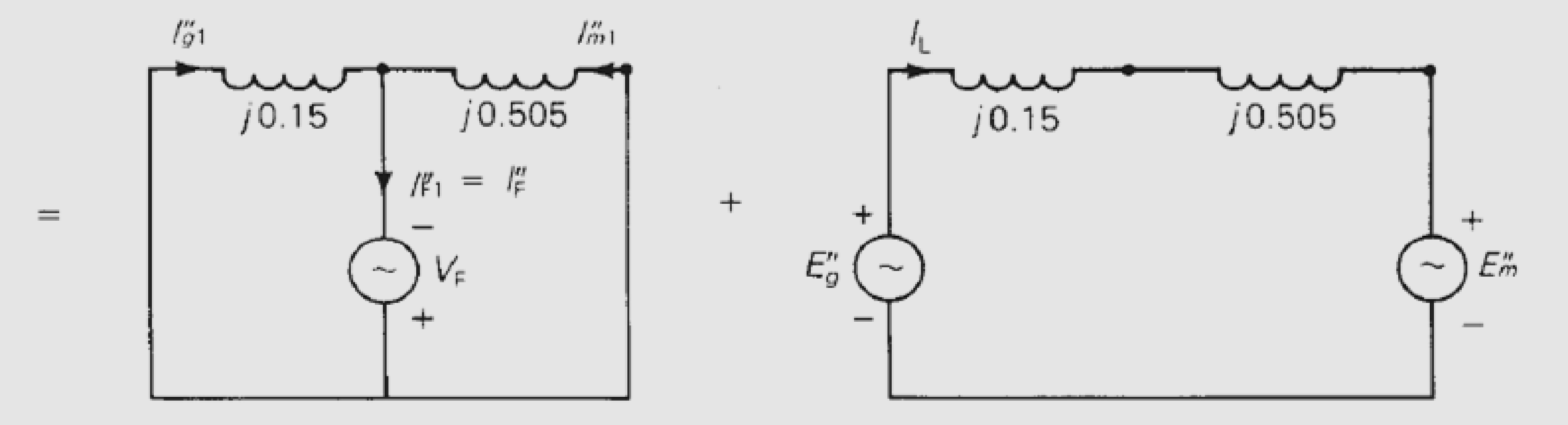


(c) Application of superposition

- if V_F equals the prefault voltage at the fault, then the second circuit represents the system before the fault occurs. $I''_{F2} = 0$

- V_F which has no efect, can be removed from the second circuit

Superposition Approach (cont'd)



(d) V_F set equal to prefault voltage at fault

$$I''_F = I''_{F1}$$

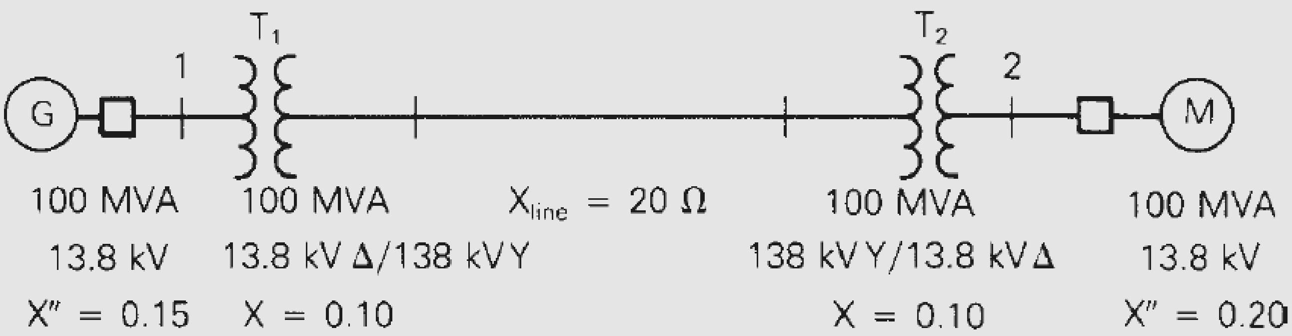
$$I''_g = I''_{g1} + I''_{g2} = I''_{g1} + I_L,$$

$$I''_m = I''_{m1} - I_L$$

where I_L is the prefault generator current

Example 5

The synchronous generator in Figure 3.4 is operating at rated MVA, 0.95 p.f. lagging and at 5% above rated voltage when a bolted three-phase short circuit occurs at bus 1. Calculate the per-unit values of (a) subtransient fault current; (b) subtransient generator and motor currents, neglecting prefault current; and (c) subtransient generator and motor currents including prefault current.

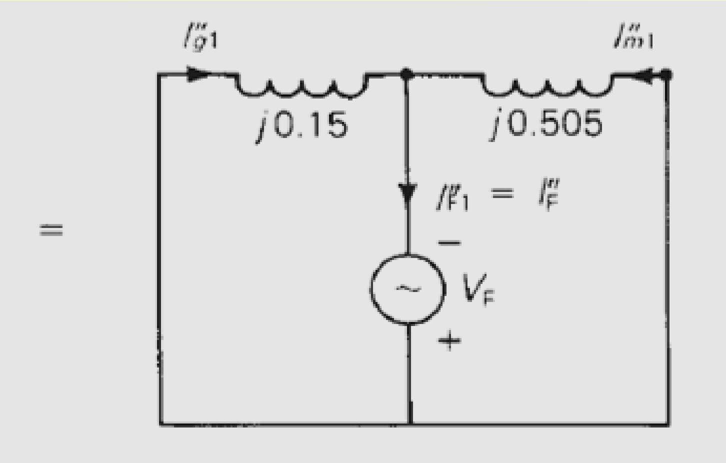


Example 5 (cont'd)

$$Z_{\text{base,line}} = \frac{(138\ \text{kV})^2}{100\ \text{MVA}} = 190.44\ \Omega$$

$$x_{\text{ine}} = \frac{20}{190.44} = 0.105\ \Omega$$

=



$$Z_{\text{Th}} = jX_{\text{Th}} = j\frac{(0.15)(0.505)}{(0.15+0.505)} = j0.11565\ \text{p.u.}$$

$$V_{\text{F}} = 1.05\underline{|0^\circ}\ \text{p.u.}$$

$$I''_{\text{F}} = \frac{V_{\text{F}}}{Z_{\text{Th}}} = \frac{1.05\underline{|0^\circ}}{j0.11565} = -j9.079\ \text{p.u.}$$

$$I''_{g1} = \left(\frac{0.505}{0.505+0.15}\right)I''_F = (0.7710)(-j9.079) = -j7.000\ \text{p.u.}$$

$$I''_{m1} = \left(\frac{0.15}{0.505+0.15}\right)I''_F = (0.2290)(-j9.079) = -j2.079\ \text{p.u.}$$

Example 5 (cont'd)

$$I_{\text{base,gen}} = \frac{100\text{ MVA}}{\sqrt{3}\left(13.8\text{ kV}\right)} = 4.1837\text{ kA}$$

$$\begin{aligned} I_{\text{L}} &= \frac{100\text{ MVA}}{\sqrt{3}\left(1.05\times 13.8\text{ kV}\right)}\underline{\left|-\cos^{-1}0.95\right.} = 3.9845\underline{\left|-18.19^\circ\right.}\text{ kA} \\ &= \frac{3.9845\underline{\left|-18.19^\circ\right.}\text{ kA}}{4.1837\text{ kA}} = 0.9542\underline{\left|-18.19^\circ\right.}\text{ p.u.} \\ &= 0.9048 - j0.2974\text{ p.u.} \end{aligned}$$

$$\begin{aligned} I_g'' &= I_{g1}'' + I_{\text{L}} \\ &= -j7.000 + 0.9048 - j0.2974 \\ &= 0.9048 - j7.297 = 7.353\underline{\left|-82.9^\circ\right.}\text{ p.u.} \end{aligned}$$

$$\begin{aligned} I_m'' &= I_{m1}'' - I_{\text{L}} \\ &= -j2.079 - 0.9048 + j0.2974 \\ &= -0.9048 - j1.7802 = 1.999\underline{\left|-243.1^\circ\right.}\text{ p.u.} \end{aligned}$$

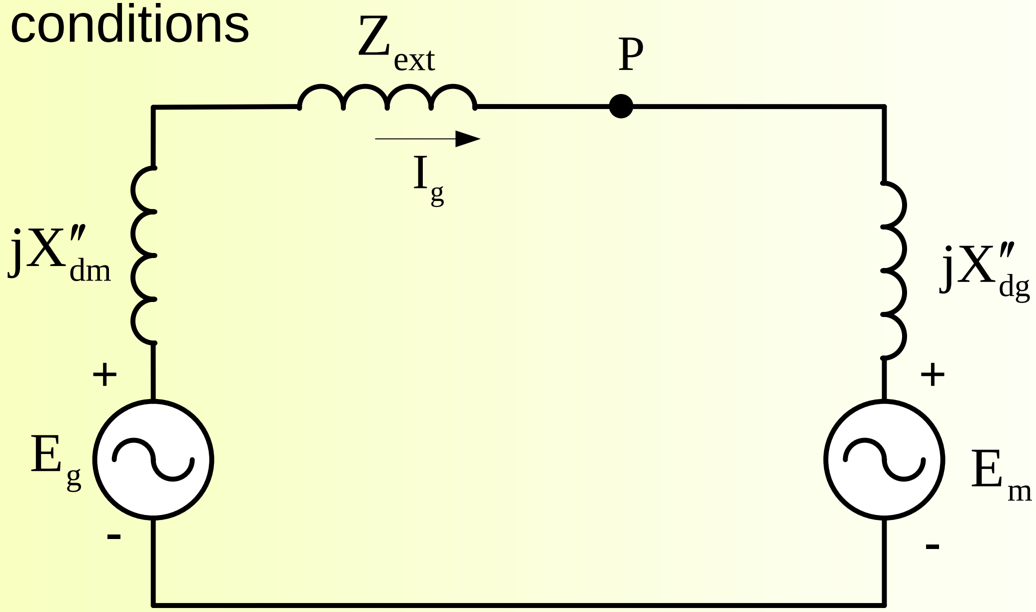
Thevenin Method

- In superpostion approach, if prefault loading conditions are ignored, that means prefault load current is zero and all buses have the same voltage.
- In a such a case, we need to know the prefault voltage V_F at the bus where fault occurs. All we need is to determine Thevenin equivalent of the system seen from the faulted bus.
- If desired, the prefault loading conditions will then be taking into account.
- If the fault impedance is zero, the fault is referred to as a “bolted fault” or “solid fault”

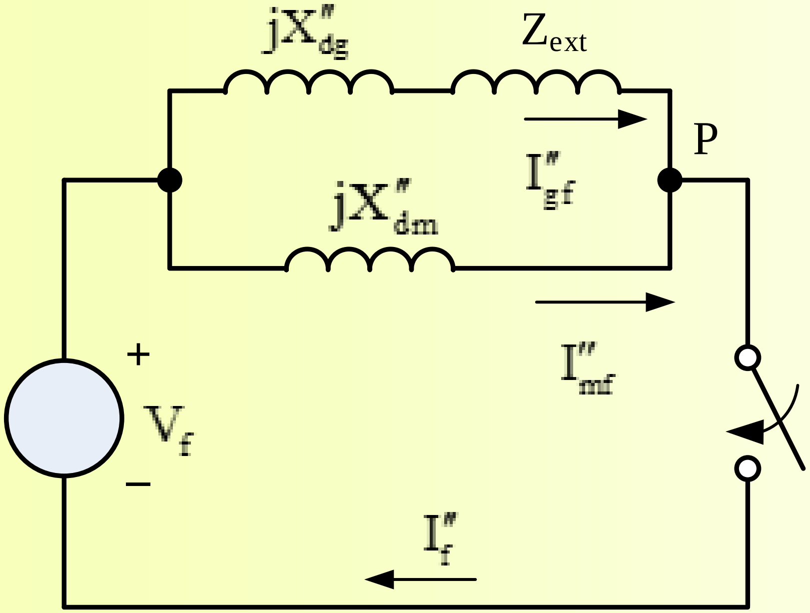
Example 6: Thevenin Method

A synchronous generator and motor are rated 30,000kVA, 13.2 kV with subtransient reactances of 20% connected thru a reactance of 10%. The motor is drawing 20,000kW at 0.8 p.f. leading @12.8kV. A 3-phase fault occurs @motor terminals.

- Find fault currents of generator and motor without considering prefault loading conditions
- Find fault currents of generator and motor considering prefault loading conditions



Example 6: Thevenin Method (cont'd)



$$Z_{th} = \frac{j0.3 \times j0.2}{j0.3 + j0.2} = j0.12$$

$$V_f = 0.97\angle 0$$

$$I''_f = \frac{V_f}{Z_{th}} = \frac{0.97}{j0.12} = -j8.08$$

$$I''_{gf} = -j8.08 \times j0.2 / j0.5 = -j3.23$$

$$I''_{mf} = -j8.08 \times j0.3 / j0.5 = -j4.85$$

$$I_L = 0.69 + j0.52\,pu$$

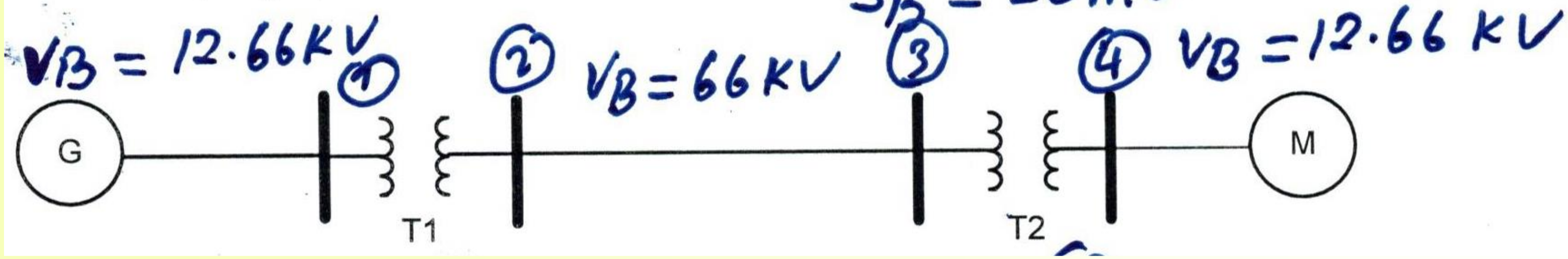
$$I''_{gf} + I_L = -j3.23 + 0.69 + j0.52 = 0.69 - j2.71\,pu$$

$$I''_{mf} - I_L = -j4.85 - (0.69 + j0.52) = -0.69 - j5.37\,pu$$

Example 7: Thevenin method

A synchronous generator and a synchronous motor each rated 20 MVA, 12.66 KV having 15 % transient reactance are connected through transformers and a line as shown below. The transformers are rated 20 MVA, 12.66/66 kV and 66/12.66 kV with leakage reactance of 10 % each. The line has a reactance of 8% on a base of 20 MVA, 66 kV. The motor is drawing 10 MW at 0.8 leading power factor and a terminal voltage 11 kV when a symmetrical three-phase fault occurs at the motor terminals. By using **Thevenin method**, determine the fault current, generator and motor fault currents **(35 pts).**

$S_B = 20\,MVA$



Example 7: Thevenin method

motor terminal voltage :

$$V_4 = \frac{11}{12.66} = 0.8688\angle 0^\circ \text{ pu}$$

$$I = \frac{P}{\sqrt{3}V\cos\phi} = \frac{10\times 10^6}{\sqrt{3}(11\times 10^3)(0.8)} = 656\text{A}$$

5

Base current

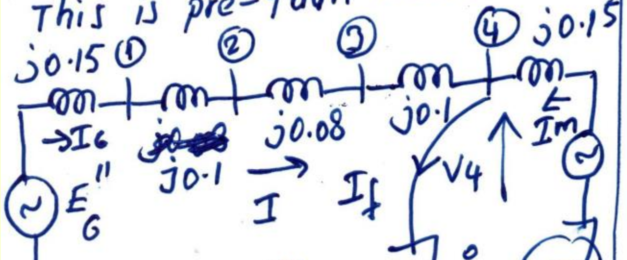
$$I_B = \frac{20\times 10^6}{\sqrt{3}(12.66)\times 10^3} = 912.086\text{A}$$

$$I_{pu} = \frac{656}{912.086}\angle 36.87^\circ \text{ pu}$$

$$I_{pu} = 0.7192\angle 36.87^\circ \text{ pu}$$

5

This is pre-fault current.



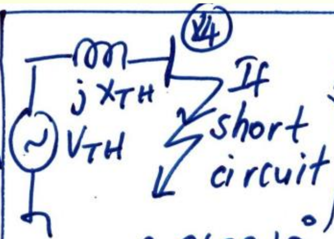
5

$$V_{TH} = V_4 = 0.8688\angle 0^\circ$$

$$X_{TH} = (j0.15) \,//\, (j0.15 + j0.1 + j0.1 + j0.08)$$

$$X_{TH} = (j0.15) \,//\, (j0.43)$$

$$X_{TH} = j0.1112 \text{ pu}$$



Fault current

$$I_f = \frac{V_{TH}}{jX_{TH}}$$

5

$$I_f = \frac{0.8688\angle 0^\circ}{j0.1112} = -j7.811 \text{ pu}$$

$$I_f = (-j7.811)(912.086)$$

$$I_f = -j7126.08 \text{ A}$$

5

$$I_G = I + \frac{j0.15}{j0.58} I_f$$

$$I_G = 0.7192\angle 36.87^\circ + \frac{j0.15}{j0.58}(-j7.811)$$

$$I_G = 1.689\angle -70.107^\circ \text{ pu}$$

$$I_G = (1.689\angle -70.107^\circ)(912.086\text{ A})$$

$$I_G = 1540.51\text{ A}\angle -70.107^\circ \text{A}$$

5

$$I_m = -I + \frac{j0.43}{j0.58} I_f$$

$$I_m = -0.7192\angle 36.87^\circ + \frac{j0.43}{j0.58}(-j7.811)$$

5

$$I_m = 6.24\angle -95.28^\circ \text{ pu}$$

$$I_m = (6.24\angle -95.28^\circ)(912.086\text{A})$$

$$I_m = 5691.4\angle -95.28^\circ \text{ A}$$

Example 8: Thevenin method

Example 3.4. Two generators are connected in parallel to the LV side of a 3-phase delta-star transformer as shown in Fig. 3.6. Generator 1 is rated 60 MVA, 11 kV. Generator 2 is rated 30 MVA, 11 kV. Each generator has a subtransient reactance of 25%. The transformer is rated 90 MVA at 11 kV Δ / 66 kV Y with a reactance of 10%. Before a fault occurred the voltage on the HV side of the transformer is 63 kV. The transformer in unloaded and there is no circulating current between the generators. Find the subtransient current in each generator when a 3-phase short circuit occurs on the HV side of the transformer.

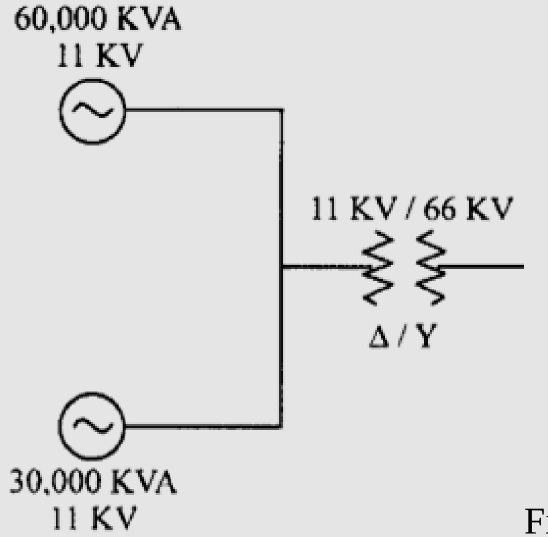


Figure 3.6

Example 8: Thevenin method

Let the line voltage on the h.v. side be the base KV = 66 KV.

Let the base KVA = 90,000 KVA

Generator 1 : $x''_d = 0.25 \times \dfrac{90,000}{60,000} = 0.375$ p.u.

For generator 2 : $x''_d = \dfrac{90,000}{30,000} = 0.75$ p.u.

The internal voltage for generator 1

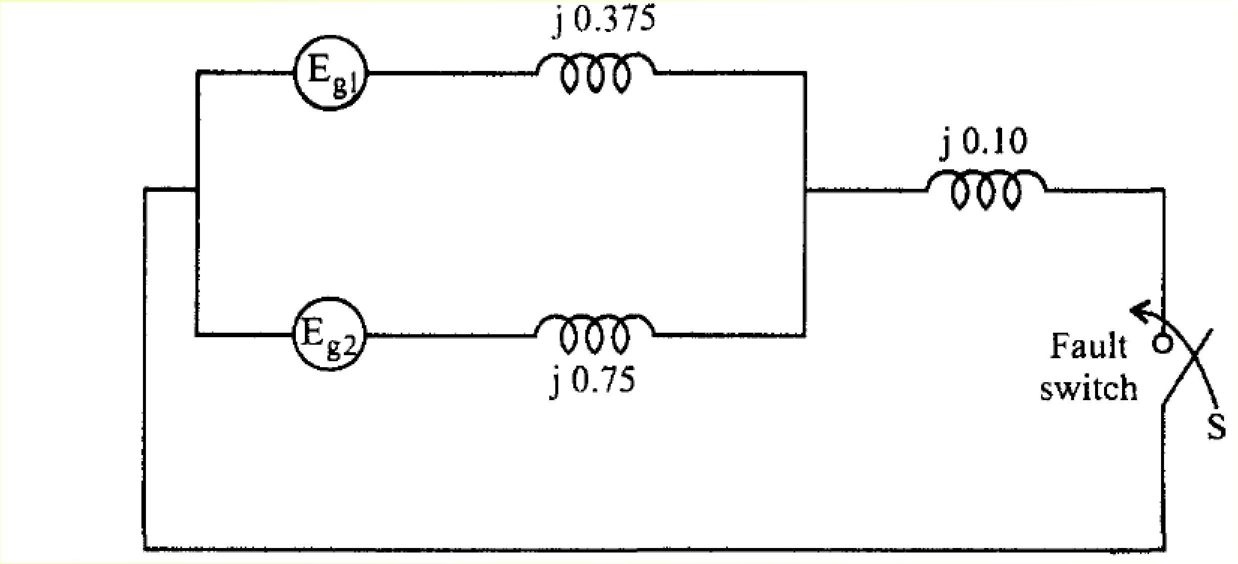
$$E_{g1} = \frac{0.63}{0.66} = 0.955 \text{ p.u.}$$

The internal voltage for generator 2

$$E_{g2} = \frac{0.63}{0.66} = 0.955 \text{ p.u.}$$

The reactance diagram is shown in Fig. E.6.7 when switch S is closed, the fault condition is simulated. As there is no circulating current between the generators, the equivalent reactance of the parallel circuit is $\dfrac{0.375 \times 0.75}{0.375 + 0.75} = 0.25$ p.u.

Example 8: Thevenin method



The subtransient current,

$$I'' = \frac{0.955}{(j0.25 + j0.10)} = -j2.7285 \text{ p.u.}$$

The voltage as the delta side of the transformer is $(-j2.7285)(j0.10) = 0.27285$ p.u.

Example 8: Thevenin method

The subtransient current flowing into fault from Generator 1 and 2,

$$I_1'' = \frac{0.955 - 0.2728}{j0.375} = -j1.8057 \text{ p.u.}$$

$$I_2'' = \frac{0.955 - 0.2728}{j0.75} = -j0.9029 \text{ p.u.}$$

The base value of current in the generator side,

$$I_B = \frac{90 \text{ MVA}}{\sqrt{3}.11 \text{ kV}} = 4.7238 \text{ kA}$$

The actual fault currents supplied in amperes are,

$$I_1'' = I_{1(\text{p.u.})}'' I_B = \left(-j1.8057\right)\left(4.7238 \text{ kA}\right) = j8.5297 \text{ kA}$$

$$I_1'' = I_{1(\text{p.u.})}'' I_B = \left(-j0.9029\right)\left(4.7238 \text{ kA}\right) = -j4.2649 \text{ kA}$$

Example 9: Thevenin method

Example 3.5. R station with two generators feeds through transformers a transmission system operating at 132 kV . The far end of the transmission system consisting of 200 km long double circuit line is connected to load from bus B. If a 3-phase fault occurs at bus B, determine the total fault current and fault current supplied by each generator. Select 75 MVA and 11 kV on LV side and 132 kV on HV side as base values.

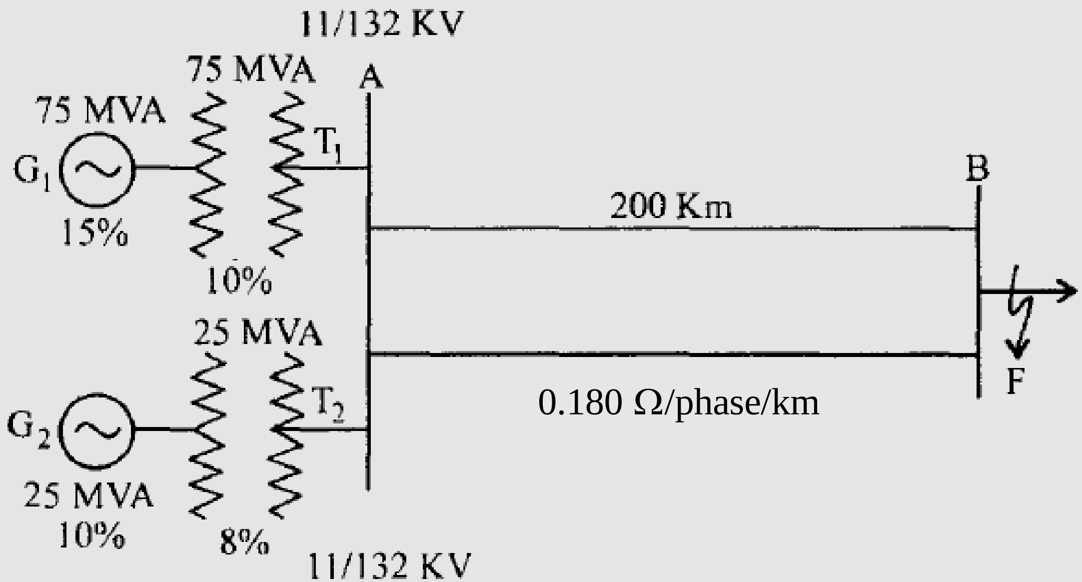


Figure 3.8

Example 9: Thevenin method

Solution.

$$x_{\text{new}} = x_{\text{old}} \left(\frac{V_{\text{baseold}}}{V_{\text{basenew}}} \right)^2 \left(\frac{S_{\text{basenew}}}{S_{\text{baseold}}} \right)$$

Generator 1: $x_{\text{newG1}} = x_{\text{oldG1}} = j0.1 \text{ p.u.}$

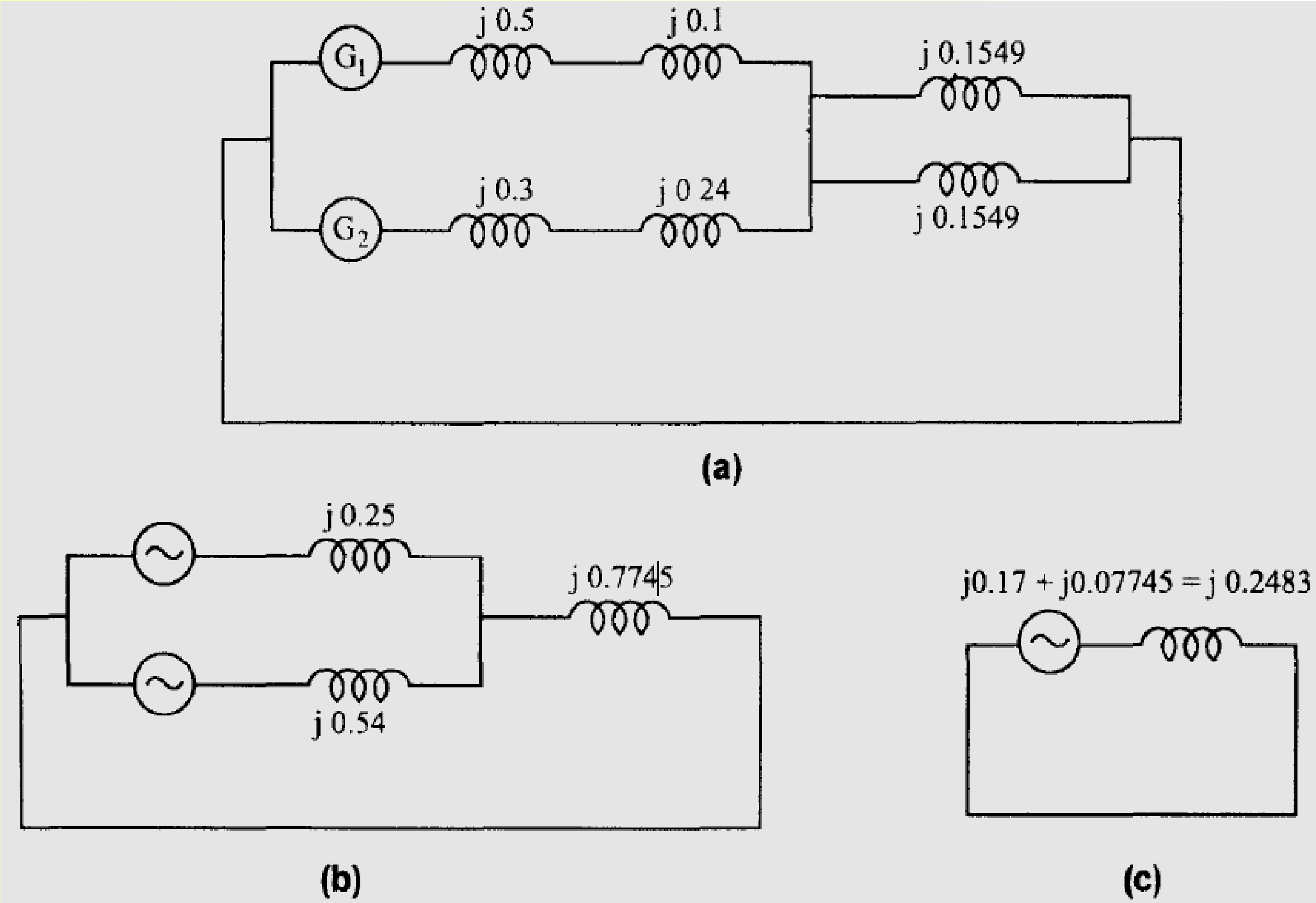
Generator 2: $x_{\text{newG2}} = x_{\text{oldG2}} \left(\frac{S_{\text{basenew}}}{S_{\text{oldnew}}} \right) = j0.1 \left(\frac{75}{25} \right) = j0.3 \text{ p.u.}$

Transformer 1: $x_{\text{newT1}} = x_{\text{oldT1}} = j0.1 \text{ p.u.}$

Transformer 2: $x_{\text{newT2}} = x_{\text{oldT2}} \left(\frac{S_{\text{basenew}}}{S_{\text{oldnew}}} \right) = j0.08 \left(\frac{75}{25} \right) = j0.24 \text{ p.u.}$

Lines: $x_{\text{line}} = \dfrac{(200 \text{ km})(0.18\ \Omega/\text{phase/km})}{\dfrac{(132 \text{ kV})^2}{75 \text{ MVA}}} = j0.1549 \text{ p.u.}$

Example 9: Thevenin method



Example 9: Thevenin method

Total fault current

$$I_F = \frac{1\angle 0^\circ}{j0.248336} = -j4.0268 \text{ p.u.}$$

$$\text{Base current for 132 kV circuit} = \frac{75 \text{ MVA}}{\sqrt{3}(132 \text{ kV})} = 328.04 \text{ A}$$

$$\text{Hence actual fault current for 132 kV side} = -j4.0268 \times 328 = 1321\angle -90^\circ \text{ A}$$

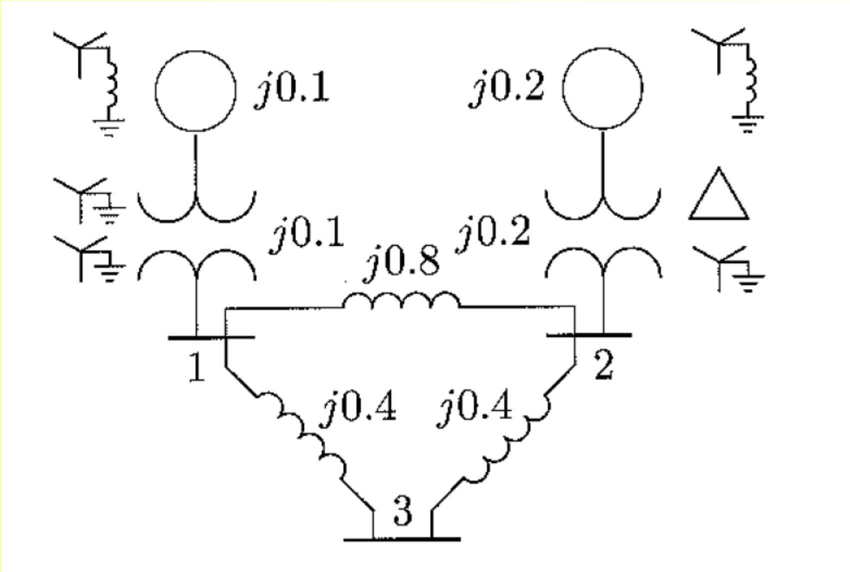
$$\text{Base current for 11 kV side of the transformer} = \frac{75 \text{ MVA}}{\sqrt{3}(11 \text{ kV})} = 3936.5 \text{ A}$$

$$\text{Actual fault current supplied from 11 kV side} = -j4.0268 \times 3936.5 = 15851.4\angle -90^\circ \text{ A}$$

$$\text{Falt current supplied by generator 1} = \frac{(15851.4\angle -90^\circ \text{ A}) \times j0.54}{j0.54 + j0.25} = -j10835.1 \text{ A}$$

$$\text{Falt current supplied by generator 2} = \frac{(15851.4\angle -90^\circ \text{ A}) \times j0.25}{j0.54 + j0.25} = -j5016.3 \text{ A}$$

Example 10: Thevenin method



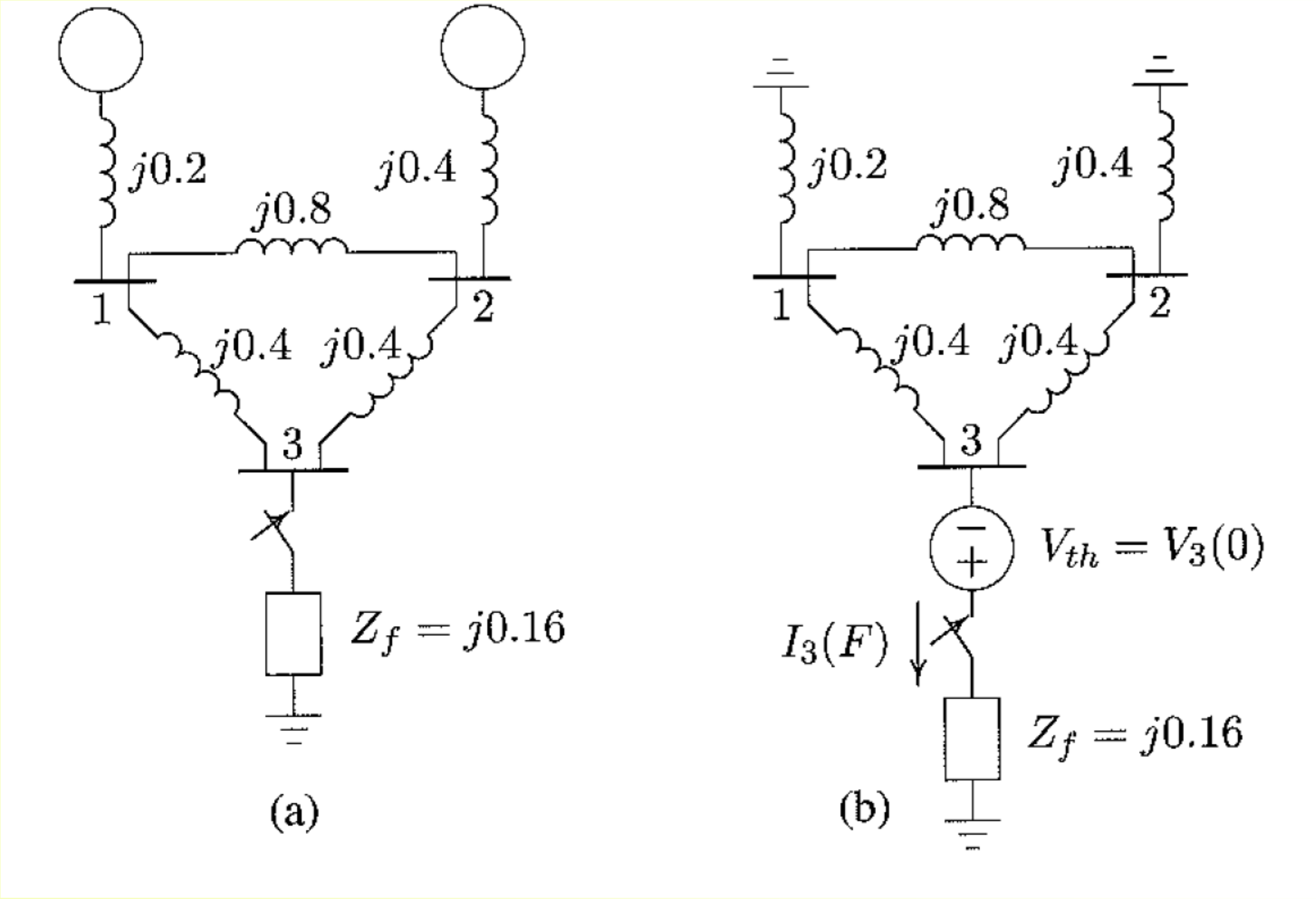
The one-line diagram of a simple three-bus power system is shown in Figure 9.1. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common 100 MVA base, and for simplicity, resistances are neglected. The following assumptions are made.

(*i*) Shunt capacitances are neglected and the system is considered on no-load.

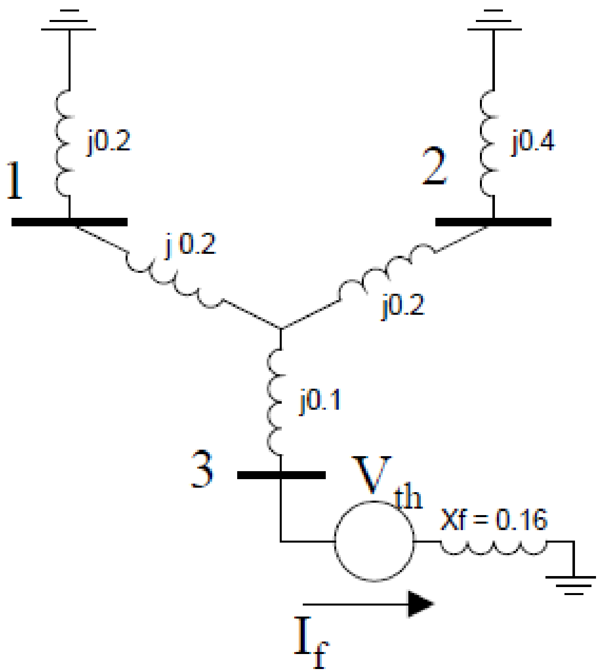
(*ii*) All generators are running at their rated voltage and rated frequency with their emfs in phase.

Determine the fault current, the bus voltages, and the line currents during the fault when a balanced three-phase fault with a fault impedance $Z_f = 0.16$ per unit

Example 10: Thevenin method



Example 10: Thevenin method



$$I_3^{[f]} = \frac{V_3^{[0]}}{Z_{33} + Z_f}$$

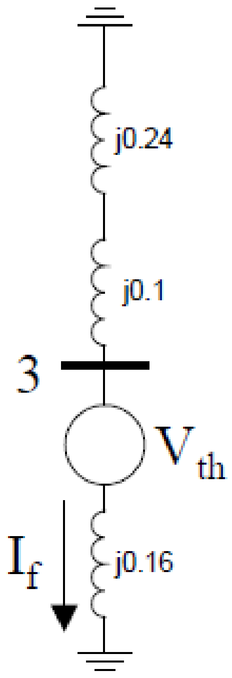
$$V_1^{[0]} = V_2^{[0]} = V_3^{[0]} = 1.0$$

$$Z_{1s} = Z_{2s} = \frac{(j0.4)(j0.8)}{(j1.6)} = j0.2$$

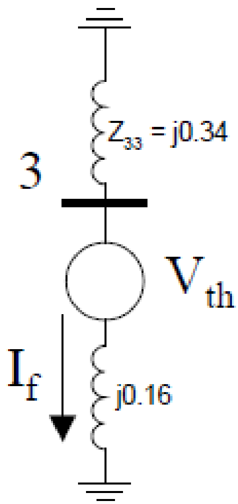
$$Z_{3s} = \frac{(j0.4)(j0.4)}{(j1.6)} = j0.1$$

$$Z_{33} = \frac{(j0.4)(j0.6)}{j0.4 + j0.6} + j0.1$$

$$Z_{33} = j0.34$$



Example 10: Thevenin method



$$Z_{33} = j0.34$$

$$I_3^{[f]} = \frac{V_3^{[0]}}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0$$

$$I_{G1} = \frac{j0.6}{j0.4 + j0.6} I_3(F) = -j1.2 \quad \text{pu}$$

$$I_{G2} = \frac{j0.4}{j0.4 + j0.6} I_3(F) = -j0.8 \quad \text{pu}$$

$$\Delta V_1 = 0 - (j0.2)(-j1.2) = -0.24 \quad \text{pu}$$

$$\Delta V_2 = 0 - (j0.4)(-j0.8) = -0.32 \quad \text{pu}$$

$$\Delta V_3 = (j0.16)(-j2) - 1.0 = -0.68 \quad \text{pu}$$

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.24 = 0.76 \quad \text{pu}$$

$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.32 = 0.68 \quad \text{pu}$$

$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.68 = 0.32 \quad \text{pu}$$

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \quad \text{pu}$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \quad \text{pu}$$

$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \quad \text{pu}$$

Short-Circuit Capacity (SCC)

- Measures the electrical strength of the bus
- Stated in MVA
- Determines the dimension of bus bars and the interrupting capacity of circuit breakers
- Definition: The SCC MVA at bus *k* $$SCC = \sqrt{3}\,V_{Lk} I_k(F) \times 10^{-3} \quad \text{MVA}$$
- The symmetrical three-phase fault current in pu: $$I_k(F)_{pu} = \frac{V_k(0)}{X_{kk}}$$
- The base current $$I_B = \frac{S_B \times 10^3}{\sqrt{3}\,V_B}$$
- Note that S_B is the base MVA and V_B is the line-to-line voltage in kV and thus, the fault current in amperes is

$$\begin{aligned} I_k(F) &= I_k(F)_{pu} I_B \\ &= \frac{V_k(0)}{X_{kk}} \frac{S_B \times 10^3}{\sqrt{3}\,V_B} \end{aligned}$$

Short-Circuit Capacity (SCC)

- Then SCC becomes

$$SCC = \sqrt{3} V_{Lk} I_k(F).10^{-3} MVA$$

$$SCC = \sqrt{3} V_{Lk} \frac{V_k(0)}{X_{kk}} \frac{S_B.10^3}{\sqrt{3} V_B}.10^{-3}$$

$$SCC = V_{Lk} \frac{V_k(0)}{X_{kk}} \frac{S_B}{V_B}$$

- If the base voltage is equal to the rated voltage $V_{Lk} = V_B$

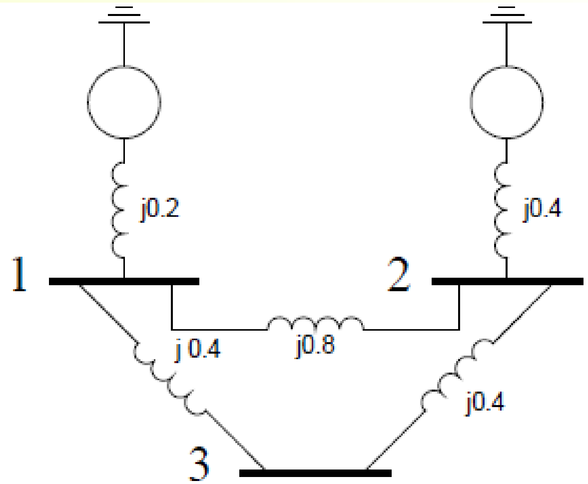
$$SCC = \frac{V_k(0) S_B}{X_{kk}}$$

- The prefault bus voltage is usually assmed to be 1.0. Then, SCC in MVA is

$$SCC = \frac{S_B}{X_{kk}}$$

Example 11: Short-Circuit Capacity

- **Find the SCC for bus #3**

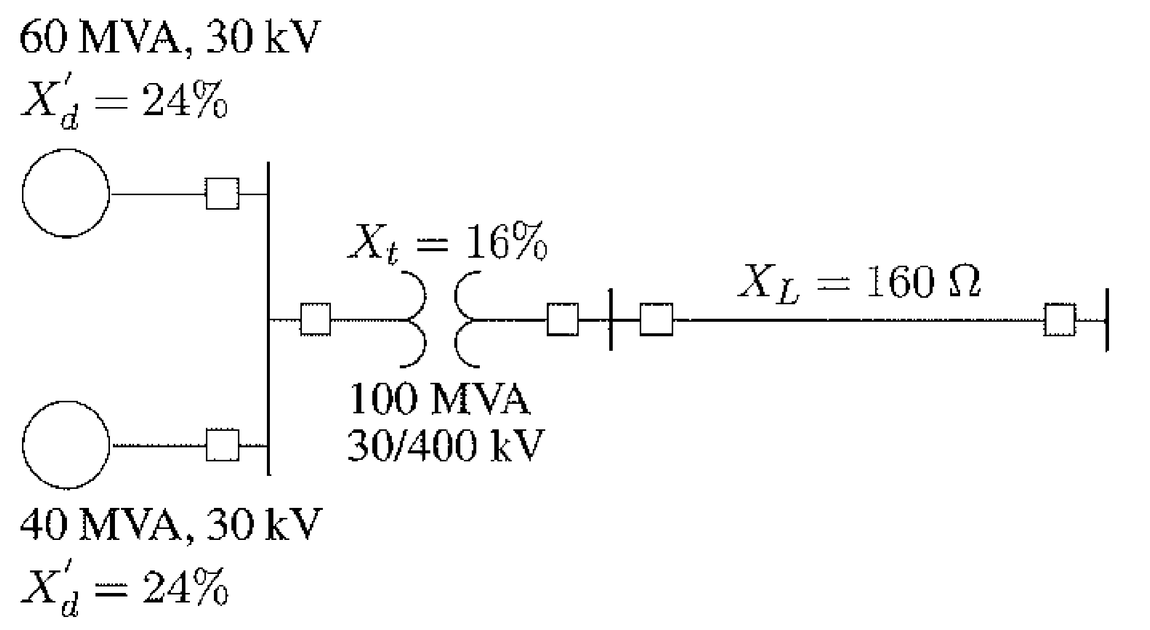


$$Z_{33} = j0.34$$

$$S_{base} = 100 \text{ MVA}$$

$$SCC_3 = \frac{S_{base}}{|Z_{33}|} = \frac{100 \text{ MVA}}{0.34} = 294 \text{ MVA}$$

Example 12: Thevenin method and SCC



The system shown above is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reactances are marked on the diagram. All resistances are neglected. The line impedance is *j*160. A three-phase balanced fault occurs at the receiving end of the transmission line. Determine the shortcircuit current and the short-circuit MVA.

Example 12: Thevenin method and SCC

The base impedance for line is

$$Z_B = \frac{(400)^2}{100} = 1,600 \ \ \Omega$$

and the base current is

$$I_B = \frac{100,000}{\sqrt{3}(400)} = 144.3375 \ \ \mathrm{A}$$

The reactances on a common 100 MVA base are

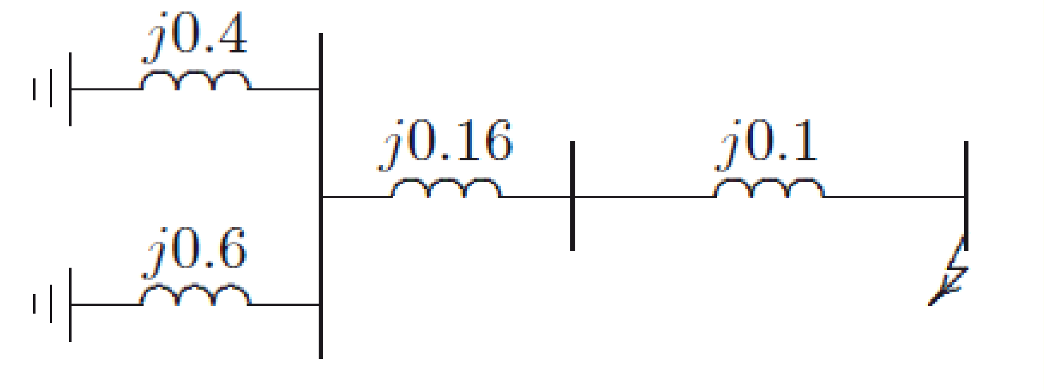
$$X'_{dg1} = \frac{100}{60}(0.24) = 0.4 \ \ \mathrm{pu}$$

$$X'_{dg2} = \frac{100}{40}(0.24) = 0.6 \ \ \mathrm{pu}$$

$$X_t = \frac{100}{100}(0.16) = 0.16 \ \ \mathrm{pu}$$

$$X_{line} = \frac{160}{1600} = 0.1 \ \ \mathrm{pu}$$

Example 12: Thevenin method and SCC



Impedance to the point of fault is

$$X = j\frac{(0.4)(0.6)}{0.4+0.6} + j0.16 + j0.1 = j0.5 \ \text{pu}$$

The fault current is

$$I_f = \frac{1}{j0.5} = 2\angle{-90^\circ} \ \text{pu}$$
$$= (144.3375)(2\angle{-90^\circ}) = 288.675\angle{-90^\circ} \ \text{A}$$

The Short-circuit MVA is

$$\text{SCMVA} = \sqrt{3}(400)(288.675)(10^{-3}) = 200 \ \text{MVA}$$