POWER SYSTEM DYNAMICS (STABILITY) AND CONTROL

Transient Stability Analysis
Equal-Area Criterion
Lecture Notes 4

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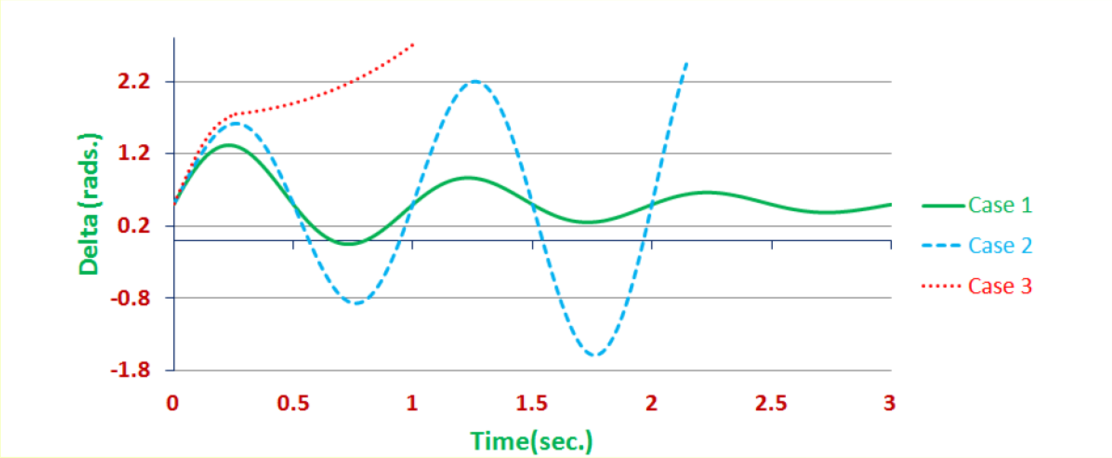
TRANSIENT STABILITY



- the disturbance on the system is quite severe and sudden and the machine is unable to maintain synchronism under the impact of this disturbance. In this case, there is a large excursion of the rotor angle (even if the generator is transiently stable).
- Figure shows various cases of stable and unstable behavior of the generator. In case 1, under the influence of the fault, the generator rotor angle increases to a maximum, subsequently decreases and settles to a steady state value following oscillations with decreasing magnitude.
- In case 2, the rotor angle decreases after attaining a maximum value. However, subsequently, it undergoes oscillations with increasing amplitude. This type of instability is not caused by the lack of synchronizing torque; rather it occurs due to lack of sufficient damping torque in the post fault system condition.
- In case 3, the rotor angle monotonically keeps on increasing due to insufficient synchronizing torque till the protective relay trips it. This type of instability, in which the rotor angle never decreases, is termed as ’first **swing instability’.**

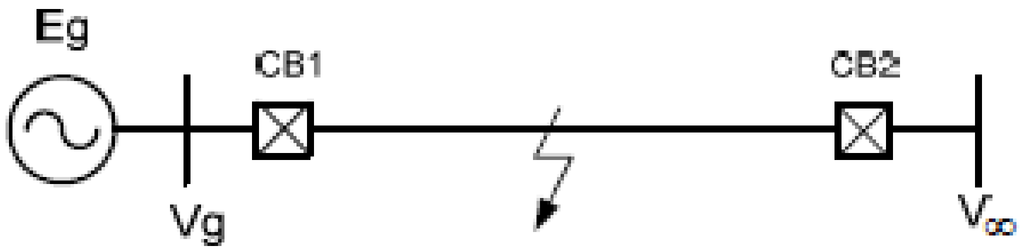
TRANSIENT STABILITY

- **The ability of the power system to remain in synchronism when subject to large disturbances**
  - Large power and voltage angle oscillations do not permit linearization of the generator swing equations
- **Lyapunov energy functions**
  - simplified energy method: the Equal Area Criterion
- **Time-domain methods**
  - numerical integration of the swing equations
  - Runga-Kutta numerical integration techniques

ROTOR ANGEL STABILITY

- Ability of interconnected synchronous machines to remain in synchronism after being subjected to a disturbance
- Depends on the ability to restore equilibrium between <u>electromagnetic torque</u> and <u>mechanical torque</u> of each synchronous machine
- Instability is a result of a runaway situation due to torque imbalance
  - Key factor: output power varies as rotor angles swing
- Instability occurs in the form of increasing angular swings of some generators → loss of synchronism

FAULT SEQUENCE

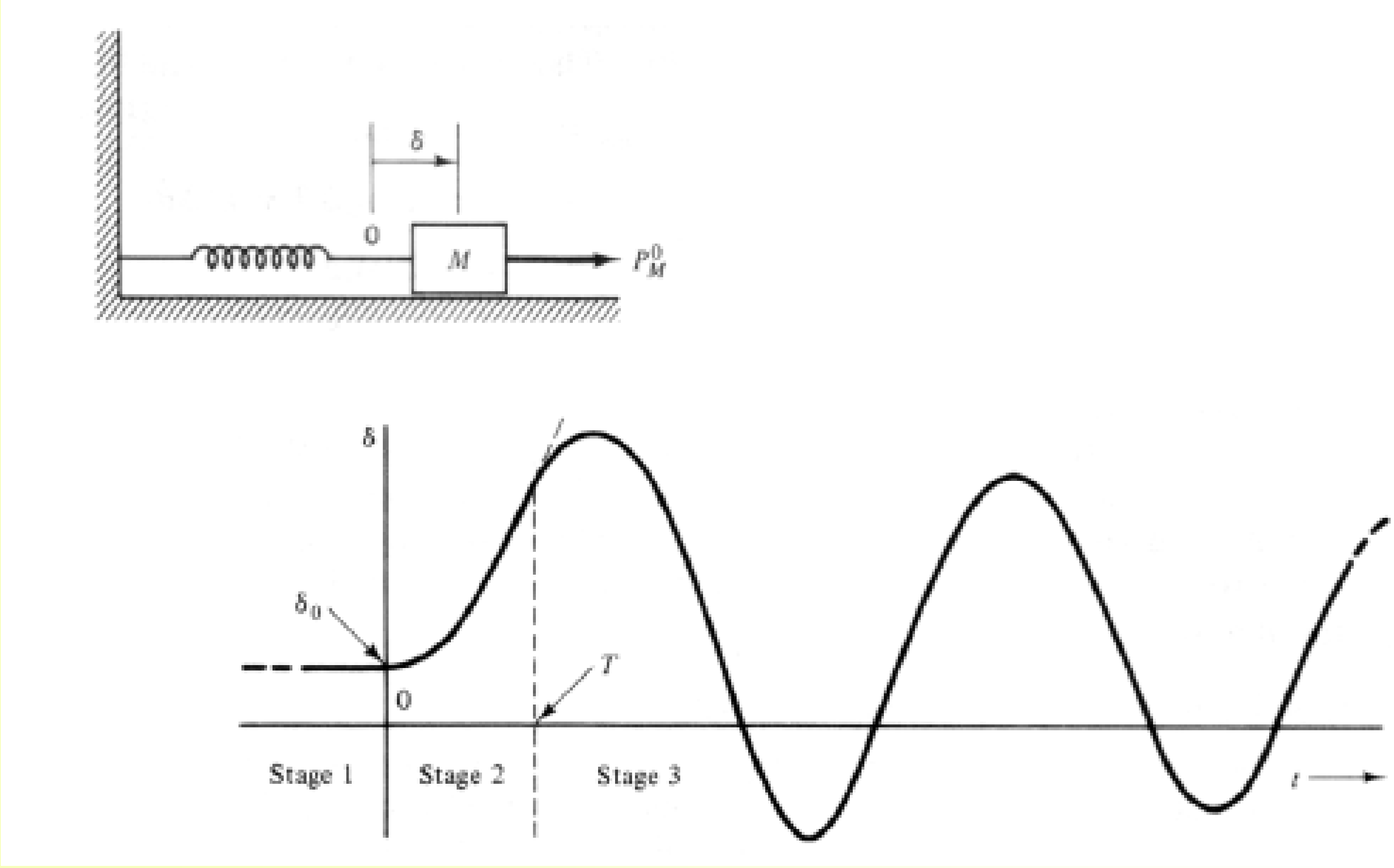


- **Prefault**: the system is in steady-state condition
- **During fault**: fault occurs → CBs open simultaneously

  → fault cleared
- **Postfault** (after fault): CBs reclose and remain closed

- Note: 2nd and 3rd steps are considered *dynamics*

SPRING MASS ANALOGY



NONLINEAR SWING EQUATION

- Recall $$M\ddot{\delta} + D\dot{\delta} + P_G(\delta) = P_M^0$$
- Neglect the very small mechanical damping (D = 0)
- Three stages of fault sequence
- **Stage 1:** $\delta = \delta^0,\ \dot{\delta} = 0,\ P_G(\delta) = P_M^0$
- **Stage 2:** With the transmission line open and D = 0

$$M\ddot{\delta} = P_M^0 \qquad 0 \le t < T$$

- By integration

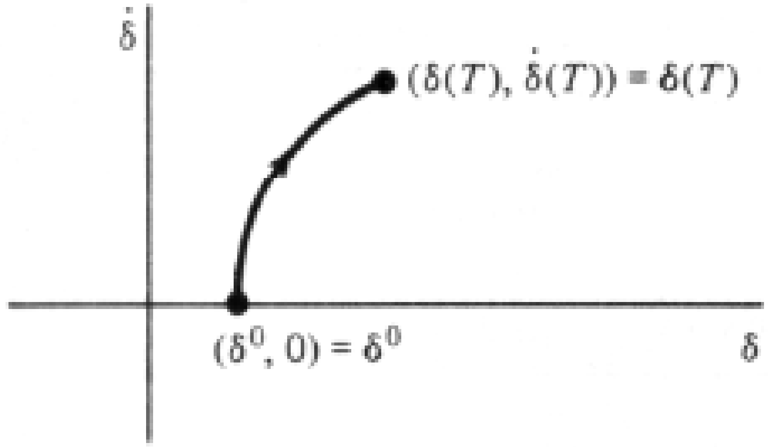
$$\dot{\delta}(t) = \frac{P_M^0}{M} t + \dot{\delta}(0) = \frac{\pi f_0 P_M^0}{H} t \ \text{ rad/sec}$$

$$\delta(t) = \frac{P_M^0}{2M} t^2 + \delta^0 = \frac{\pi f_0 P_M^0}{2H} t^2 + \delta^0 \ \text{ rad}$$

NONLINEAR SWING EQUATION

- State-space description

$$\delta - \delta^0 = \frac{P_M^0}{2M} t^2 = \frac{M}{2P_M^0}\left(\frac{P_M^0}{M} t\right)^2 = \frac{M}{2P_M^0}\dot{\delta}^2$$



$$0 \le t < T$$

- **Stage 3:** The transmission line is reconnected at t = T

$$M\ddot{\delta} + P_G(\delta) = P_M^0 \quad \Rightarrow \quad M\ddot{\delta} + P(\delta) = 0 \quad \text{where} \quad P(\delta) = P_G(\delta) - P_M^0$$

NONLINEAR SWING EQUATION

- Total energy

$$V(\boldsymbol{\delta}) = \frac{1}{2} M \dot{\delta}^2 + \int_{\delta^0}^{\delta} P(u)du$$

Note: 1. Both kinetic energy and potential energy at equilibrium are zero

2. The kinetic energy of the actual turbine is $J\omega_0^2 / 2$

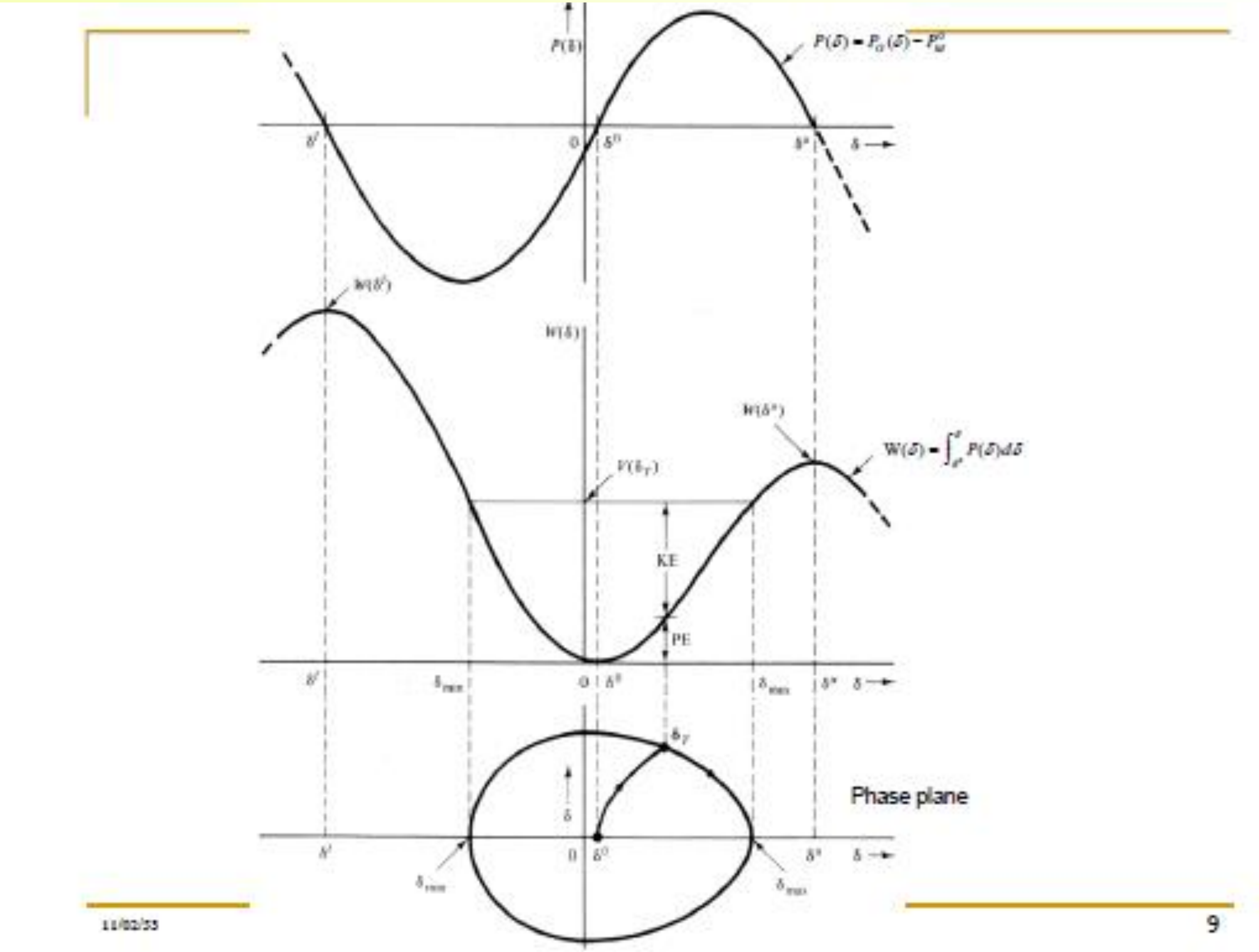
- At the beginning of stage 3

$$V(\boldsymbol{\delta}) = \frac{1}{2} M \dot{\delta}(t)^2 + \int_{\delta^0}^{\delta(t)} P(u)du = \frac{1}{2} M \dot{\delta}_T^2 + \int_{\delta^0}^{\delta_T} P(u)du = V(\boldsymbol{\delta}_T)$$

- Let define the potential energy

$$W(\delta) = \int_{\delta^0}^{\delta} P(u)du$$

KE AND PE /PHASE DIAGRAM



NONLINEAR SWING EQUATION

- Let bring back the damping ($D \neq 0$) → the energy is slowly drained
- Using the chain rule

$$\dot{\mathrm{V}}(\boldsymbol{\delta}(t)) = \frac{\partial V}{\partial \dot{\delta}}\frac{d\dot{\delta}}{dt} + \frac{\partial V}{\partial \delta}\frac{d\delta}{dt} = M\dot{\delta}\ddot{\delta} + P(\delta)\dot{\delta}$$

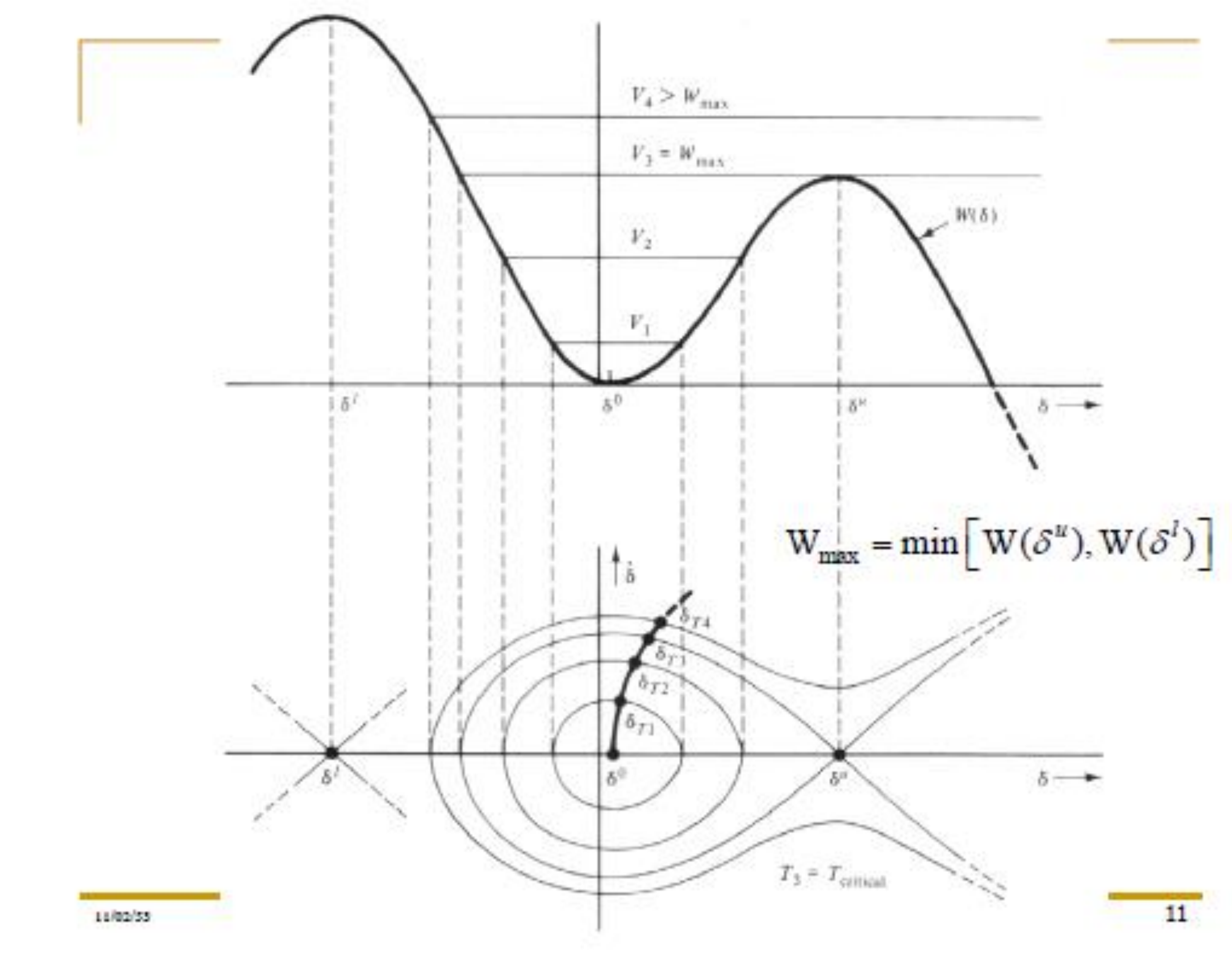
$$= \left[M\ddot{\delta} + P(\delta)\right]\dot{\delta}$$

- Hence

$$\dot{\mathrm{V}}(\delta(t)) = -D\dot{\delta}^2 \leq 0$$

- Instead of the closed curve, the trajectory will spiral inward toward the equilibrium point

KE AND PE /PHASE DIAGRAM



TRANSIENT STABILITY

- Is $V(\delta_T) < W_{max}$?
- $V(\delta_T) < W_{max}$ implies

$$\frac{1}{2}M\dot{\delta}_T^2 + \int_{\delta^0}^{\delta_T} P(u)du < \int_{\delta^0}^{\delta_u} P(u)du \quad \Rightarrow \quad \frac{1}{2}M\dot{\delta}_T^2 < \int_{\delta_T}^{\delta_u} P(u)du$$

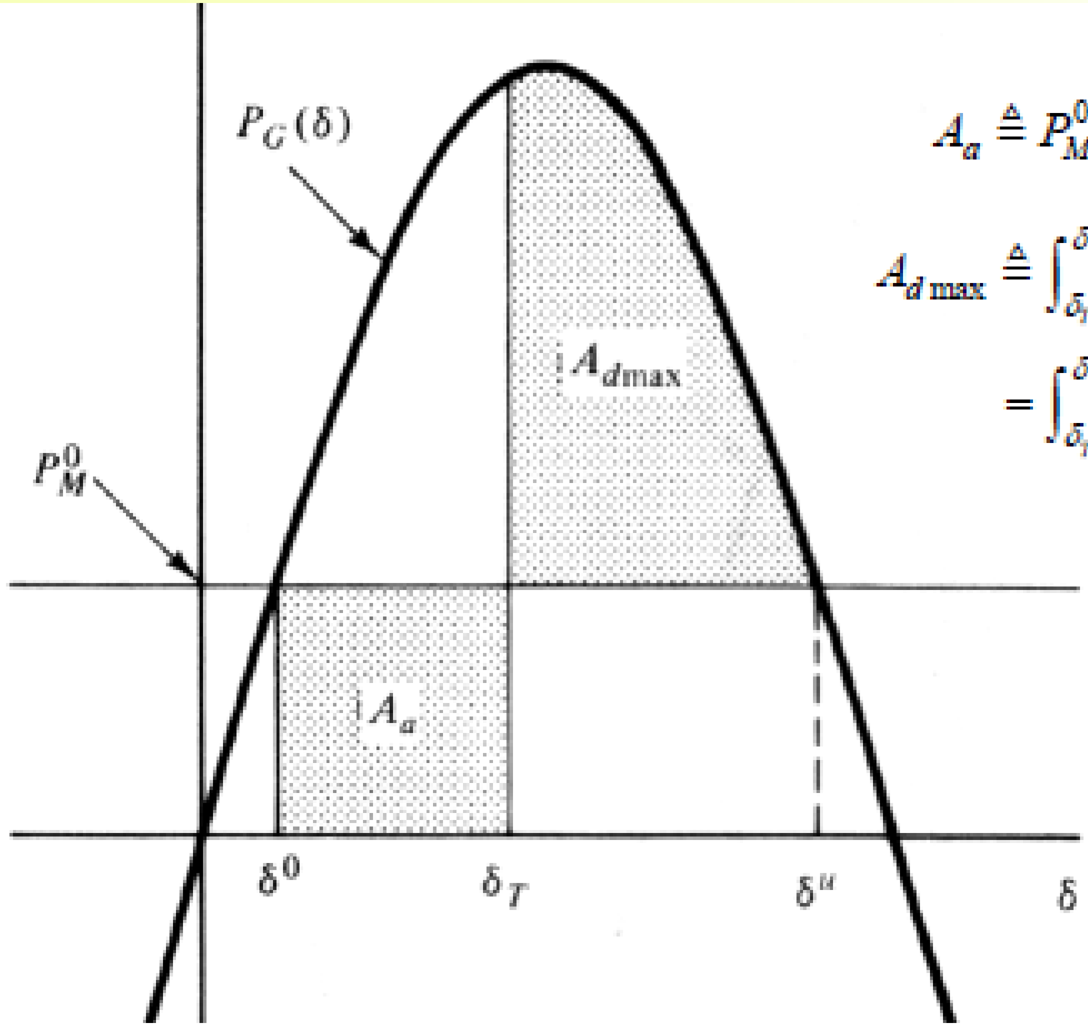
- Recall

$$\delta - \delta^0 = \frac{P_M^0}{2M}t^2 = \frac{M}{2P_M^0}\left(\frac{P_M^0}{M}t\right)^2 = \frac{M}{2P_M^0}\dot{\delta}^2 \quad \Rightarrow \quad P_M^0(\delta_T - \delta^0) < \int_{\delta_T}^{\delta_u} P(u)du$$

- We can calculate

$$\delta_T = \frac{P_M^0}{2M}T^2 + \delta^0 \ \text{ rad}$$

EQUAL-AREA STABILITY CRITERION



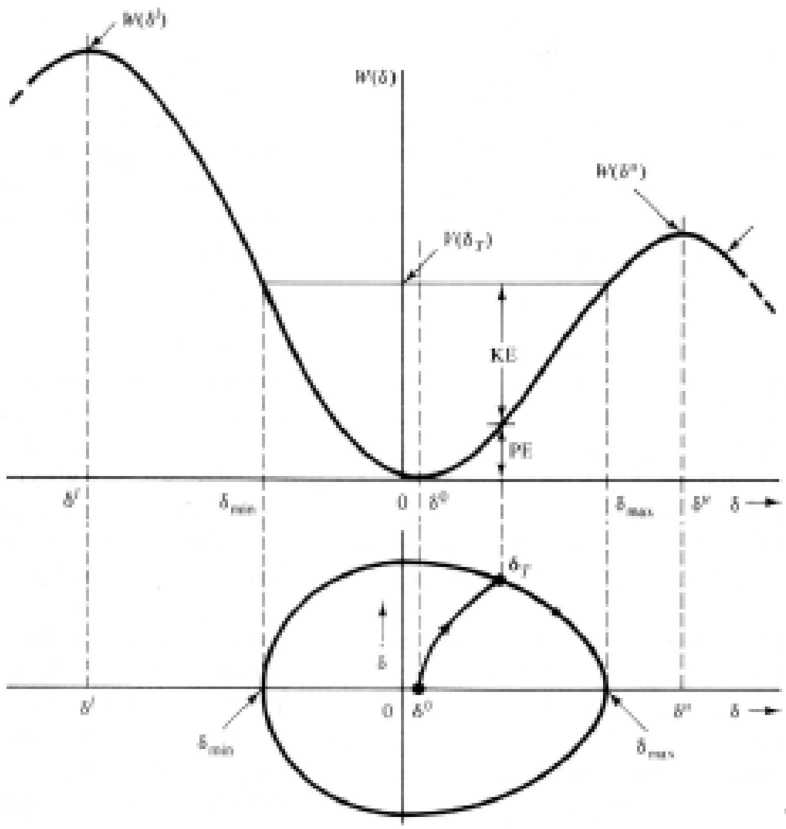
$$A_a \triangleq P_M^0(\delta_T - \delta^0)$$

$$A_{d\max} \triangleq \int_{\delta_T}^{\delta^u} P(\delta)\,d\delta$$

$$= \int_{\delta_T}^{\delta^u} (P_G(\delta) - P_M^0)\,d\delta$$

$$\boxed{A_a < A_{d\max}}$$

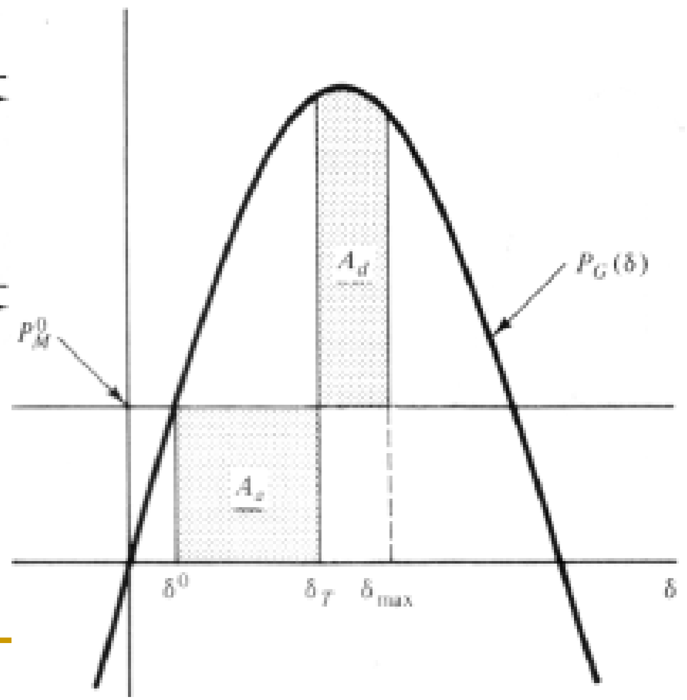
EQUAL-AREA STABILITY CRITERION



11/02/33

$$V(\delta_T) = W(\delta_{max}) = \int_{\delta_0}^{\delta^{max}} P(\delta)d\delta$$

$$P_M^0(\delta_T - \delta^0) = \int_{\delta_0}^{\delta^{max}} P(\delta)d\delta$$



EQUAL-AREA STABILITY CRITERION

- **Quickly predicts the stability after a major disturbance**
  - graphical interpretation of the energy stored in the rotating masses
  - method only applicable to a few special cases:
    - one machine connected to an infinite bus
    - two machines connected together
- **Method provides physical insight to the dynamic behavior of machines**
  - relates the power angle with the acceleration power

EQUAL-AREA STABILITY CRITERION

- **For a synchronous machine connected to an infinite bus**

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e = P_{accel}$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}\left(P_m - P_e\right) = \frac{\pi f_0}{H}\cdot P_{accel}$$

- **The energy form of the swing equation is obtained by multiplying both sides by the system frequency (shaft rotational speed)**

$$\left(2\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right) = \frac{\pi f_0}{H}\left(P_m - P_e\right)\left(2\frac{d\delta}{dt}\right)$$

EQUAL-AREA STABILITY CRITERION

$$2\left(\frac{d^2\delta}{dt^2}\right)\left(\frac{d\delta}{dt}\right)=\frac{\pi f_0}{H}(P_m-P_e)\left(2\frac{d\delta}{dt}\right)$$

- **The left hand side can be reworked as the derivative of the square of the system frequency (shaft speed)**

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right]=\frac{2\pi f_0}{H}(P_m-P_e)\frac{d\delta}{dt}$$

$$d\left[\left(\frac{d\delta}{dt}\right)^2\right]=\frac{2\pi f_0}{H}(P_m-P_e)d\delta$$

EQUAL-AREA STABILITY CRITERION

- Integrating both sides with respect to time,

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)d\delta$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)d\delta}$$

- The equation gives the relative speed of the machine. For stability, the speed must go to zero over time

$$\left.\frac{d\delta}{dt} = 0\right|_{t\to\infty}$$

$$0 = \int_{\delta_0}^{\delta}(P_m - P_e)d\delta$$

EQUAL-AREA STABILITY CRITERION

- **Consider a machine operating at equilibrium**
  - the power angle, $\delta = \delta_0$
  - the electrical load, $P_{\mathrm{e0}} = P_{\mathrm{m0}}$
- **Consider a sudden increase in the mechanical power input**
  - $P_{\mathrm{m1}} > P_{\mathrm{e0}}$; the acceleration power is positive
  - excess energy is stored in the rotor and the power frequency increases, driving the relative power angle larger over time

$$U_{Potential} = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta > 0$$

$$\frac{d\delta}{dt} = \omega = \sqrt{\frac{2\pi f_0}{H} \int_{\delta_0}^{\delta} (P_m - P_e)\, d\delta} > 0$$

EQUAL-AREA STABILITY CRITERION

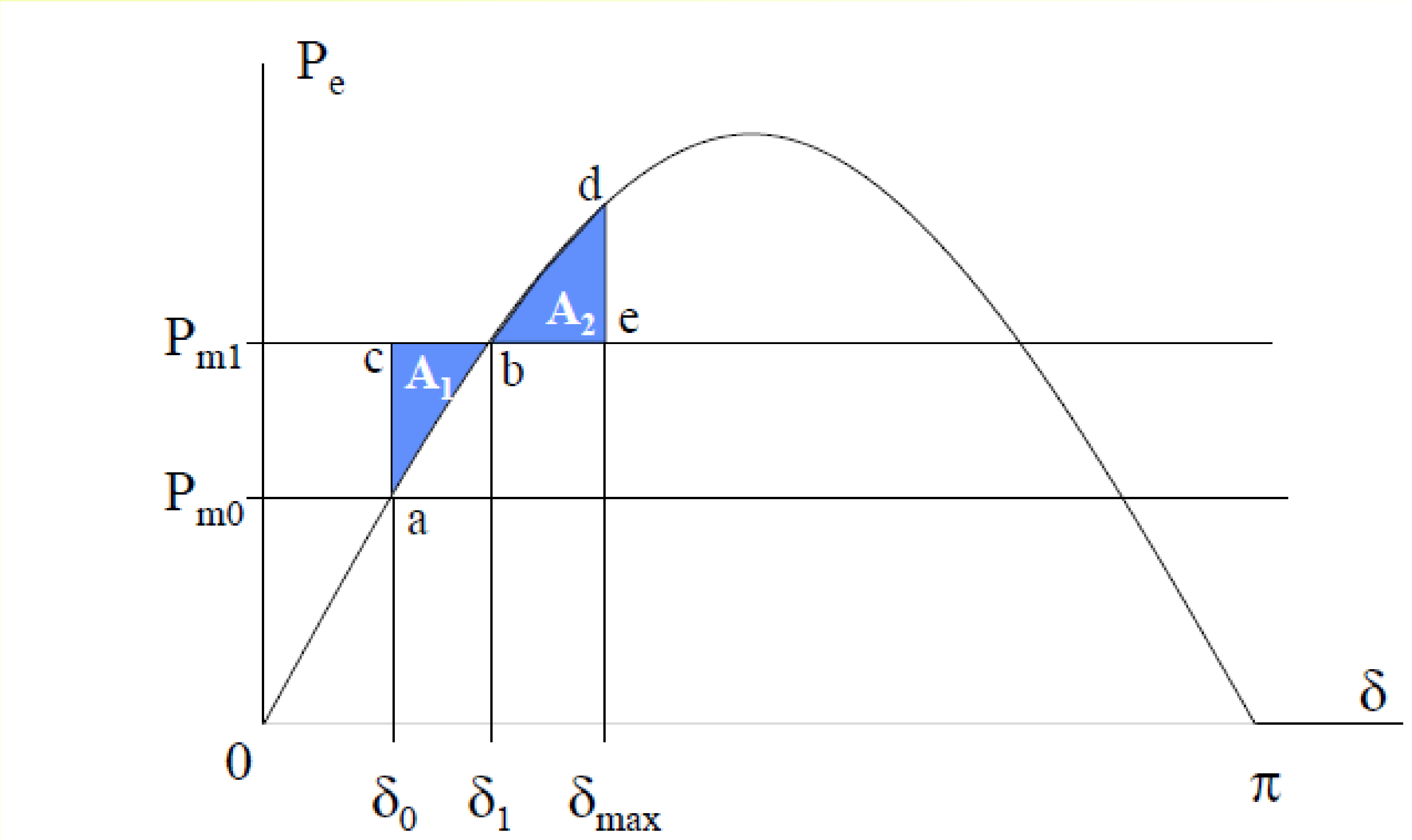
- with increase in the power angle, δ, the electrical power increases

$$P_e = P_{\max} \sin \delta$$

- when $\delta = \delta_1$, the electrical power equals the mechanical power, P_{m1}
- acceleration power is zero, but the rotor is running above synchronous speed, hence the power angle, δ, continues to increase
- now $P_{\mathrm{m1}} < P_{\mathrm{e}}$; the acceleration power is negative (deceleration), causing the rotor to decelerate to synchronous speed at $\delta = \delta_{\max}$
- an equal amount of energy must be given up by the rotating masses

$$U_{Potential} = \int_{\delta_0}^{\delta_1} \left(P_{m1} - P_e\right) d\delta - \int_{\delta_1}^{\delta_{\max}} \left(P_{m1} - P_e\right) d\delta = 0$$

EQUAL-AREA STABILITY CRITERION



EQUAL-AREA STABILITY CRITERION

- **The result is that the rotor swings to a maximum angle**
  - at which point the acceleration energy area and the deceleration energy area are equal

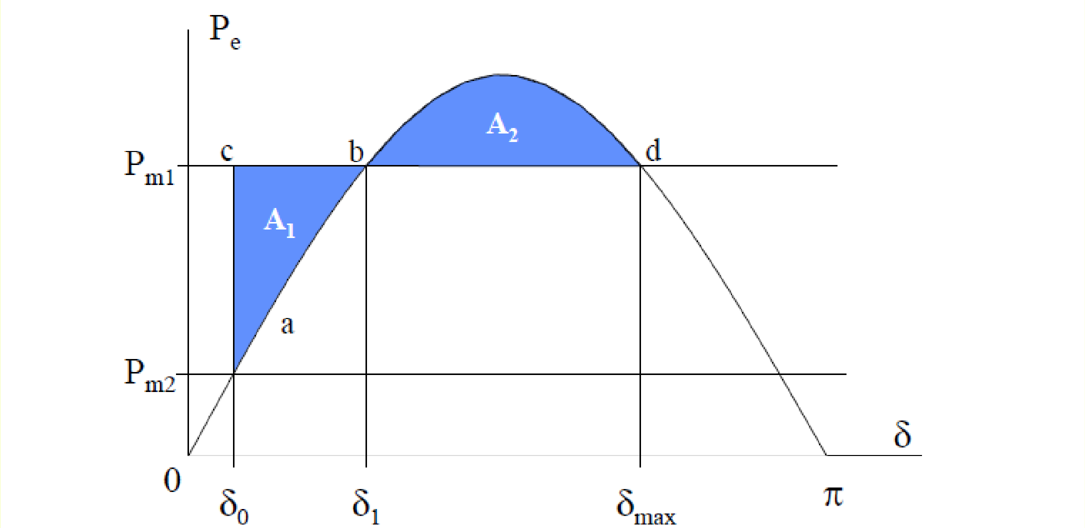
$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta = \textbf{area}\, abc = \textbf{area}\, A_1$$

$$\int_{\delta_1}^{\delta_{\max}} (P_{m1} - P_e) d\delta = \textbf{area}\, bde = \textbf{area}\, A_2$$

$$|\textbf{area}\, A_1| = |\textbf{area}\, A_2|$$

  - this is known as the equal area criterion
  - the rotor angle will oscillate back and forth between δ and δ_{max} at its natural frequency

EQUAL-AREA STABILITY CRITERION
ΔP MECHANICAL



The equal-area criterion is used to determine the maximum additional power P_m which can be applied for stability to be maintained. With a sudden change in the power input, the stability is maintained only if area A_2 at least equal to A_1 can be located above P_m. If area A_2 is less than area A_1, the accelerating momentum can never be overcome. The limit of stability occurs when δ_{max} is at the intersection of line P_m and the power-angle curve for $90° < \delta < 180°$, as shown

EQUAL-AREA STABILITY CRITERION ΔP MECHANICAL

$$P_{m1}(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{\max} \sin\delta \, d\delta = \int_{\delta_1}^{\delta_{\max}} P_{\max} \sin\delta \, d\delta - P_{m1}(\delta_{\max} - \delta_1)$$

$$P_{m1}(\delta_{\max} - \delta_0) = P_{\max}(\cos\delta_0 - \cos\delta_{\max})$$

$$P_{m1} = P_{\max} \sin\delta_{\max}$$

$$(\delta_{\max} - \delta_0)\sin\delta_{\max} = \cos\delta_0 - \cos\delta_{\max}$$

$$\rightarrow P_{m1} = P_{\max} \sin\delta_1$$

Function is nonlinear in $\delta_{\max}$
Solve using Newton-Raphson

Solution by Newton-Raphson Algorithm

$$(\delta_{max} - \delta_0)\sin\delta_{max} + \cos\delta_{max} = \cos\delta_0 \quad \Longrightarrow \quad f(\delta_{max}) = c$$

Newton-Raphson Algorithm:

$$\Delta\delta_{max}^{(k)} = \frac{c - f(\delta_{max}^{(k)})}{\left.\frac{df}{d\delta_{max}}\right|_{\delta_{max}^{(k)}}} \quad \Longrightarrow \quad \left.\frac{df}{d\delta_{max}}\right|_{\delta_{max}^{(k)}} = (\delta_{max}^{(k)} - \delta_0)\cos\delta_{max}^{(k)}$$

$$\pi/2 < \delta_{max}^{(k)} < \pi,$$

Iterative procedure:

$$\delta_{max}^{(k+1)} = \delta_{max}^{(k)} + \Delta\delta_{max}^{(k)}$$

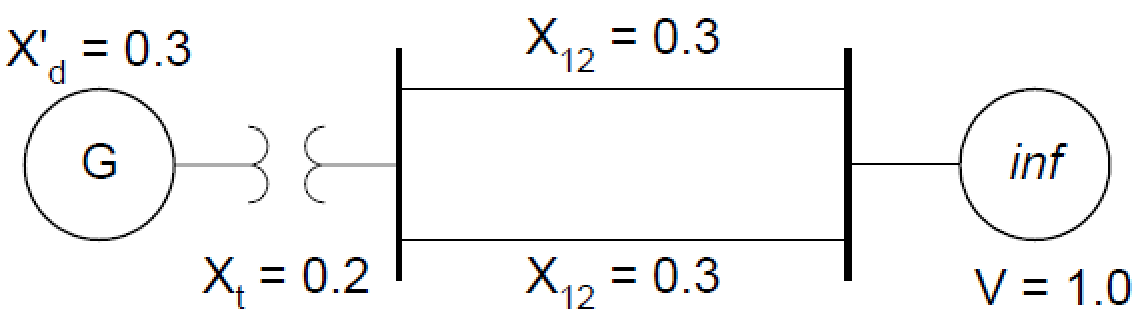
Convergence Criterion:

$$|\delta_{max}^{(k+1)} - \delta_{max}^{(k)}| \le \epsilon$$

Example

A 60-Hz synchronous generator having inertia constant $H = 9.94$ MJ/MVA and a transient reactance $X'_d = 0.3$ per unit is connected to an infinite bus through a purely reactive circuit as shown in Figure 11.7. Reactances are marked on the diagram on a common system base. The generator is delivering real power of 0.6 per unit, 0.8 power factor lagging to the infinite bus at a voltage of $V = 1$ per unit.

(a) The maximum power input that can be applied without loss of synchronism.
(b) Repeat (a) with zero initial power input. Assume the generator internal voltage remains constant at the value computed in (a).



Solution

The transfer reactance between the generated voltage and the infinite bus is

$$X = 0.3 + 0.2 + \frac{0.3}{2} = 0.65$$

The per unit apparent power is

$$S = \frac{0.6}{0.8}\angle\cos^{-1} 0.8 = 0.75\angle 36.87^\circ$$

The current is

$$I = \frac{S^*}{V^*} = \frac{0.75\angle -36.87^\circ}{1.0\angle 0^\circ} = 0.75\angle -36.87^\circ$$

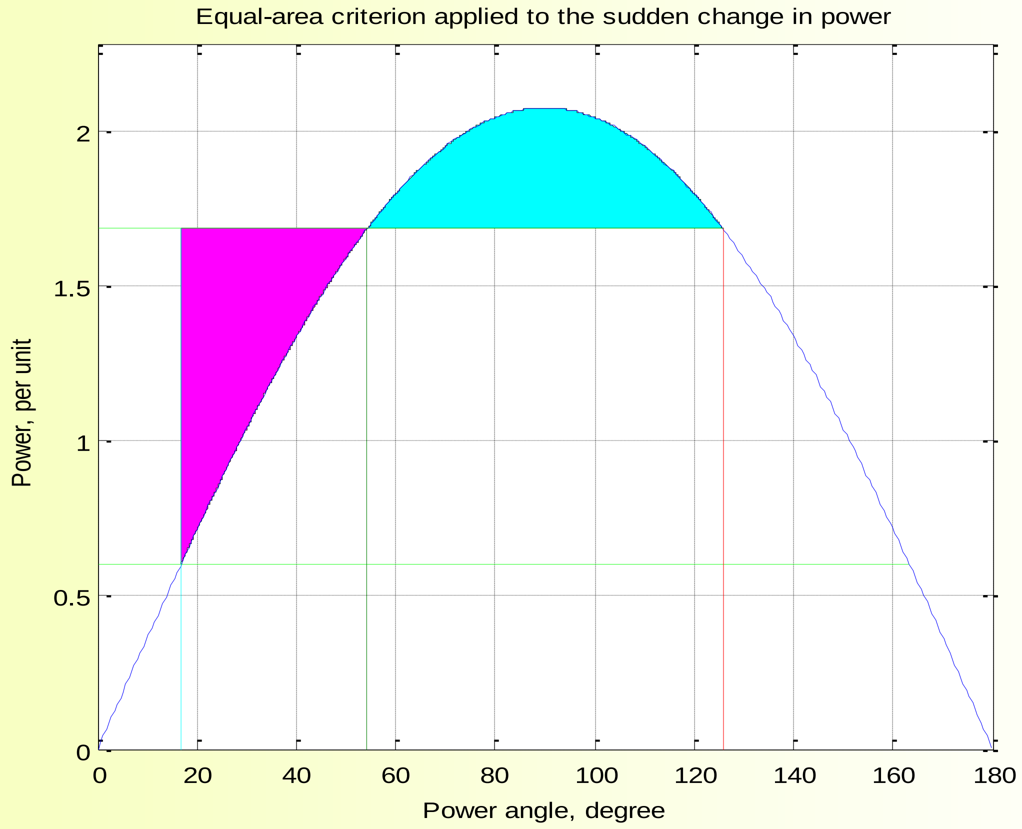
The excitation voltage is

$$E' = V + jXI = 1.0\angle 0^\circ + (j0.65)(0.75\angle -36.87^\circ) = 1.35\angle 16.79^\circ$$

Solution by Matlab

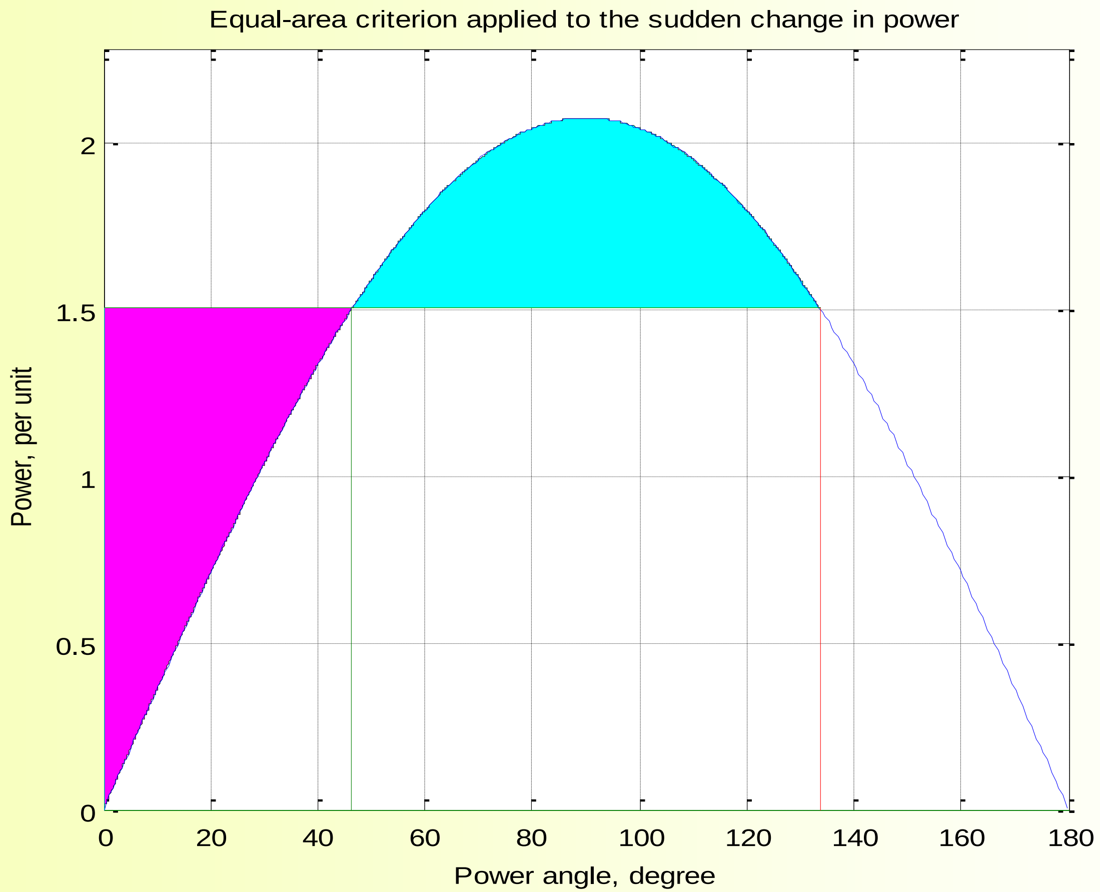
```
P0 = 0.6; E = 1.35; V = 1.0; X = 0.65;
eacpower(P0, E, V, X)
```

Solution by Matlab



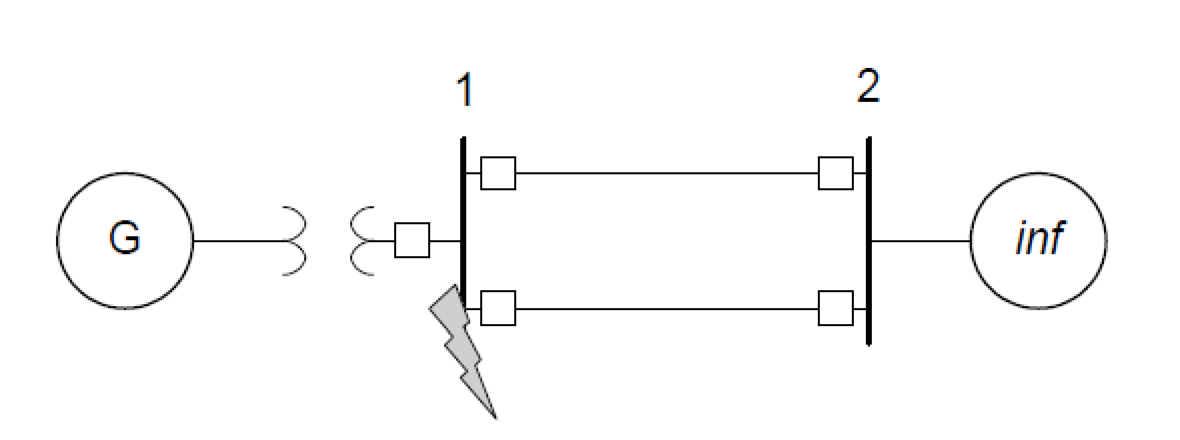
Initial power	=	0.600 pu
Initial power angle	=	16.791 degree
Sudden initial power	=	1.084 pu
Total power for critical stability	=	1.684 pu
Maximum angle swing	=	125.840 pu
New operating angle	=	54.160 degree

Solution by Matlab



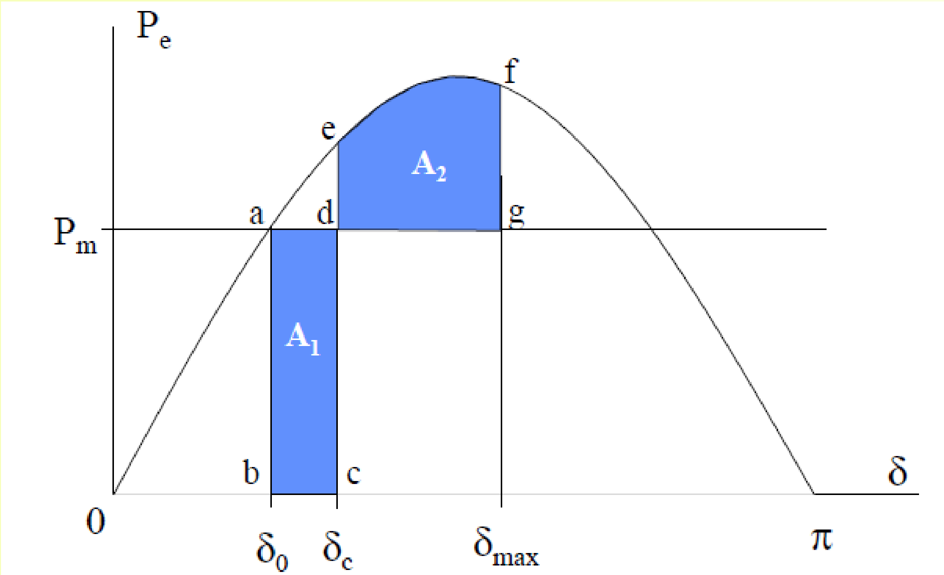

```
Initial power                      =   0.00 pu
Initial power angle                =   0.00 degree
Sudden initial power               =   1.505 pu
Total power for critical stability =   1.505 pu
Maximum angle swing                = 133.563 pu
New operating angle                =  46.437 degree
```

3-PHASE FAULT AT BUS 1



A generator is connected to an infinite bus bar through two parallel lines. Assume that the input power Pm is constant and the machine is operating steadily, delivering power to the system with a power angle δ_o. A temporary three-phase bolted fault occurs at the sending end of one of the line at bus 1.

EQUAL-AREA CRITERION 3-PHASE FAULT



- When the fault is at the sending end of the line, point F, no power is transmitted to the infinite bus.
- Since the resistances are neglected, the electrical power Pe is zero, and the power-angle curve corresponds to the horizontal axis.
- The machine accelerates with the total input power as the accelerating power, thereby increasing its speed, storing added kinetic energy, and increasing the angle δ.

- When the fault is cleared, both lines are assumed to be intact. The fault is cleared at δ_c which shifts the operation to the original power-angle curve at point e. The net power is now decelerating, and the previously stored kinetic energy will be reduced to zero at point f when the shaded area (defg), shown by A_2 equals the shaded area (abcd), shown by A_1.
- Since P_e is still greater than P_m the rotor continues to decelerate and the path is retraced along the power-angle curve passing through points e and a, The rotor angle would then oscillate back and forth around δ_o at its natural frequency.
- Because of the inherent damping, oscillation subsides and the operating point returns to the original power angle δ_o.

EQUAL-AREA CRITERION 3-PHASE FAULT

$$|\text{area } A_1| = |\text{area } A_2|$$

$$\int_{\delta_0}^{\delta_c} P_m \, d\delta = \int_{\delta_c}^{\delta_{max}} \left(P_{max} \sin\delta - P_m \right) d\delta$$

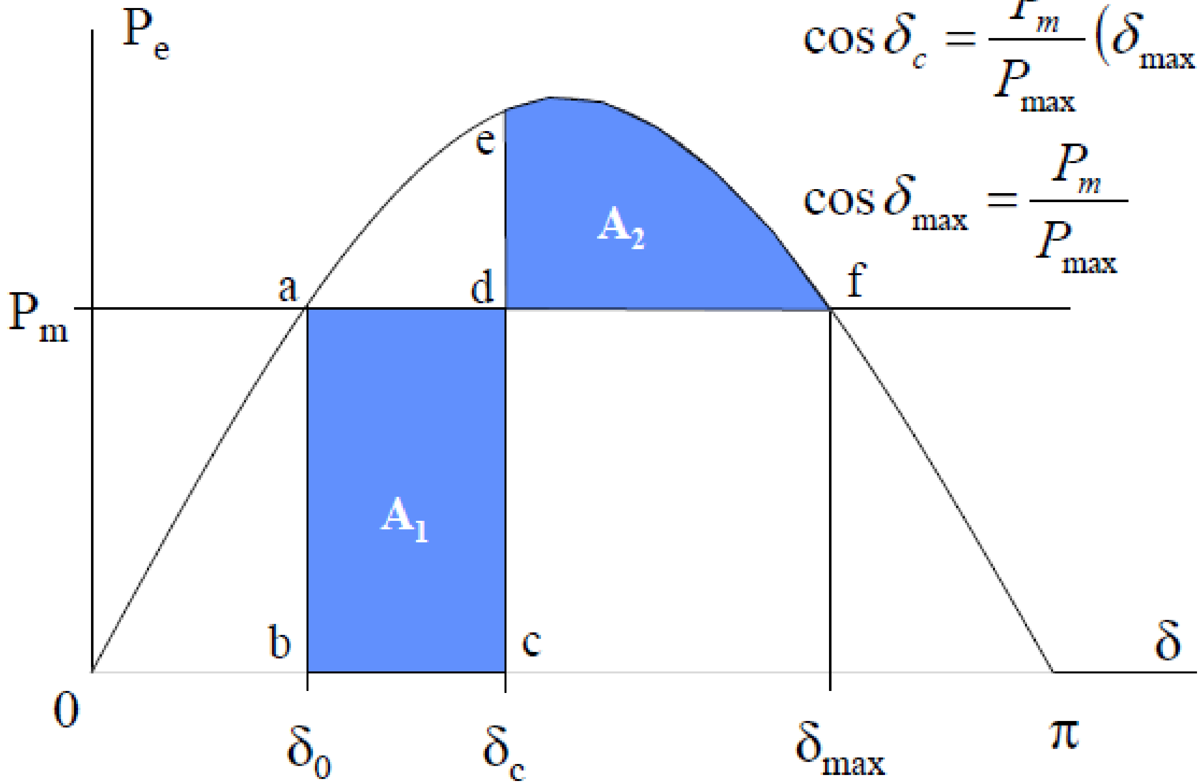
$$P_m\left(\delta_c - \delta_0\right) = P_{max}\left(\cos\delta_c - \cos\delta_{max}\right) - P_m\left(\delta_{max} - \delta_c\right)$$

$$\cos\delta_c = \frac{P_m}{P_{max}}\left(\delta_{max} - \delta_0\right) + \cos\delta_{max}$$

CRITICAL CLEARING TIME

$$\cos\delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_c) + \cos\delta_{max}$$

$$\cos\delta_{max} = \frac{P_m}{P_{max}}$$



CRITICAL CLEARING TIME

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e = P_m \leftarrow P_e = 0$$

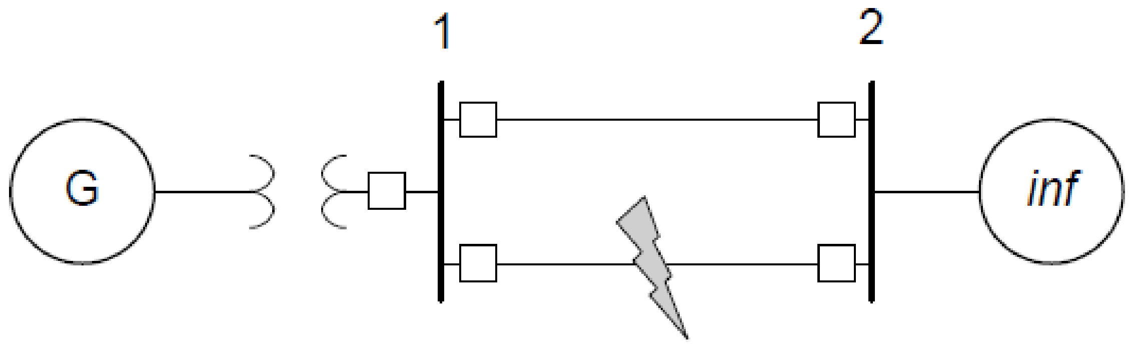
$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H} P_m$$

$$\frac{d\delta}{dt} = \frac{\pi f_0}{H} P_m \int_0^t dt = \frac{\pi f_0}{H} P_m t$$

$$\delta = \frac{\pi f_0}{2H} P_m t^2 + \delta_0$$

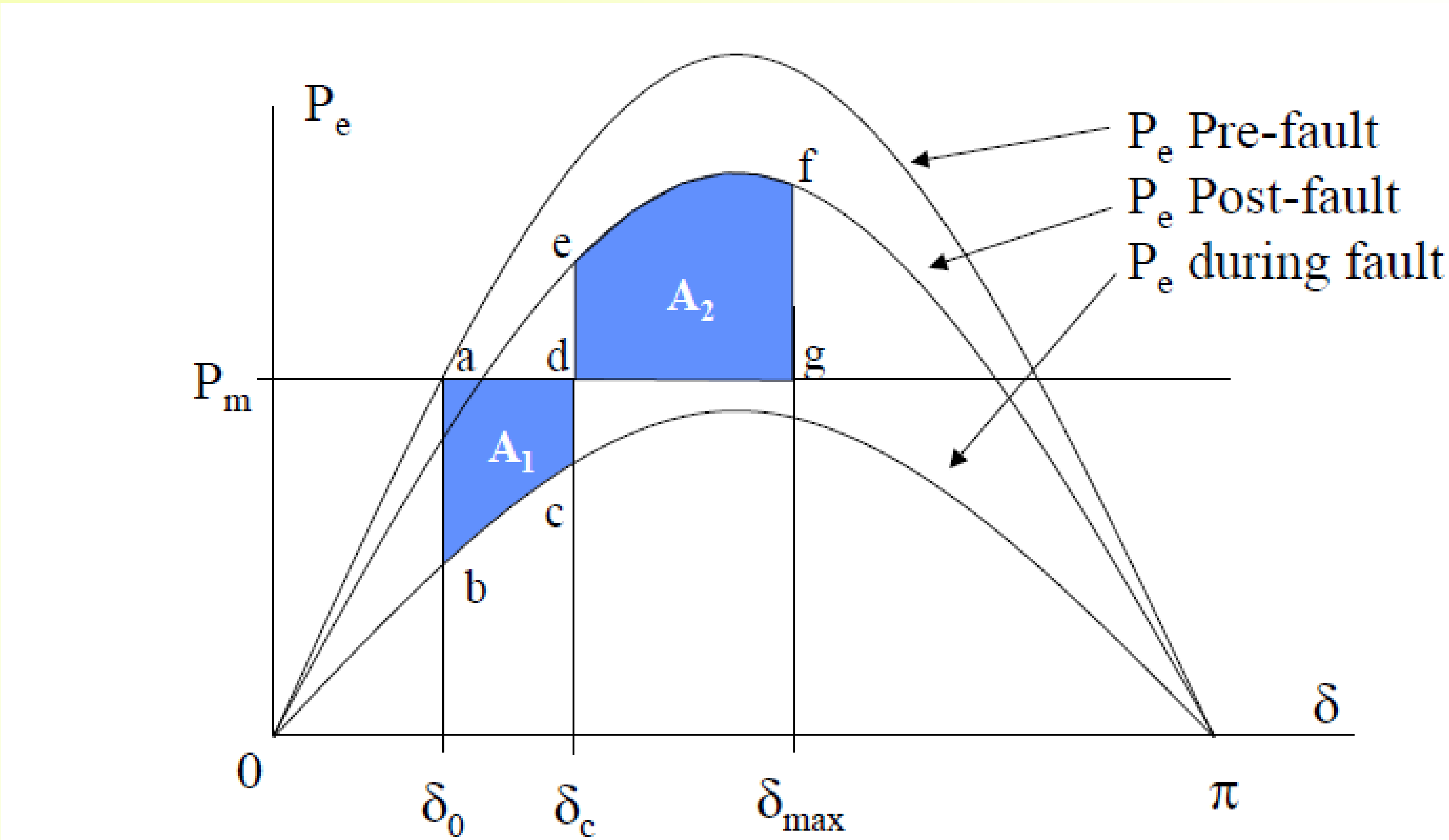
$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}}$$

3-PHASE FAULTS ALONG THE LINE

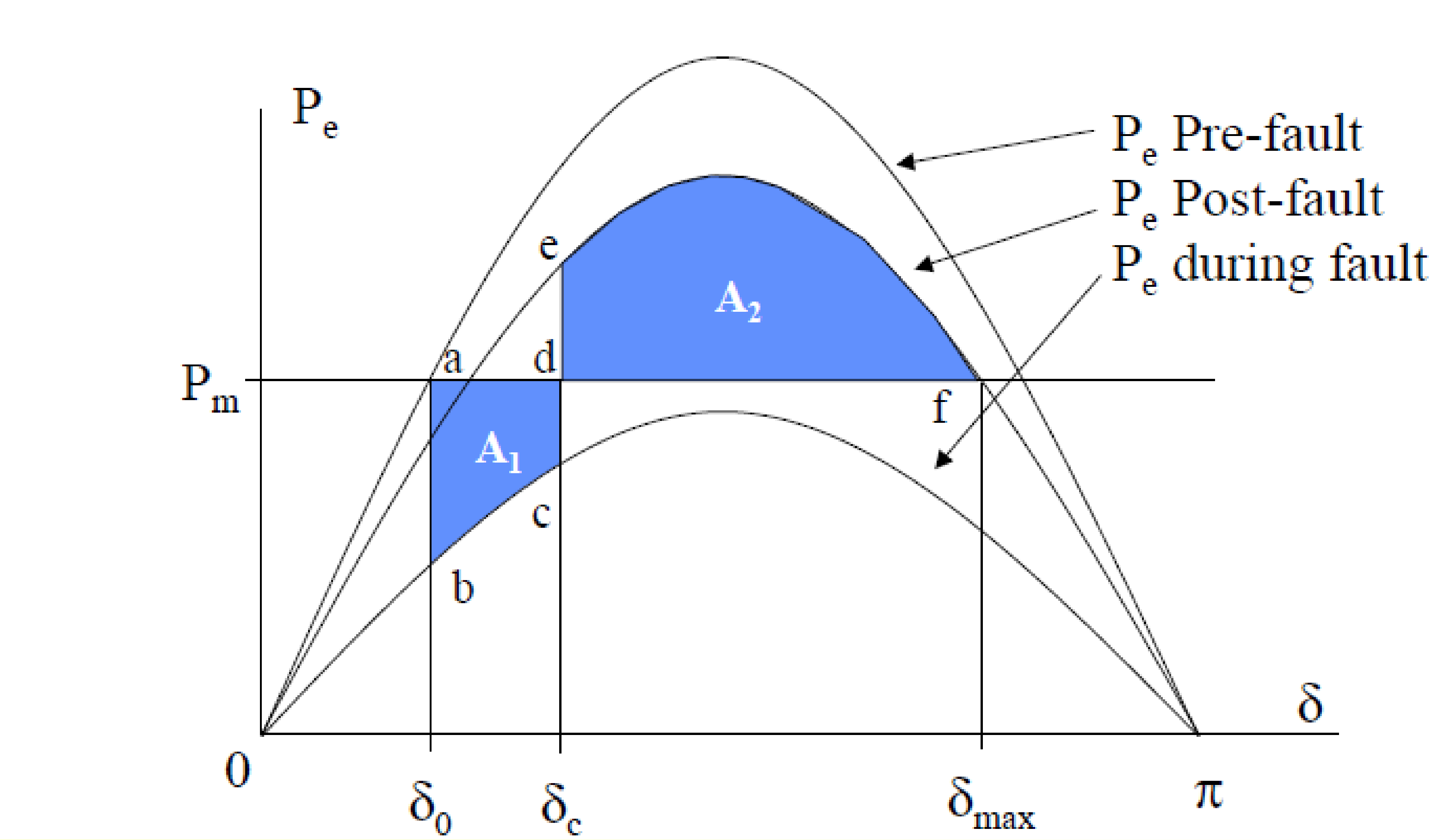


- Now consider a three-phase fault along one of the line away from the sending end of the line.
- The faulted line is opened and remians opened. One line is active only

EQUAL-AREA CRITERION



CRITICAL CLEARING TIME



CRITICAL CLEARING ANGLE

- Critical clearing angle

$$|\textbf{area } A_1| = |\textbf{area } A_2|$$

$$P_m(\delta_c - \delta_0) - \int_{\delta_0}^{\delta_c} P_{2\max} \sin\delta \, d\delta = \int_{\delta_c}^{\delta_{\max}} P_{3\max} \sin\delta \, d\delta - P_m(\delta_{\max} - \delta_c)$$

- Finally

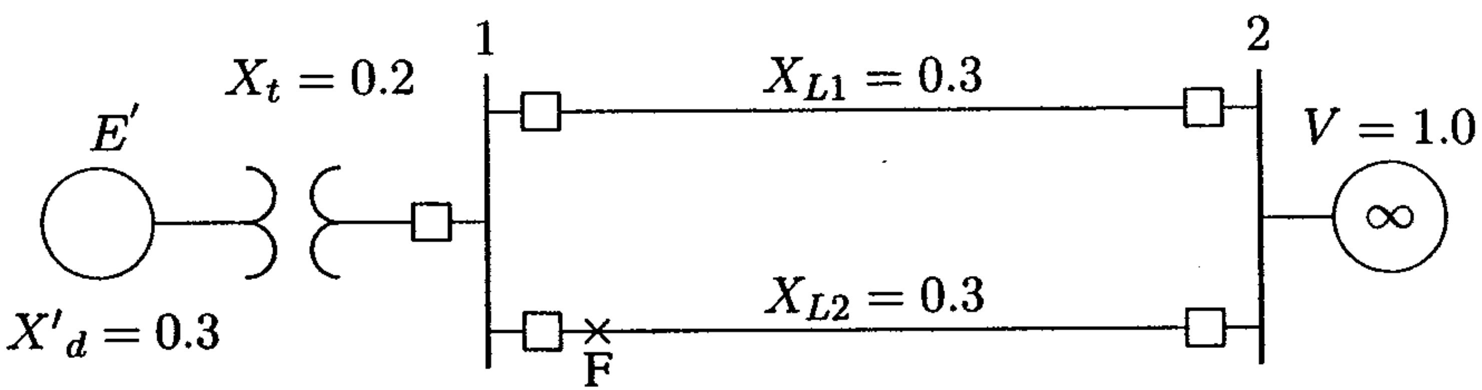
$$\cos\delta_c = \frac{P_m(\delta_{\max} - \delta_0) + P_{3\max}\cos\delta_{\max} - P_{2\max}\cos\delta_0}{P_{3\max} - P_{2\max}}$$

Example 1

A 60-Hz synchronous generator having inertia constant $H = 5$ MJ/MVA and a direct axis transient reactance $X'_d = 0.3$ per unit is connected to an infinite bus through a purely reactive circuit as shown in Figure 11.21. Reactances are marked on the diagram on a common system base. The generator is delivering real power $P_e = 0.8$ per unit and $Q = 0.074$ per unit to the infinite bus at a voltage of $V = 1$ per unit.

(a) A temporary three-phase fault occurs at the sending end of the line at point F. When the fault is cleared, both lines are intact. Determine the critical clearing angle and the critical fault clearing time.

(b) A three-phase fault occurs at the middle of one of the lines, the fault is cleared, and the faulted line is isolated. Determine the critical clearing angle.



Solution

The current flowing into the infinite bus is

$$I = \frac{S^*}{V^*} = \frac{0.8 - j0.074}{1.0\angle 0^\circ} = 0.8 - j0.074 \quad \text{pu}$$

The transfer reactance between internal voltage and the infinite bus before fault is

$$X_1 = 0.3 + 0.2 + \frac{0.3}{2} = 0.65$$

The transient internal voltage is

$$E' = V + jX_1 I = 1.0 + (j0.65)(0.8 - j0.074) = 1.17\angle 26.387^\circ \quad \text{pu}$$

Solution

(a) Since both lines are intact when the fault is cleared, the power-angle equation before and after the fault is

$$P_{max} \sin\delta = \frac{(1.17)(1.0)}{0.65} \sin\delta = 1.8 \sin\delta$$

The initial operating angle is given by

$$1.8 \sin\delta_0 = 0.8$$

or

$$\delta_0 = 26.388^\circ = 0.46055 \quad \text{rad}$$

and referring to Figure 11.17

$$\delta_{max} = 180^\circ - \delta_0 = 153.612^\circ \quad = 2.681 \quad \text{rad}$$

Solution

Since the fault is at the beginning of the transmission line, the power transfer during fault is zero, and the critical clearing angle as given by (11.91) is

$$\cos\delta_c = \frac{0.8}{1.8}(2.681 - 0.46055) + \cos 153.61^\circ = 0.09106$$

Thus, the critical clearing angle is

$$\delta_c = \cos^{-1}(0.09106) = 84.775^\circ \quad = 1.48 \;\; \text{rad}$$

From (11.92), the critical clearing time is

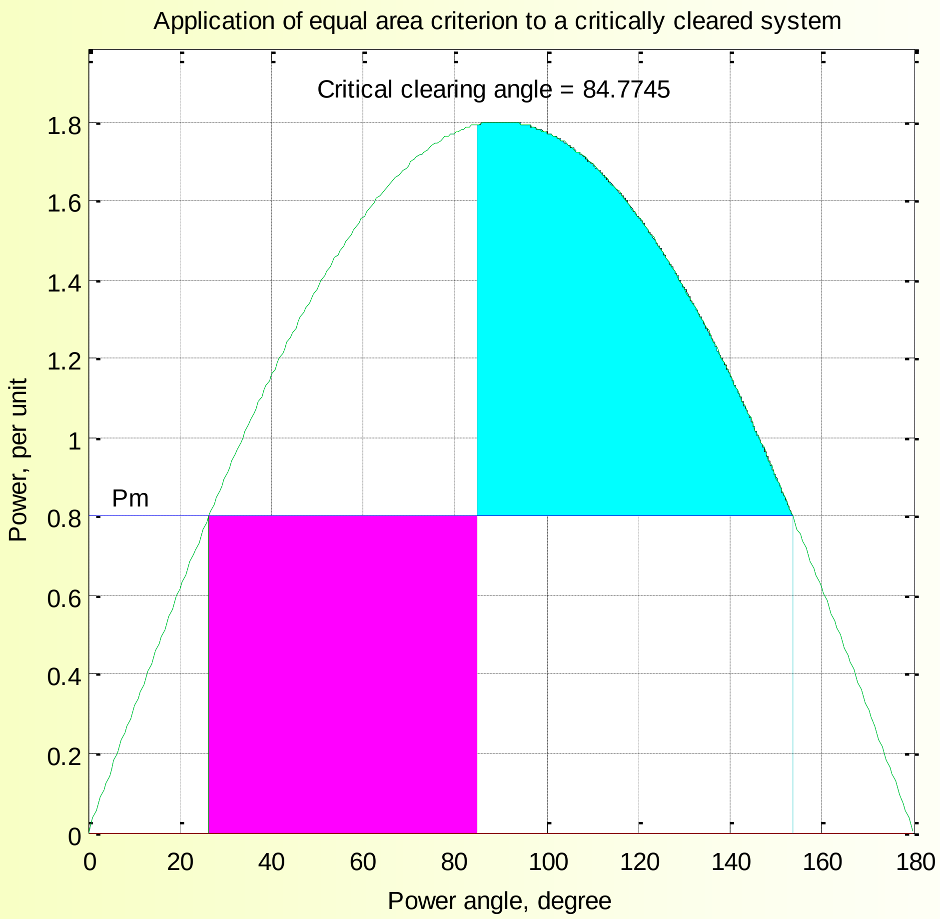
$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}} = \sqrt{\frac{(2)(5)(1.48 - 0.46055)}{(\pi)(60)(.8)}} = 0.26 \;\; \text{second}$$

Solution by Matlab

The use of function **eacfault**(P_m, E, V, X_1, X_2, X_3) to solve the above problem and to display power-angle plot with the shaded equal-areas is demonstrated below. We use the following commands

```
Pm = 0.8; E = 1.17; V = 1.0;
X1 = 0.65; X2 = inf; X3 = 0.65;
eacfault(Pm, E, V, X1, X2, X3)
```

Solution by Matlab



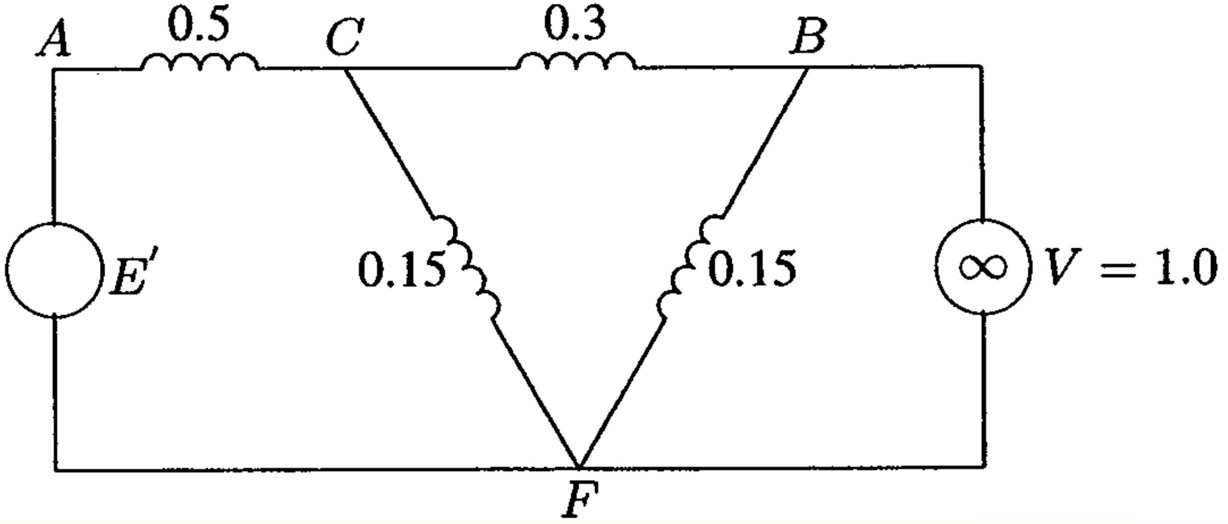
Initial power angle = 26.388
Maximum angle swing = 153.612
Critical clearing angle = 84.775
Critical clearing time = 0.260 sec.

Solution

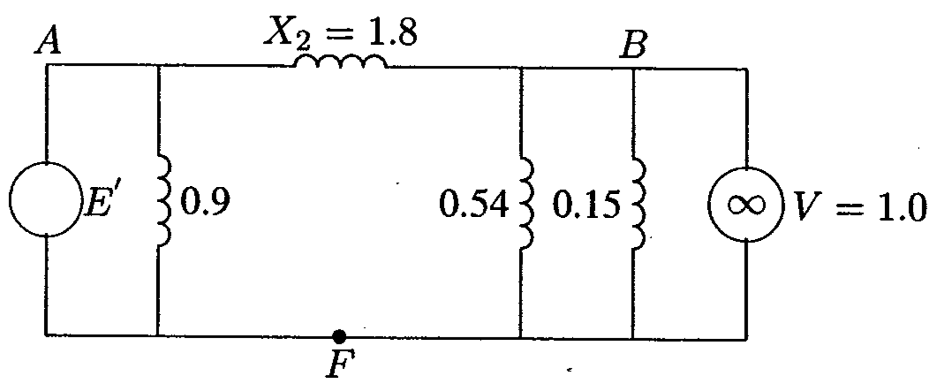
(b) The power-angle curve before the occurrence of the fault is the same as before, given by

$$P_{1\,max} = 1.8 \sin \delta$$

and the generator is operating at the initial power angle $\delta_0 = 26.4° = 0.4605$ rad. The fault occurs at point F at the middle of one line, resulting in the circuit shown in Figure 11.23. The transfer reactance during fault may be found most readily by converting the Y-circuit ABF to an equivalent delta, eliminating junction C. The resulting circuit is shown in Figure 11.24.



Solution



The equivalent reactance between generator and the infinite bus is

$$X_2 = \frac{(0.5)(0.3) + (0.5)(0.15) + (0.3)(0.15)}{0.15} = 1.8 \text{ pu}$$

Thus, the power-angle curve during fault is

$$P_{2\,max} \sin\delta = \frac{(1.17)(1.0)}{1.8} \sin\delta = 0.65 \sin\delta$$

When fault is cleared the faulted line is isolated. Therefore, the postfault transfer reactance is

$$X_3 = 0.3 + 0.2 + 0.3 = 0.8 \text{ pu}$$

and the power-angle curve is

$$P_{3\,max} \sin\delta = \frac{(1.17)(1.0)}{0.8} \sin\delta = 1.4625 \sin\delta$$

Solution

$$\delta_{max} = 180^\circ - \sin^{-1}\left(\frac{0.8}{1.4625}\right) = 146.838^\circ \;\; = 2.5628 \;\; \text{rad}$$

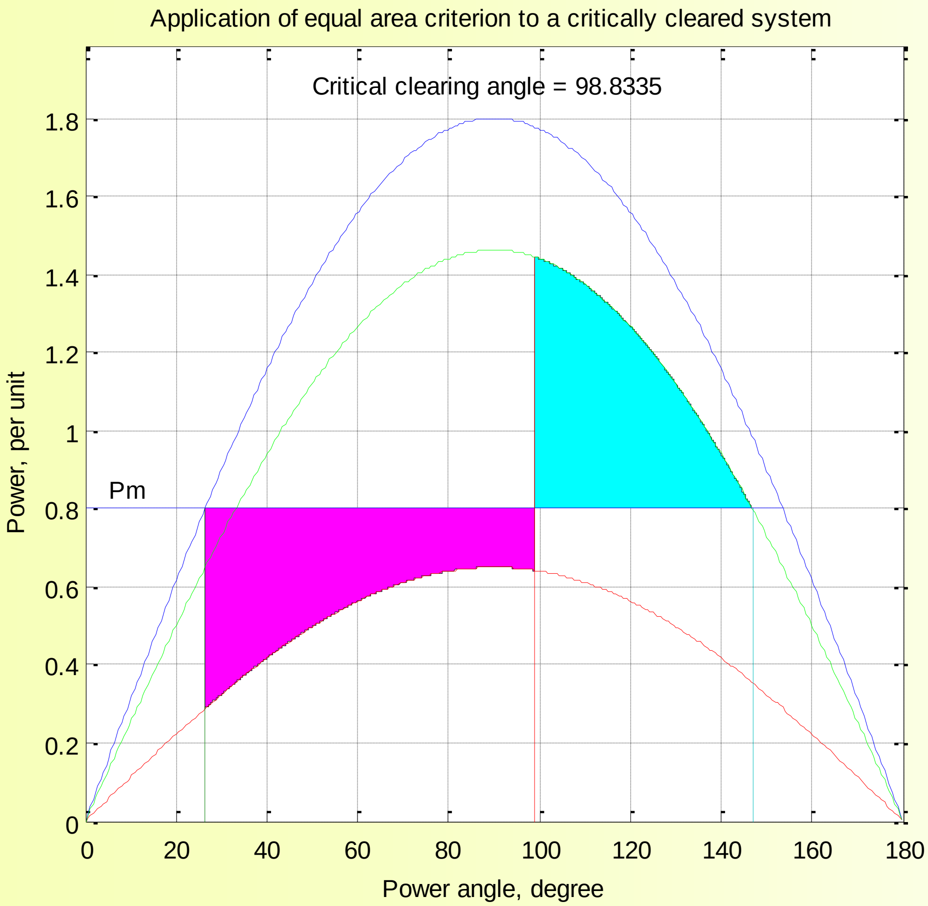
Applying (11.93), the critical clearing angle is given by

$$\begin{aligned} \cos\delta_c &= \frac{0.8(2.5628 - 0.46055) + 1.4625\cos 146.838^\circ - 0.65\cos 26.388^\circ}{1.4625 - 0.65} \\ &= -0.15356 \end{aligned}$$

Thus, the critical clearing angle is

$$\delta_c = \cos^{-1}(-0.15356) = 98.834^\circ$$

Solution by Matlab




```
Pm = 0.8; E = 1.17; V = 1.0;
X1 = 0.65; X2 = 1.8; X3 = 0.8;
eacfault(Pm, E, V, X1, X2, X3)
```

```
Initial power angle        =  26.388
Maximum angle swing        = 146.838
Critical clearing angle    =  98.834
```

Example 2

Example 11.8: *A 50Hz, synchronous generator capable of supplying 400 MW of power is connected to a large power system and is delivering 80 MW when a three phase fault occurs at its terminals, determine,*

(*a*) The time in which the fault must be cleared if the maximum power angle is to be –85°. Assume H = 7 MJ/MVA on a 100 MVA base.

(*b*) The critical clearing angle.

$$P_i = P_{max} \sin\delta_0$$

$$P_{max} = \frac{400}{3} \text{ MW}, \; P_i = \frac{80}{3} \text{ MW}$$

$$\sin \delta_0 = \frac{\left(\frac{80}{3}\right)}{\left(\frac{400}{3}\right)} = 0.2$$

$$\delta_0 = 11.54^\circ = 0.2 \text{ radian}$$

$$\delta_1 = 85^\circ = 1.48 \text{ radian}$$

$$\cos \delta_c = \cos \delta_1 + (\delta_1 - \delta_0) \sin \delta_0$$

$$\cos \delta_c = \cos(1.48) + (1.48 - 0.2) \sin (0.2)$$

$$\cos \delta_c = 0.343$$

$$\delta_c = 1.22 \text{ radian.}$$

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f P_i}}$$

$$P_i\,(3\phi) = 80 \text{ MW} = \frac{80}{100} = 0.8 \text{ pu}$$

$$H = 7 \text{ MJ/MVA}$$

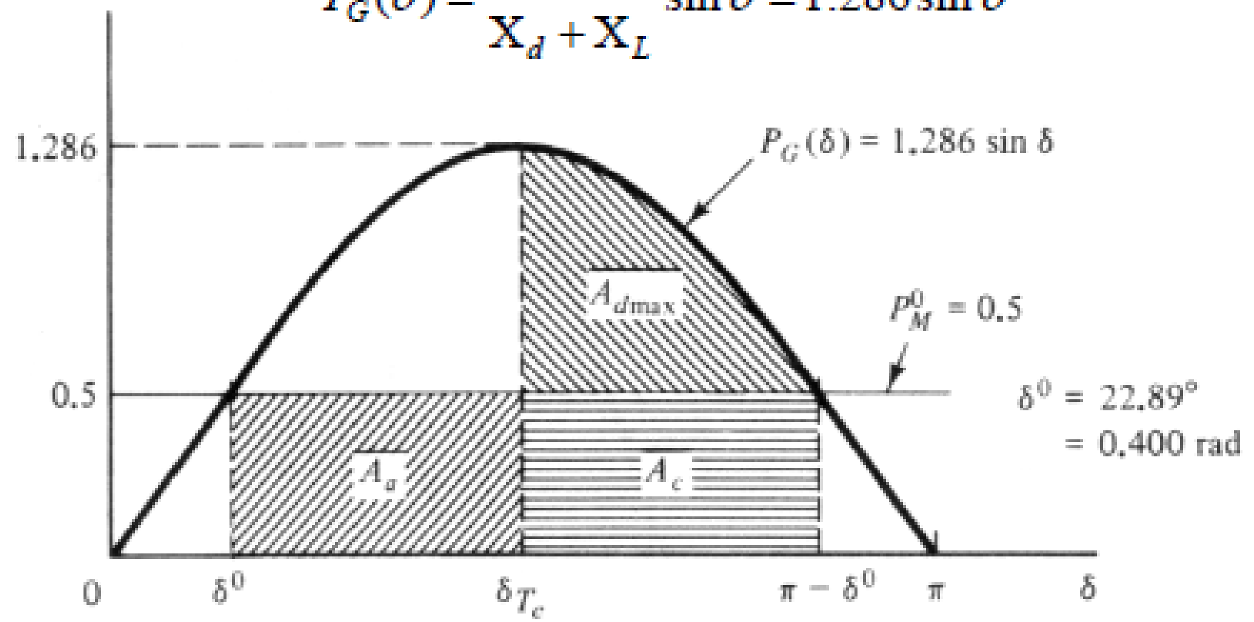
$$t_c = \sqrt{\frac{2 \times 7 \times (1.48 - 0.2)}{\pi \times 50 \times 0.8}}$$

$$t_c = 0.377 \text{ secs} = 377 \text{ ms.}$$

Example 3

- A round-rotor generator with X_L=0.4 and X_d=X_q=1.0. Assume $|E_a|$ = 1.8, $|V_\infty|$ = 1.0, H = 5 sec and $P_G^0 = 0.5$
- Critical clearing time ($T_{critical}$)

$$P_G(\delta) = \frac{|E_a||V_\infty|}{X_d + X_L}\sin\delta = 1.286\sin\delta$$



Example 3

- Initially, $P_G(\delta^0) = 1.286 \sin \delta^0 = 0.5 \quad \Rightarrow \quad \delta^0 = 0.400\text{rad} = 22.89^\circ$
- Equal area

$$A_a + A_c = A_{d\max} + A_c$$

$$0.5(\pi - 2\delta^0) = \int_{\delta_{Tc}}^{\pi-\delta^0} 1.286 \sin \delta \, d\delta$$

$$1.1708 = 1.286(\cos \delta_{Tc} + 0.9211)$$

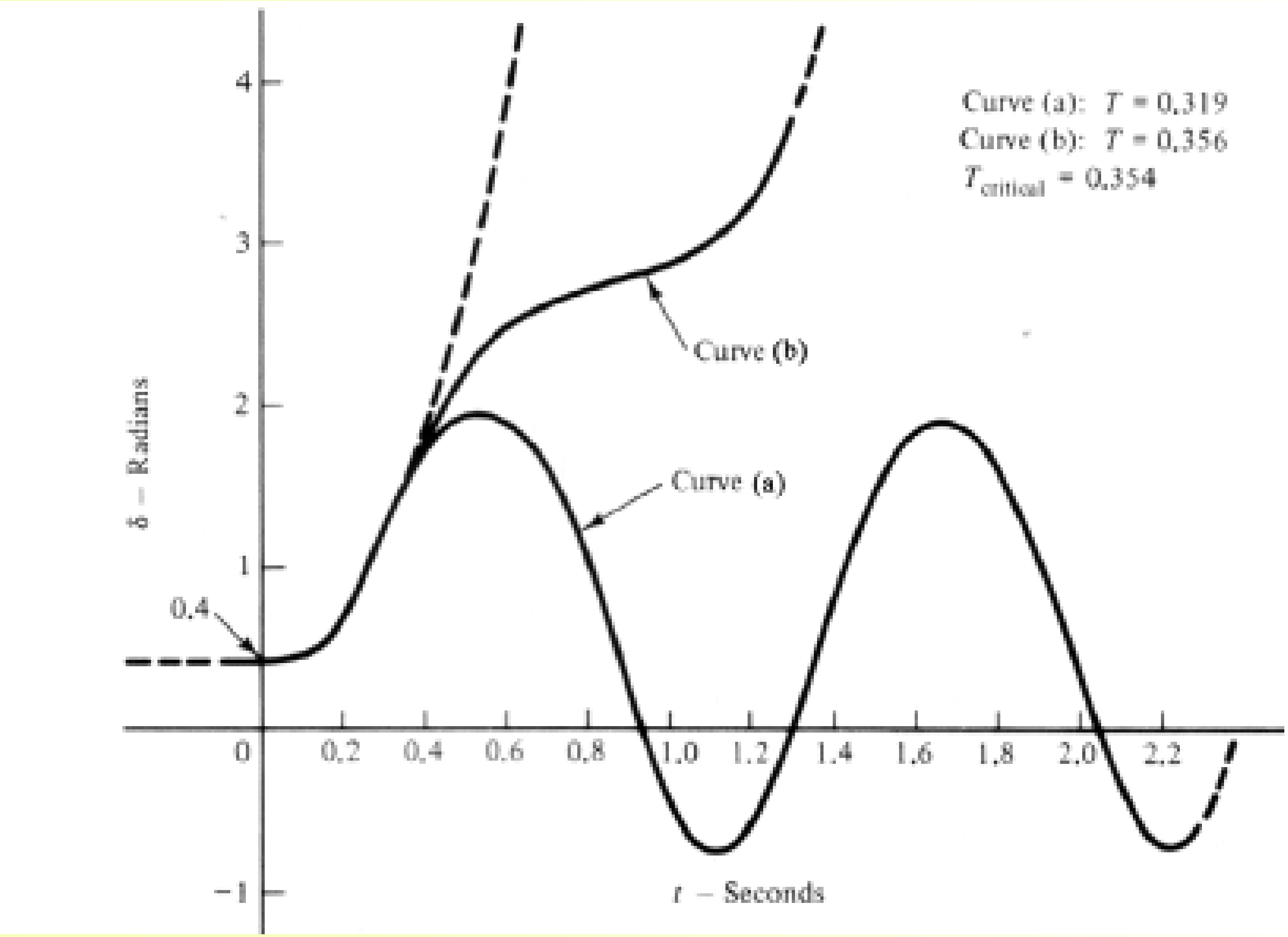
$$\delta_{Tc} = 1.581 rad = 90.61^\circ$$

- Critical clearing time

$$\delta_{Tc} = 1.581 = \frac{\pi \times 60 \times 0.5}{2 \times 5} T_{critical}^2 + 0.40$$

$$T_{critical} = 0.354 \sec$$

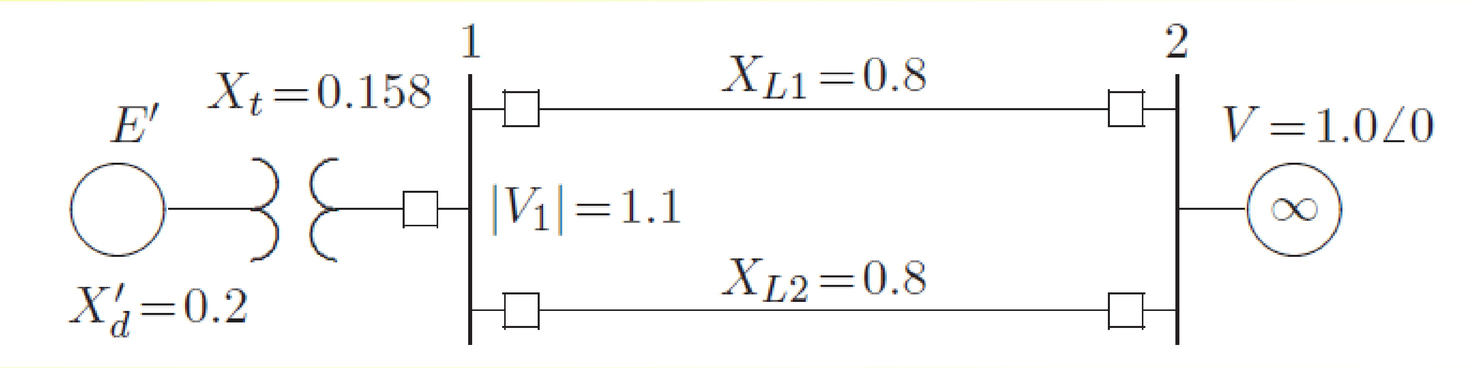
Example 3



Curve (a): T = 0.319
Curve (b): T = 0.356
$T_{critical}$ = 0.354
Curve (b)
Curve (a)
δ – Radians
t – Seconds
0.4
4
3
2
1
0
−1
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2

Homework Questions 1 and 2

A 60-Hz synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 5.66 MJ/MVA. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown below. Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator is delivering a real power of 0.77 per unit to bus bar 1. Voltage magnitude at bus 1 is $|V_1| = 1.1pu$ pu. The infinite bus voltage $V_2 = 1.0\angle 0^0\, pu$ pu.



Homework 1 (20 points)

- The generator in the previous is delivering a real power input of 0.77 pu to the infinite bus at a voltage of 1.0 pu. The generator excitation voltages is E_0 = 1.25 pu. Use **eacpower(***Pm,E, V,X***)** to find

  (a) The maximum power input that can be added without loss of synchronism.

  (b) Repeat (a) with zero initial power input. Assume the generator internal voltages remains constant at the value computed in (a).

Homework 2 (40 points)

- The same generator is delivering a real power input of 0.77 pu to the infinite bus at a voltage of 1.0 pu. The generator excitation voltages is E_0 = 1.25 pu.

(a) A temporary three-phase fault occurs at the sending end of one of the transmission lines. When the fault is cleared, both lines are intact. Using equal area criterion, determine the critical clearing angle and the critical fault clearing time. Use **eacfault(***Pm,E, V,X*1*,X*2*,X*3**)** to check the result and to display the power angle plot.

(b) A three-phase fault occurs at the middle of one of the lines, the fault is cleared, and the faulted line is isolated. Determine the critical clearing angle. Use **eacfault (***Pm,E,V,X*1*,X*2*,X*3**)** to check the results and to display the power-angle plot.

POWER SYSTEM DYNAMICS (STABILITY) AND CONTROL

Lecture Notes 5

Transient Stability: Numerical Integration Methods and Multi-Machine System

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Gazi University

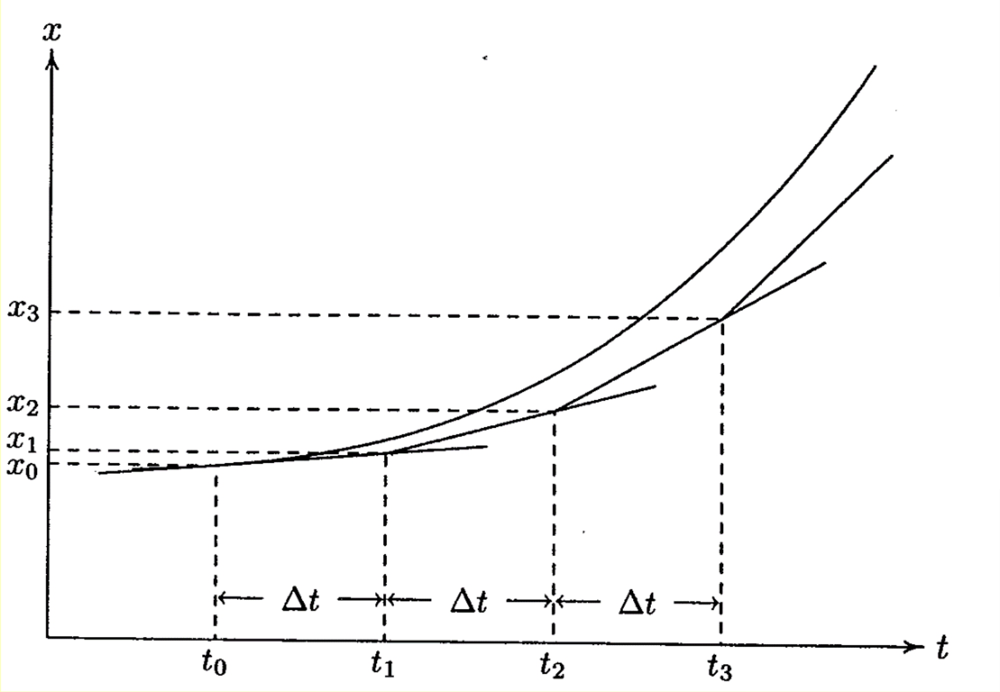
SOLVING NONLNEAR ODE

- **Objective**
  - Time domain solution of a system of differential equations
    - Given a function or a system of functions: $f(x)$ or $\mathbf{F}(\mathbf{x})$
    - Seek a time domain solution $x(t)$ or $\mathbf{x}(t)$ which satisfy $f(x)$ or $\mathbf{F}(\mathbf{x})$
- **Integration of the differential equations**
  - Linear equations - Closed form solutions:
    - Laplace transforms
  - Non-linear equations - Frequently no closed form solutions:
    - Numerical integration
      - Taylor Series
      - Euler
      - Runga-Kutta

Euler Method

Consider the first-order differential equation

$$\frac{dx}{dt} = f(x)$$



$$\Delta x \approx \left.\frac{dx}{dt}\right|_{x_0} \Delta t$$

the value of x at $t_0 + \Delta t$ is

$$x_1 = x_0 + \Delta x = x_0 + \left.\frac{dx}{dt}\right|_{x_0} \Delta t$$

$$x_{i+1} = x_i + \left.\frac{dx}{dt}\right|_{x_i} \Delta t$$

Modified Euler Method

By using the derivative at the beginning of the step, the value at the end of the step ($t_1 = t_0 + \Delta t$) is predicted from

$$x_1^p = x_0 + \left.\frac{dx}{dt}\right|_{x_0} \Delta t$$

Using the predicted value of x_1^p, the derivative at the end of interval is determined by

$$\left.\frac{dx}{dt}\right|_{x_1^p} = f(t_1, x_1^p)$$

Then, the average value of the two derivatives is used to find the corrected value

$$x_1^c = x_0 + \left(\frac{\left.\frac{dx}{dt}\right|_{x_0} + \left.\frac{dx}{dt}\right|_{x_1^p}}{2} \right) \Delta t$$

$$x_{i+1}^c = x_i + \left(\frac{\left.\frac{dx}{dt}\right|_{x_i} + \left.\frac{dx}{dt}\right|_{x_{i+1}^p}}{2} \right) \Delta t$$

MATLAB
ORDINARY DIFFERENTIAL EQUATION (ODE) SOLVER

Introduction

- Analytical solutions of linear time-invariant equations are obtained through the Laplace transform and its inversion.
- The analytical methods are normally restricted to linear differential equations with constant coefficients.
- Numerical techniques solve differential equations directly in the time domain; they apply not only the linear time-invariant, but also to nonlinear and time varying differential equations.
- The value of the function obtained at any step is an approximation of the value which would have been obtained analytically; whereas, the analytical solution is exact
- However, an analytical solution may be difficult, lime consuming, or even impossible to find.

Matlab ODE Solvers

- *MATLAB* provides two functions for numerical solutions of differential equations employing the Runge-Kutta method.
- These are ode23 and ode45, based on the Fehlberg-second and third-order pair of formulas for medium accuracy and fourth- and fifth-order pair for high accuracy.
- The nth-order differential equation must be transformed into n first-order differential equations and must be placed in an M-file that returns the state derivatives of the equations.

State-Space Equation Model

the second-order differential equation

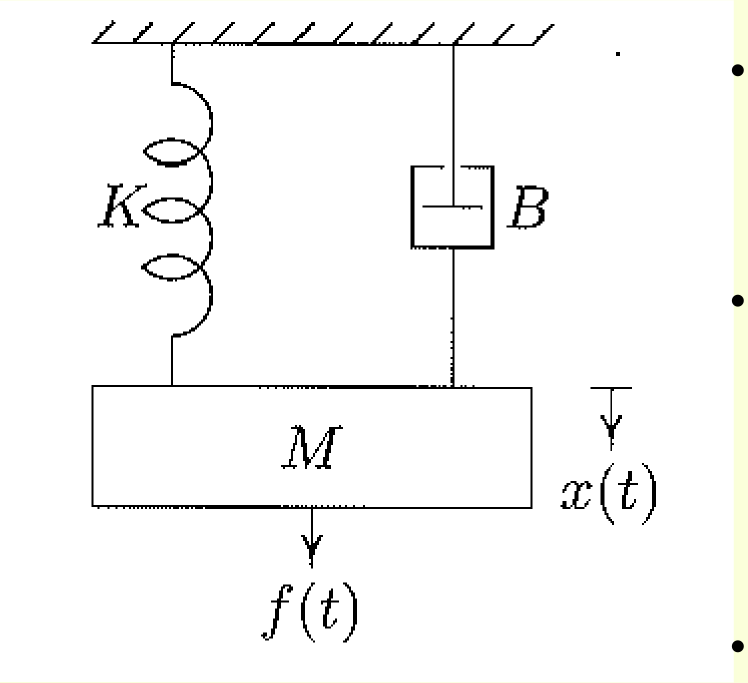
$$a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = c$$

the initial conditions x_0 and $\left.\frac{dx}{dt}\right|_{x_0}$ at t_0.

$$x_1 = x$$
$$x_2 = \frac{dx}{dt}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{c}{a_2} - \frac{a_0}{a_2} x_1 - \frac{a_1}{a_2} x_2$$

Example 1: A Simple Mechanical System



- Three forces influence the motion of the mass, namely, the applied force, the frictional force, and the spring force.
- Applying Newton's law of motion, the force equation of the system is

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f(t)$$

- Define states $x_1 = x$ $\quad x_2 = \frac{dx}{dt}$

$$\frac{dx_1}{dt} = x_2$$
$$\frac{dx_2}{dt} = \frac{1}{M}[f(t) - Bx_2 - Kx_1]$$

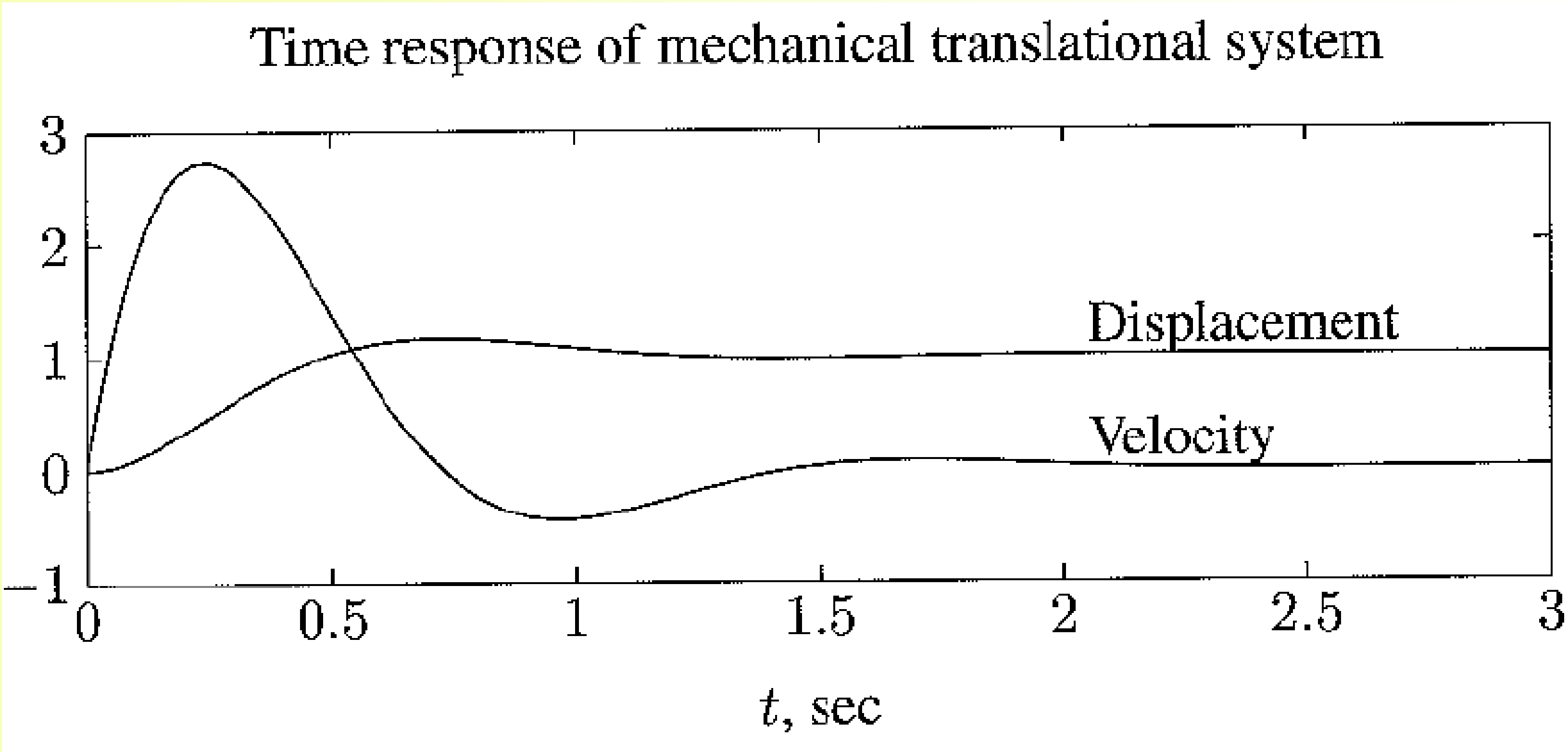
Example 1: M-File and Function

```
function xdot = mechsys(t, x); % returns the state derivatives
```

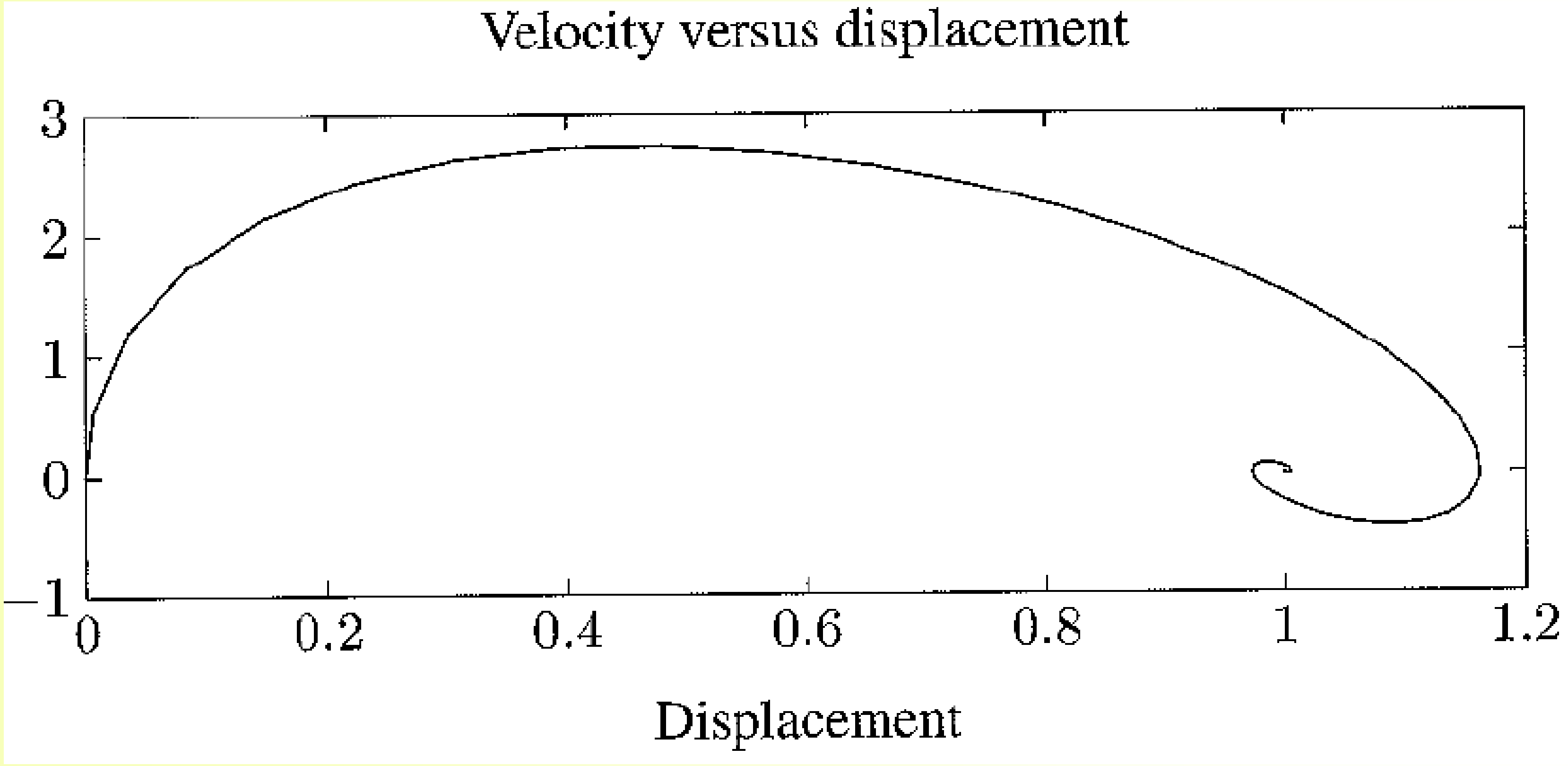
```
F = 25;                                          % Step input
M =1; B = 5; K = 25; xdot = [x(2); 1/M*(F - B*x(2)-K*x(1) ) ];
```

```
tspan = [0, 3];                                  % time interval
x0 = [0, 0];                                     % initial conditions
[t,x] = ode23('mechsys', tspan, x0); subplot(2, 1, 1), plot(t, x),
xlabel('t,  sec') title('Time response of mechanical translational
system') text(2, 1.2, 'Displacement'), text(2, 0.2, 'Velocity') d
= x(:, 1);    v = x(:, 2); subplot(2, 1, 2), plot(d, v)
title('Velocity versus displacement ') xlabel('Displacement'),
ylabel('Velocity'), subplot(111)
```

Example 1: Results

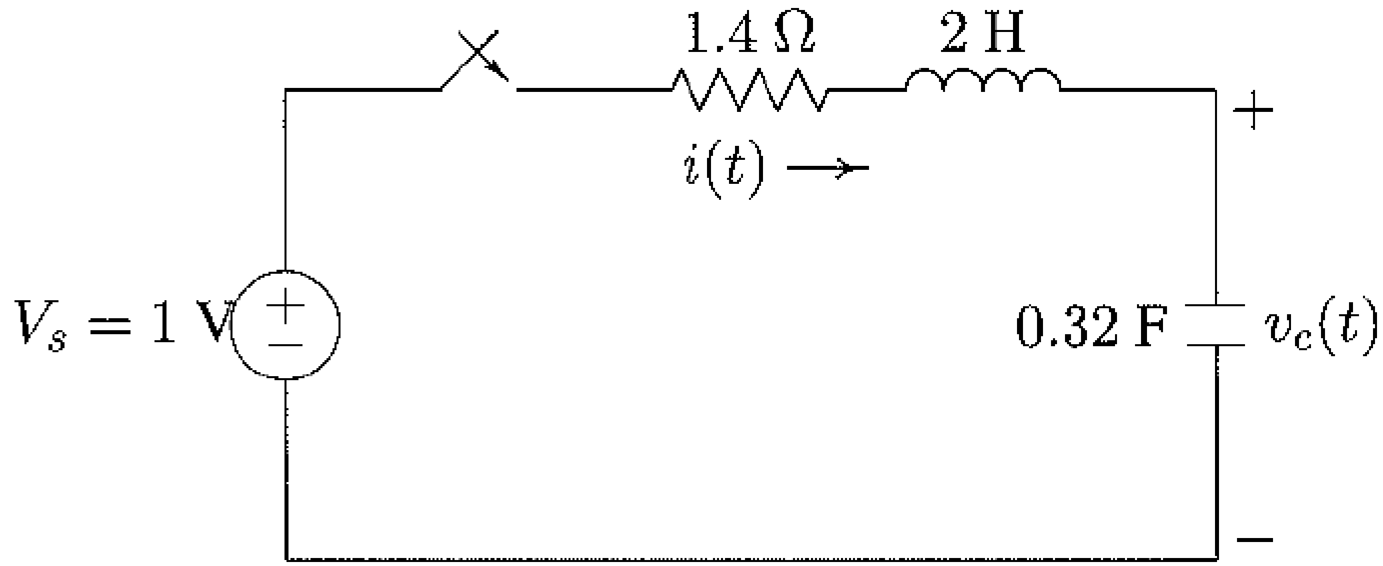


Example 1: Phase-Potrait



Example 2: Electric Circuit

The circuit elements in Figure A.12 are $R = 1.4\ \Omega$, $L = 2$ H, and $C = 0.32$ F. The initial inductor current is zero, and the initial capacitor voltage is 0.5 volts. A step voltage of 1 volt is applied at time $t = 0$. Determine $i(t)$ and $v(t)$ over the range $0 < t < 15$ sec. Also, obtain a plot of current versus capacitor voltage.



Example 2: State-Space Equations

- Apply KVL

$$Ri + L\frac{di}{dt} + v_c = V_s$$

$$i = C\frac{dv_c}{dt}$$

- Define states

$$x_1 = v_c \qquad x_2 = i$$

- Obtain state-space equations

$$\dot{x}_1 = \frac{1}{C}x_2$$

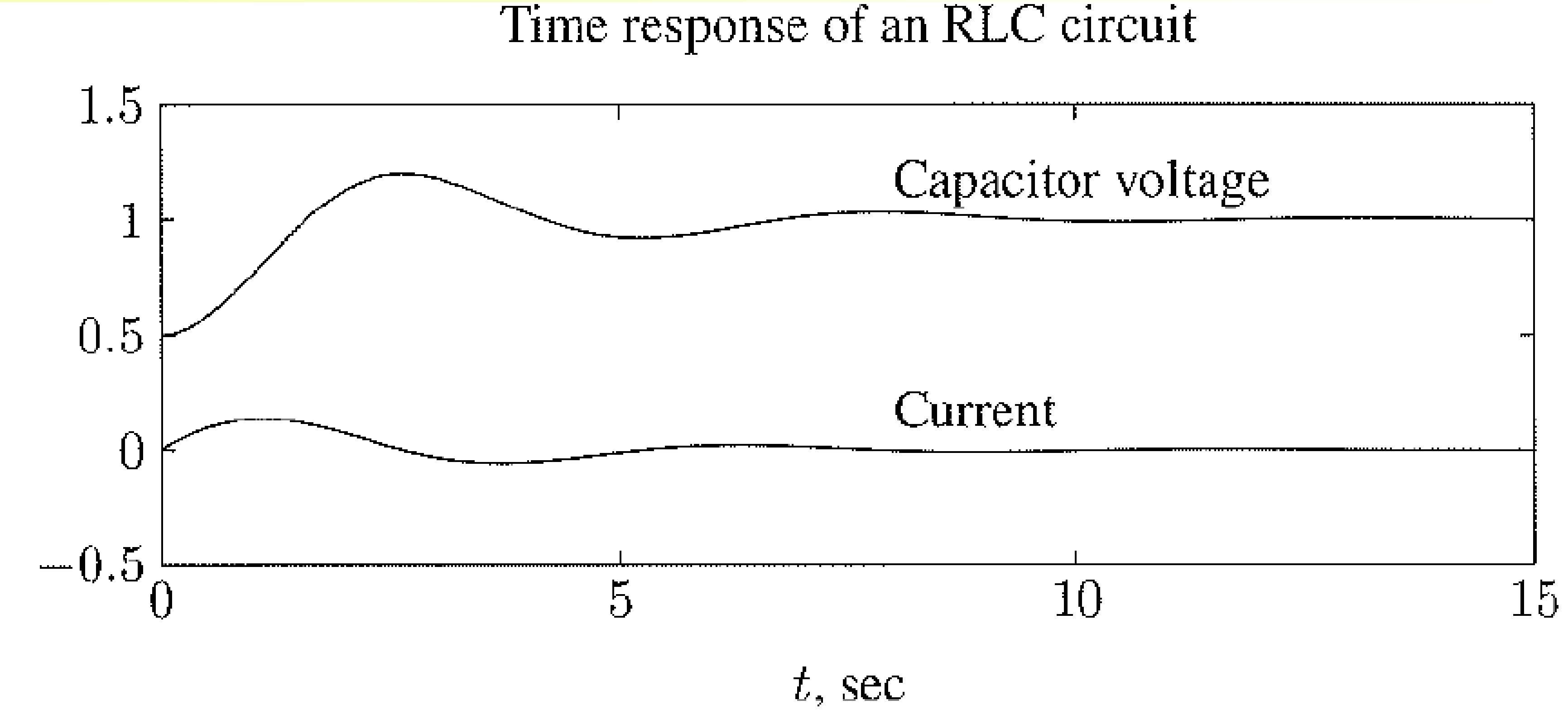
$$\dot{x}_2 = \frac{1}{L}(V_s - x_1 - Rx_2)$$

Example 2: M-file and Function

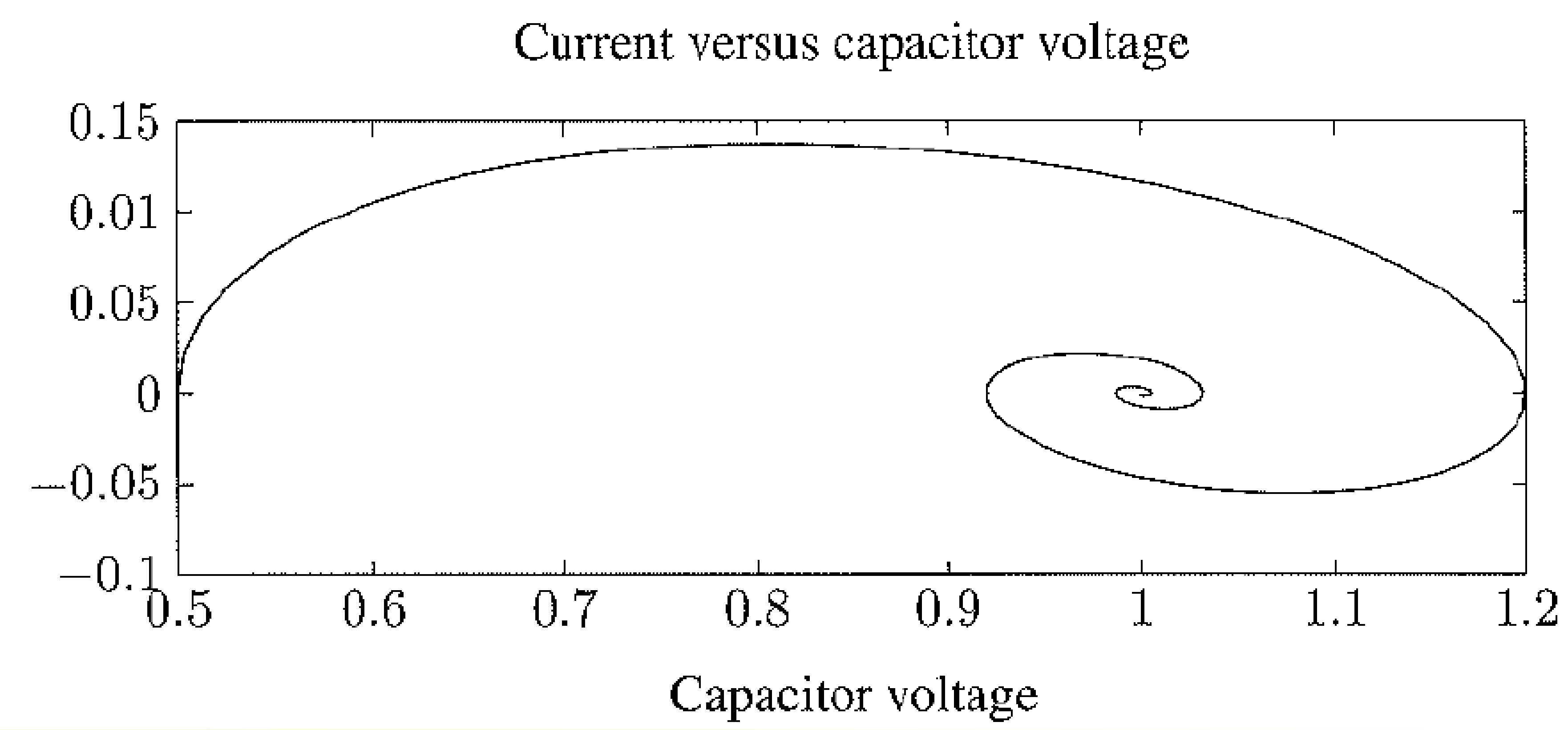
```
function xdot = electsys(t, x);
                                 % returns the state derivatives
V = 1;                                             % Step input
R =1.4; L = 2; C = 0.32; xdot = [x(2)/C; 1/L*( V - x(1) - R*x(2)
)];
```

```
tspan = [0, 15];                                  % time interval
x0 = [0.5, 0];                               % initial conditions
[t,x] = ode23('electsys', tspan, x0); subplot(2, 1, 1), plot(t, x)
title('Time response of an RLC series circuit') xlabel('t,  sec')
text(8,1.05,'Capacitor voltage'), text(8, .05,'Current') vc= x(:,
1);   i = x(:, 2); subplot(2, 1, 2), plot(vc, i) title('Current
versus capacitor voltage ') xlabel('Capacitor voltage'),
ylabel('Current') subplot(111)
```

Example 2: Results

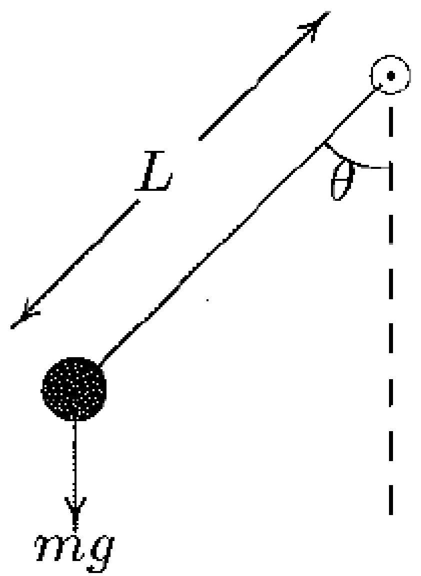


Example 2: Phase-Potrait



Example 3: Pendulum

Consider the simple pendulum illustrated in Figure A.14, where a weight of $W = mg$ kg is hung from a support by a weightless rod of length L meters. While usually approximated by a linear differential equation, the system really is nonlinear and includes viscous damping with a damping coefficient of B kg/m/sec.



$$mL\ddot{\theta} + BL\dot{\theta} + W \sin\theta = 0$$

Let $x_1 = \theta$ and $x_2 = \dot{\theta}$ (angular velocity), then

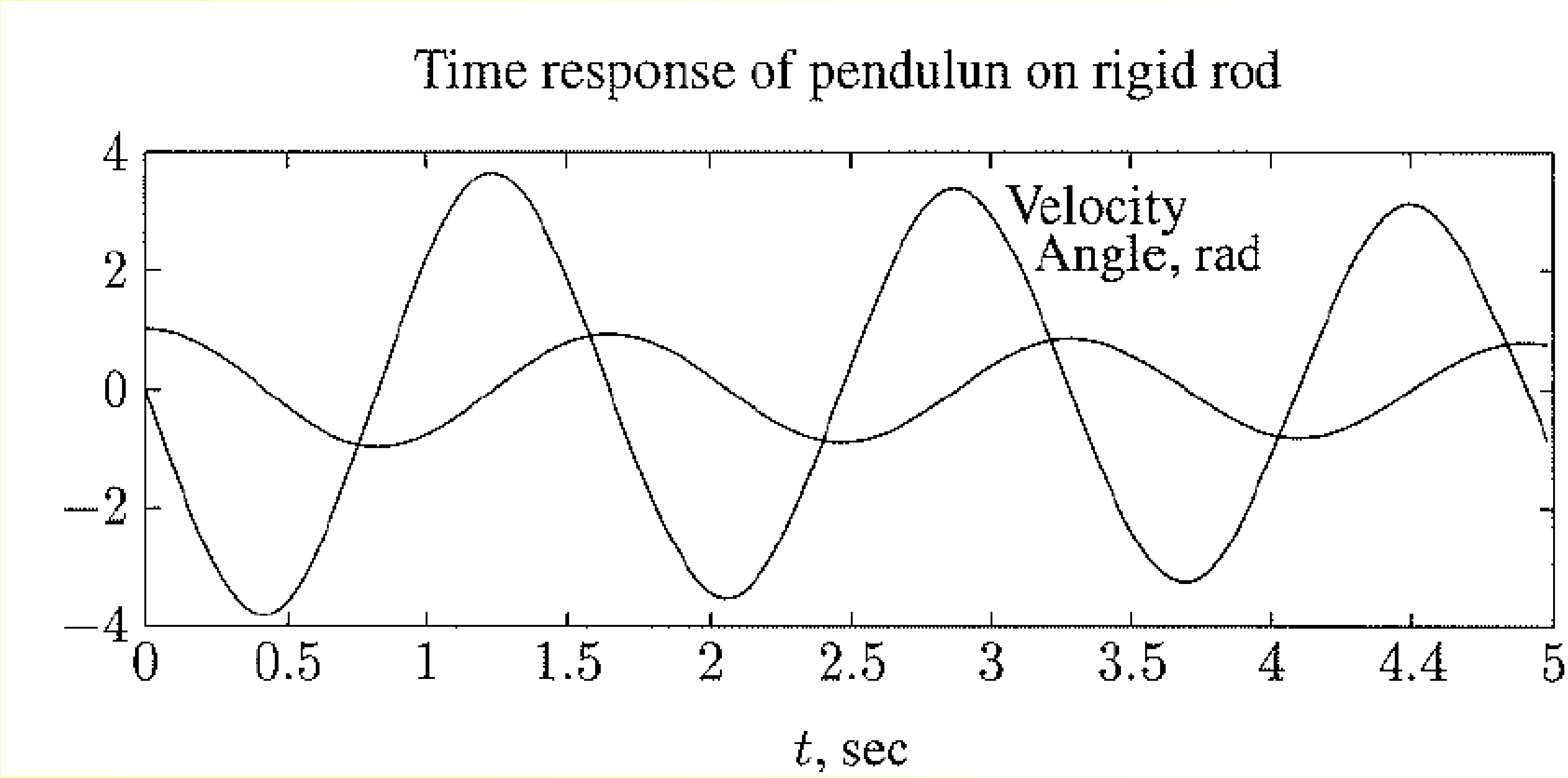
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{B}{m}x_2 - \frac{W}{mL}\sin x_1$$

Example 3: M-file and Function

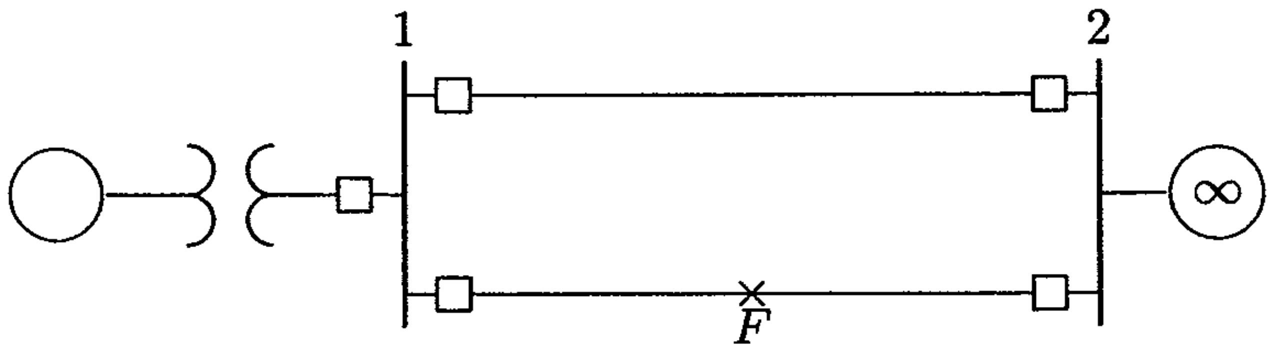
```
function xdot = pendulum(t,x);%returns the state derivatives
W = 2; L = .6; B = 0.02; g = 9.81;  m = W/g; xdot = [x(2) ;
-B/m*x(2)-W/(m*L)*sin(x(1)) ];
```

```
tspan = [0, 5];                                          % time interval
x0 = [1, 0];                                        % initial conditions
[t,x] = ode23('pendulum', tspan, x0); subplot(2, 1, 1), plot(t, x)
title('Time response of a rigid pendulum') xlabel('t, sec')
text(3.2, 3.5, 'Velocity') , text(3.2, 1.2, 'Angle-rad.') th= x(:,
1);   w = x(:, 2); subplot(2, 1, 2), plot(th, w) title('Phase
plane plot of pendulum') xlabel('Position, rad'), ylabel('Angular
velocity') subplot(111)
```

Example 3: Results



Numerical Solution of Swing Equation



the input power P_m is constant.

Under steady state operation $P_e = P_m$, and the initial power angle is given by

$$\delta_0 = \sin^{-1} \frac{P_m}{P_{1\,max}}$$

$$P_{1\,max} = \frac{|E'||V|}{X_1}$$

and X_1 is the transfer reactance before the fault. The rotor is running at synchronous speed, and the change in the angular velocity is zero, i.e.,

$$\Delta\omega_0 = 0$$

Numerical Solution of Swing Equation

Now consider a three-phase fault at the middle of one line

The equivalent transfer reactance between bus bars is increased, lowering the power transfer capability, and the amplitude of the power-angle equation becomes

$$P_{2\,max} = \frac{|E'||V|}{X_2}$$

where X_2 is the transfer reactance during fault.

The swing equation given by

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(P_m - P_{2\,max}\sin\delta) = \frac{\pi f_0}{H}P_a$$

Numerical Solution of Swing Equation

The above swing equation is transformed into the state variable form as

$$\frac{d\delta}{dt} = \Delta\omega$$
$$\frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H} P_a$$

$$\delta_{i+1}^{p} = \delta_i + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} \Delta t$$
$$\Delta\omega_{i+1}^{p} = \Delta\omega_i + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} \Delta t$$

Using the predicted value of δ_{i+1}^{p}, and $\Delta\omega_{i+1}^{p}$ the derivatives at the end of interval are determined by

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{i+1}^{p}} = \Delta\omega_{i+1}^{p}$$
$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{i+1}^{p}} = \left.\frac{\pi f_0}{H} P_a\right|_{\delta_{i+1}^{p}}$$

Numerical Solution of Swing Equation

the average value of the two derivatives is used to find the corrected value

$$\delta_{i+1}^{c} = \delta_i + \left(\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{i+1}^{p}}}{2} \right) \Delta t$$

$$\Delta\omega_{i+1}^{c} = \Delta\omega_i + \left(\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{i+1}^{p}}}{2} \right) \Delta t$$

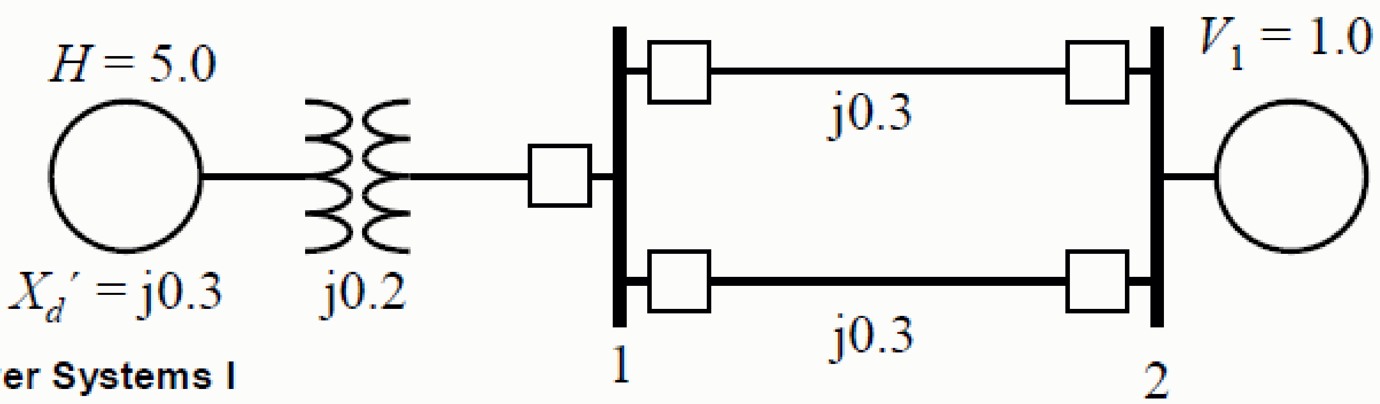
Matlab Function

Based on the above algorithm, a function named **swingmeu**(P_m, E, V, X_1, X_2, X_3, H, f, t_c, t_f, Dt) is written for the transient stability analysis of a one-machine system. The function arguments are

$\mathbf{P_m}$	Per unit mechanical power input, assumed to remain constant
E	Constant voltage back of transient reactance in per unit
V	Infinite bus bar voltage in per unit
$\mathbf{X_1}$	Per unit reactance between buses E and V before fault
$\mathbf{X_2}$	Per unit reactance between buses E and V during fault
$\mathbf{X_3}$	Per unit reactance between buses E and V after fault clearance
H	Generator inertia constant in second, (MJ/MVA)
f	System nominal frequency
$\mathbf{t_c}$	Fault clearing time
$\mathbf{t_f}$	Final time for integration
Dt	Integration time interval, required for modified Euler

EXAMPLE

- **Consider the following system**
  - a three-phase fault at the middle of one line is cleared by isolating the faulted circuit simultaneously at both ends.
  - The fault is cleared in 0.3 seconds, perform several steps of the numerical solution of the swing equation using the modified Euler method with a step size of
    $\Delta t = 0.01$ seconds.
  - graph the swing equation for clearing times of 0.3 s, 0.4 s, and 0.5 s.



Power Systems I

EXAMPLE

$$H = 5$$ Machine parameters

$$P_m = 0.8$$

$$E = V + jX_1 I = 1.0 + (j0.65)\frac{0.8 - j0.074}{1.0} = 1.17\angle 26^\circ$$

$$P_m = P_{\max}\sin\delta = \frac{(1.17)(1.0)}{0.65}\sin\delta = 1.8\sin\delta$$ Pre-fault equation

$$\delta_0 = 26.4^\circ = 0.4606\ \text{rad}$$ Initial conditions

$$P_m = 0.8 \qquad \Delta\omega = 0\ \text{rad/s}$$

$$P_{\max}^{[fault]} = \frac{(1.17)(1.0)}{1.8}\sin\delta = 0.65\sin\delta$$

fault parameters

$$P_a = P_m - P_e^{[fault]} = 0.8 - 0.65\sin\delta$$

EXAMPLE

$$\frac{d\delta}{dt} = \Delta\omega$$

$$\frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H} P_a$$

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_0} = 0 \text{ rad/s}$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_0} = \frac{\pi \cdot 60}{5}\left(0.8 - 0.65\sin\left(0.4606 \text{ rad}\right)\right) = 19.27 \text{ rad/s}^2$$

$$\delta_1^p = 0.4606 + (0)(0.01) = 0.4606 \text{ rad}$$

$$\Delta\omega_1^p = 0 + (19.27)(0.01) = 0.1927 \text{ rad/s}$$

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_1^p} = 0.1927 \text{ rad/s}$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_1^p} = \frac{\pi \cdot 60}{5}\left(0.8 - 0.65\sin\left(0.4606 \text{ rad}\right)\right) = 19.27 \text{ rad/s}^2$$

EXAMPLE

$$\delta_1^c = 0.4606 + \tfrac{1}{2}(0 + 0.1927)(0.01) = 0.4615 \text{ rad}$$

$$\Delta\omega_1^c = 0 + \tfrac{1}{2}(19.27 + 19.27)(0.01) = 0.1927 \text{ rad/s}$$

End of first step. Next step:

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_1} = 0.1927 \text{ rad/s}$$

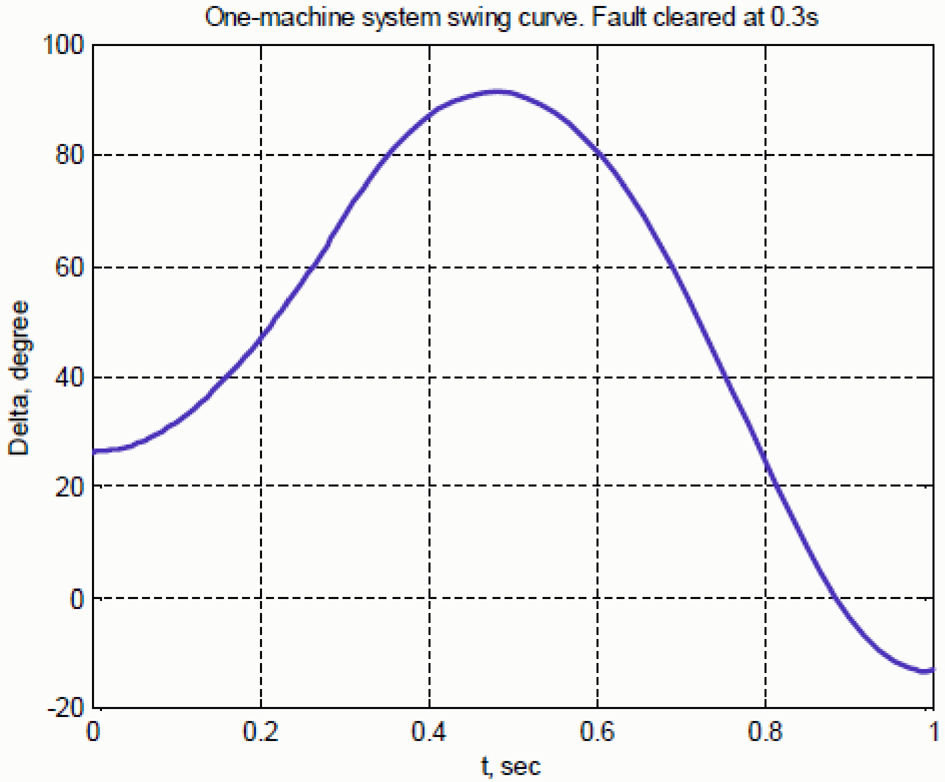
$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_1^p} = \frac{\pi \cdot 60}{5}\left(0.8 - 0.65\sin\left(0.4615 \text{ rad}\right)\right) = 19.25 \text{ rad/s}^2$$

The process is continued for the successive steps, until at $t = 0.3$ second when the fault is cleared. From Example 11.5, the postfault accelerating power equation is

$$P_a = 0.8 - 1.4625 \sin \delta$$

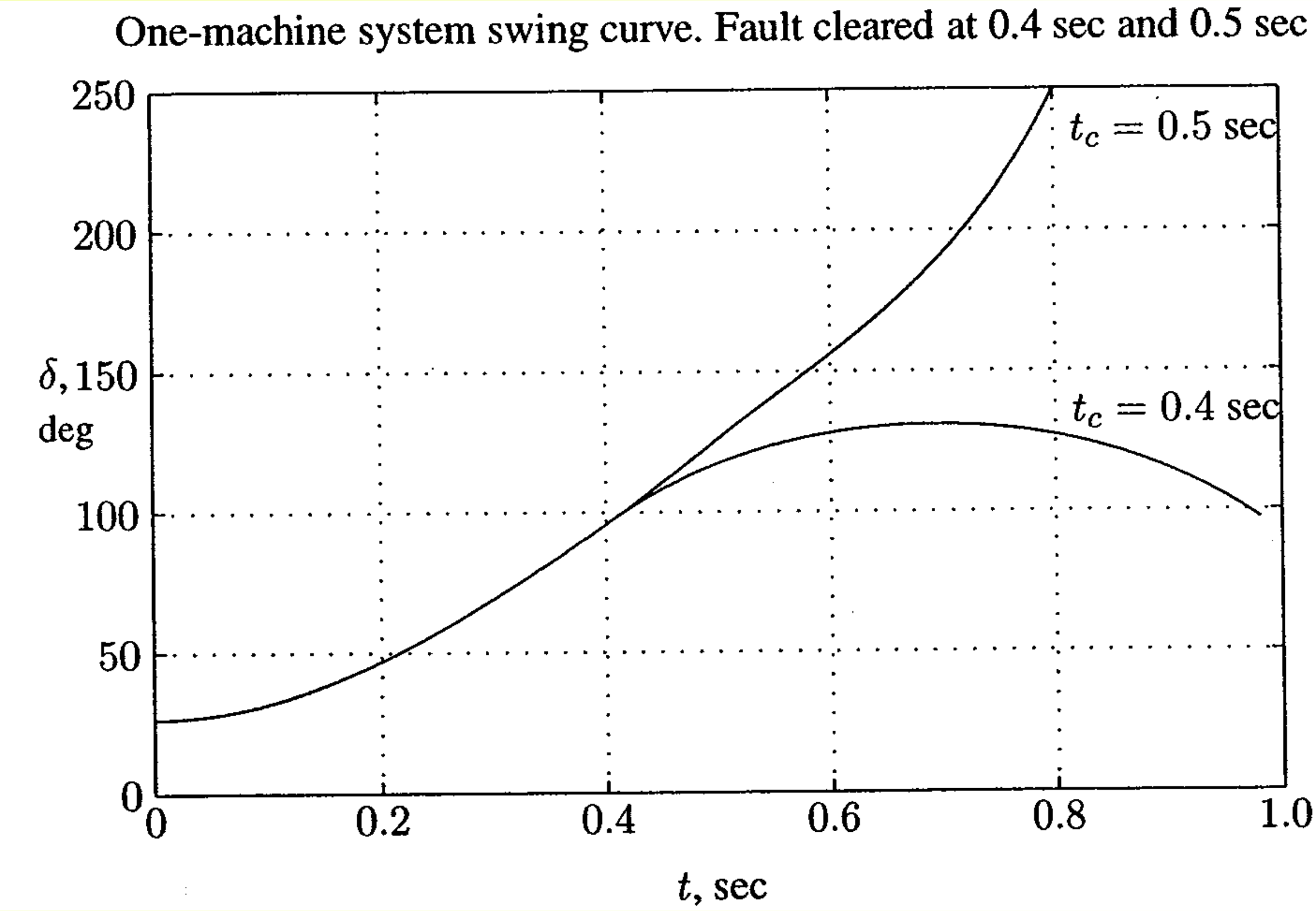
EXAMPLE

Power angle / time
fault clearing in 0.3 s

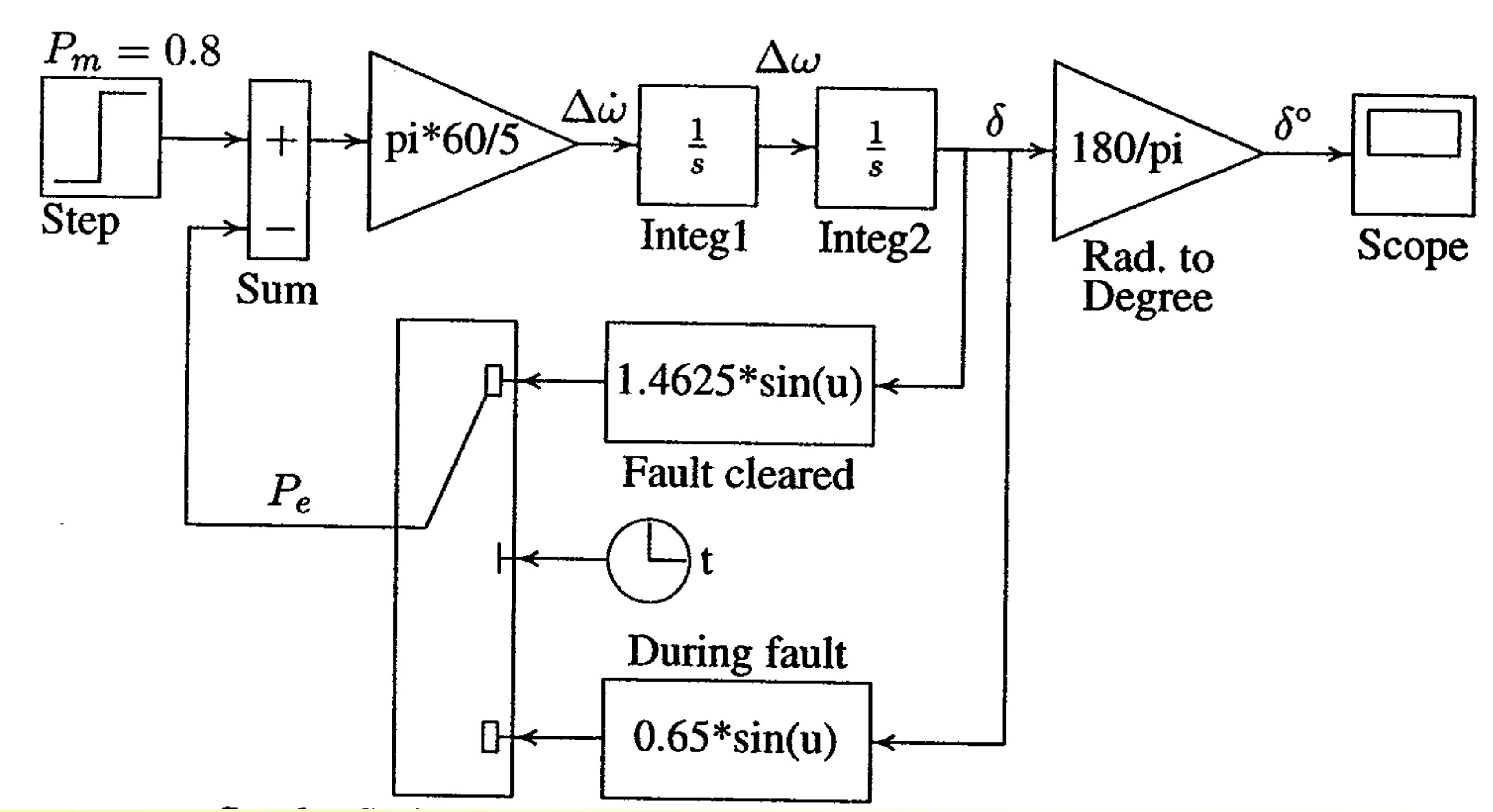



```
Pm = 0.80;  E = 1.17;  V = 1.0;
X1 = 0.65; X2 = 1.80; X3 = 0.8;
H = 5; f = 60; tc = 0.3; tf = 1.0; Dt = 0.01;
swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)
```

EXAMPLE


One-machine system swing curve. Fault cleared at 0.4 sec and 0.5 sec
$t_c = 0.5$ sec
$t_c = 0.4$ sec
δ, 150
deg
250
200
100
50
0
0
0.2
0.4
0.6
0.8
1.0
t, sec

Simulink Model



MULTI-MACHINE SYSTEMS

- Each synchronous machine is represented by a constant voltage source behind the direct axis transient reactance
- The input powers are assumed to remain constant
- Using the pre-fault voltage, all loads are converted to equivalent admittances to ground and are assumed to remain constant
- Damping or asynchronous powers are ignored
- The mechanical rotor angle of each machine coincides with the electrical angle of the excitation voltage source

MODELING STEPS

- Solve the initial load flow and obtain the initial bus voltage magnitude and phase angle
- Calculate the machine currents prior to the disturbance

$$I_{mach-i} = \frac{S^*_{mach-i}}{V^*_{mach-i}}$$

- Obtain the voltages behind the transient reactances

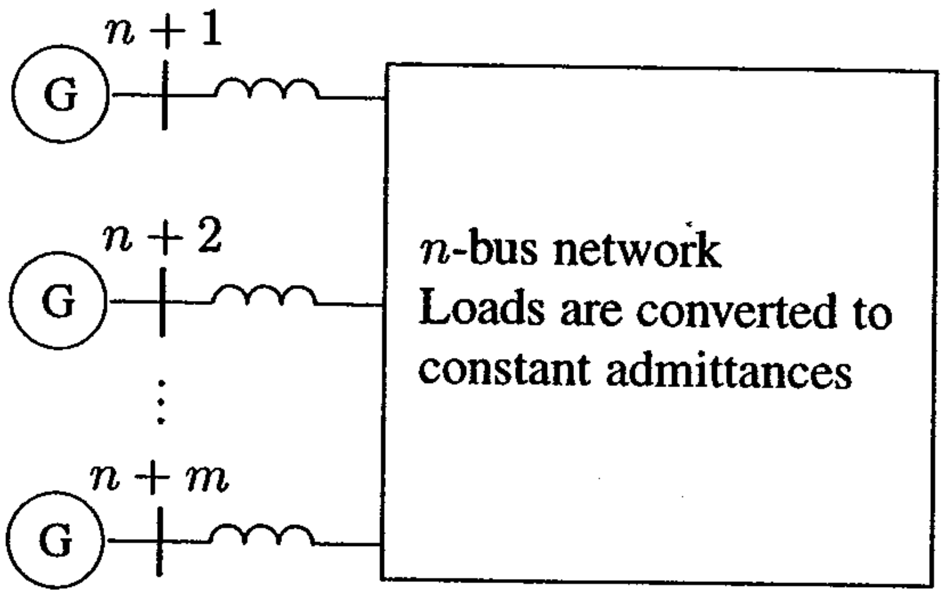
$$E'_{mach-i} = V_{mach-i} + j\,X'_d\, I_{mach-i}$$

- Convert all loads to equivalent admittances

$$y_{i0} = \frac{S^*_i}{|V_i|^2} = \frac{P_i - jQ_i}{|V_i|^2}$$

Power Systems I

Multi-Machine System



$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\,\mathbf{V}_{bus}$$

$$\left[\begin{array}{c} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_{n+1} \\ \vdots \\ I_{n+m} \end{array}\right] = \left[\begin{array}{ccc|ccc} Y_{11} & \cdots & Y_{1n} & Y_{1(n+1)} & \cdots & Y_{1(n+m)} \\ Y_{21} & \cdots & Y_{2n} & Y_{2(n+1)} & \cdots & Y_{2(n+m)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & \cdots & Y_{nn} & Y_{n(n+1)} & \cdots & Y_{n(n+m)} \\ \hline Y_{(n+1)1} & \cdots & Y_{(n+1)n} & Y_{(n+1)(n+1)} & \cdots & Y_{(n+1)(n+m)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Y_{(n+m)1} & \cdots & Y_{(n+m)n} & Y_{(n+m)(n+1)} & \cdots & Y_{(n+m)(n+m)} \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline E'_{n+1} \\ \vdots \\ E'_{n+m} \end{array}\right]$$

Modeling Steps

- Combine the generator models with the network's bus admittance matrix with converted loads

$$\begin{bmatrix} 0 \\ \mathbf{I}_{mach} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{nn} & \mathbf{Y}_{n-mach} \\ \mathbf{Y}_{n-mach}^{T} & \mathbf{Y}_{mach-mach} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{n} \\ \mathbf{E}'_{mach} \end{bmatrix}$$

The voltage vector $\mathbf{V}_n$ may be eliminated by substitution as follows

$$\mathbf{0} = \mathbf{Y}_{nn}\mathbf{V}_n + \mathbf{Y}_{nm}\mathbf{E'}_m$$
$$\mathbf{I}_m = \mathbf{Y}_{nm}^{t}\mathbf{V}_n + \mathbf{Y}_{mm}\mathbf{E'}_m$$

$$\mathbf{V}_n = -\mathbf{Y}_{nn}{}^{-1}\mathbf{Y}_{nm}\mathbf{E'}_m$$

$$\begin{aligned} \mathbf{I}_m &= [\mathbf{Y}_{mm} - \mathbf{Y}_{nm}^{t}\mathbf{Y}_{nn}{}^{-1}\mathbf{Y}_{nm}]\mathbf{E'}_m \\ &= \mathbf{Y}_{bus}^{red}\mathbf{E'}_m \end{aligned}$$

Modeling Steps

- **Express in terms of the machines' excitation voltages, the power output**

$$S^{*}_{mach-i} = E'^{*}_{mach-i} I_{mach-i}$$

$$P_{mach-i} = \Re\left[E'^{*}_{mach-i} I_{mach-i}\right]$$

$$I_{mach-i} = \sum_{j=1}^{m} E'_{mach-j} Y_{ij}$$

$$P_{mach-i} = \sum_{j=1}^{m} \left|E'_{mach-i}\right| \left|E'_{mach-j}\right| \left|Y_{ij}\right| \cos\left(\theta_{ij} - \delta_i + \delta_j\right)$$

Multi-Machine Transient Stability

- A solid three-phase fault at bus *k* (V*k* = 0)
- Remove kth row and column from prefault bus admittance matrix to simulate the fault
- Reduce the bus admittance matrix by eliminating all nodes except the internal generator nodes
- The generator excitation voltages during fault and post faults modes are assumed to remain constant

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{m} |E_i'||E_j'||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$H_i = \frac{S_{Gi}}{S_B} H_{Gi}$$

Multi-Machine Transient Stability

- **Solve the system of ODE's of the faulted network**

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{m} |E_i'||E_j'||Y_{2-ij}| \cos(\theta_{2-ij} - \delta_i + \delta_j)$$

- **Solve the system of ODE's of the post-fault network**

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{m} |E_i'||E_j'||Y_{3-ij}| \cos(\theta_{3-ij} - \delta_i + \delta_j)$$

$$\frac{d\delta_i}{dt} = \Delta\omega_i \qquad i = 1, \ldots, m$$

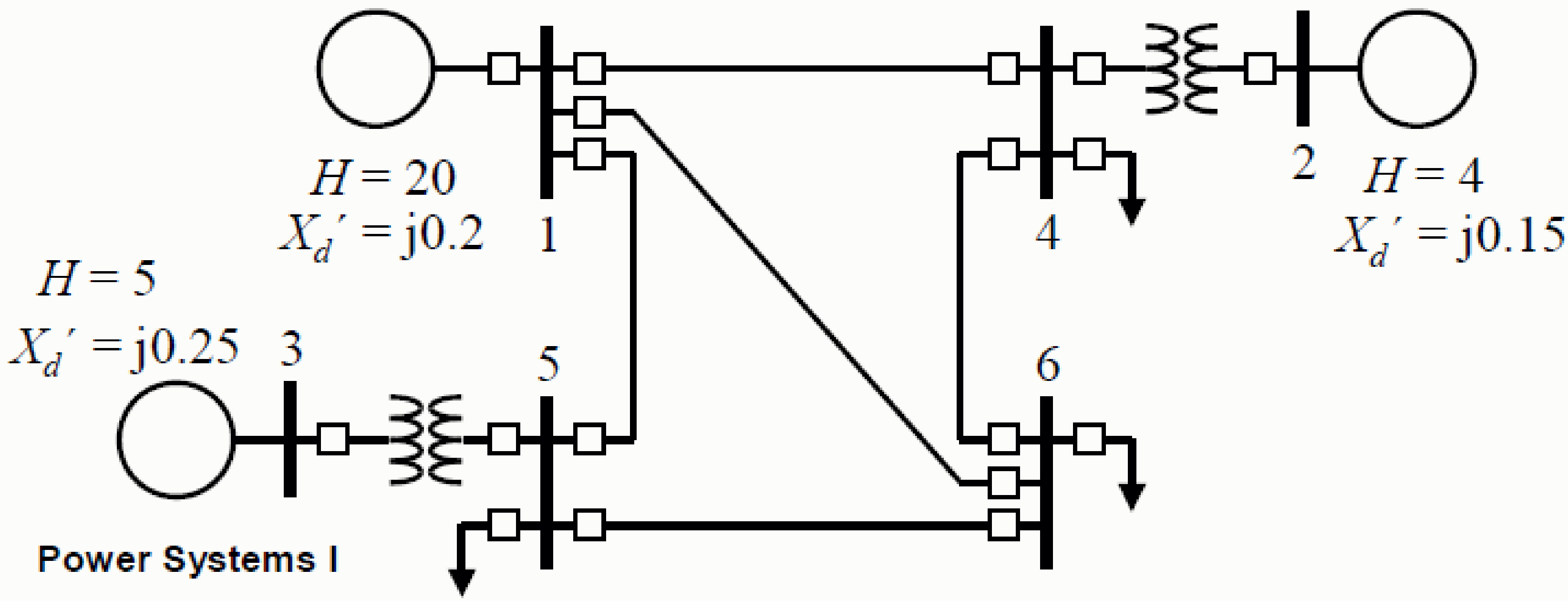
$$\frac{d\Delta\omega_i}{dt} = \frac{\pi f_0}{H_i}(P_m - P_e^f)$$

Multi-Machine Transient Stability: Steps

- We have two state equations for each generator
- Matlab ode23 solver is employed to solve 2m first-order differential equations
- When the fault is cleared, which may involve the removal of the faulty line, the bus admittance matrix is recomputed to reflect the change in the network
- The postfault reduced bus admittance matrix is evaluated and the postfault electrical power of the ith generator is determined
- Using the postfault power, the simulation is continued to determine stability
- Usually, the solution is carried out for two swings to show that the second swing is not greater than the first one

EXAMPLE

- **Consider the 3 machine system below**
  - select generator #1 as the swing machine with a constant angle of 0 degrees
  - determine the system stability when a fault on the line 5-6 near bus 6 is cleared in 0.4 and 0.5 seconds



Power Systems I

EXAMPLE

- **Load Data**

	P	Q
1	0	0
2	0	0
3	0	0
4	1.0	0.7
5	0.9	0.3
6	1.6	1.1

- **Generator Data**

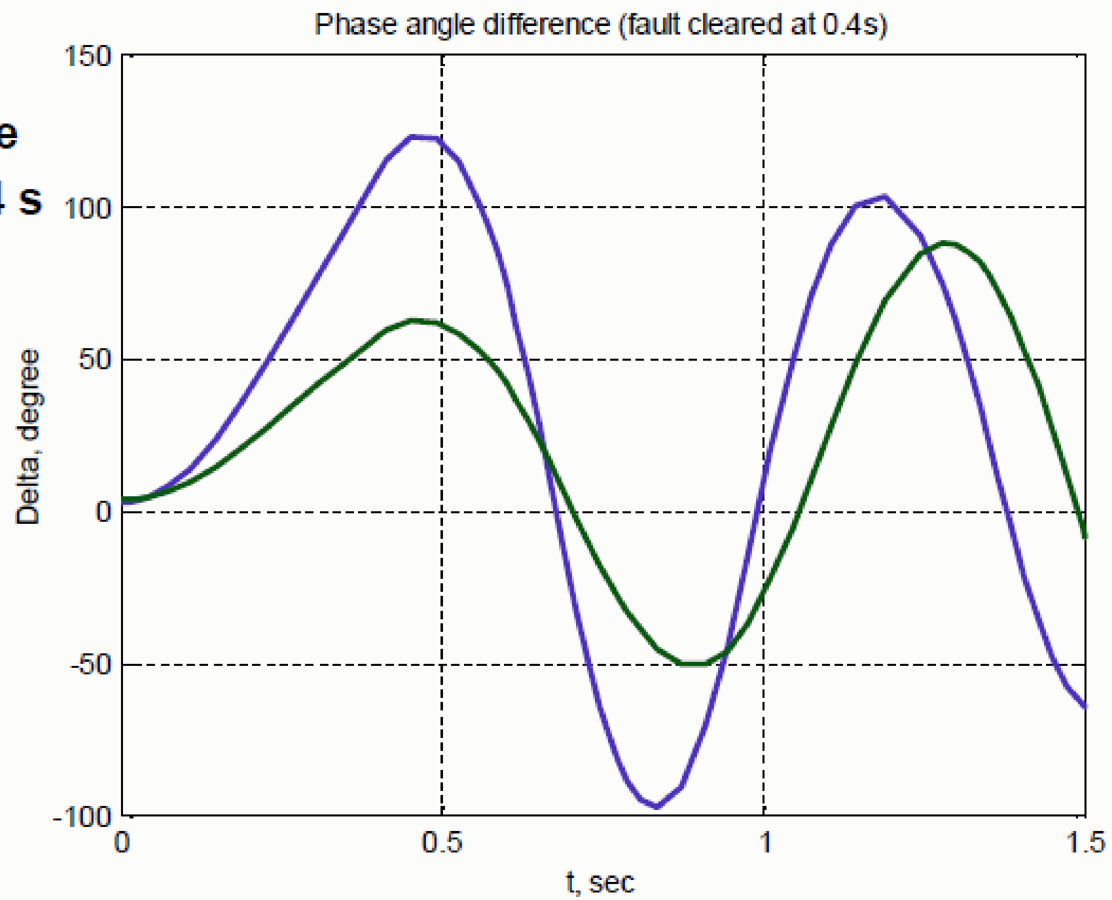
	V	P
1	1.06	---
2	1.04	1.5
3	1.03	1.0

- **Line Data**

		R	X	½B
1	4	0.035	0.225	0.0065
1	5	0.025	0.105	0.0045
1	6	0.040	0.215	0.0055
2	4	0.000	0.035	0.0000
3	5	0.000	0.042	0.0000
4	6	0.028	0.125	0.0035
5	6	0.026	0.175	0.0300

EXAMPLE

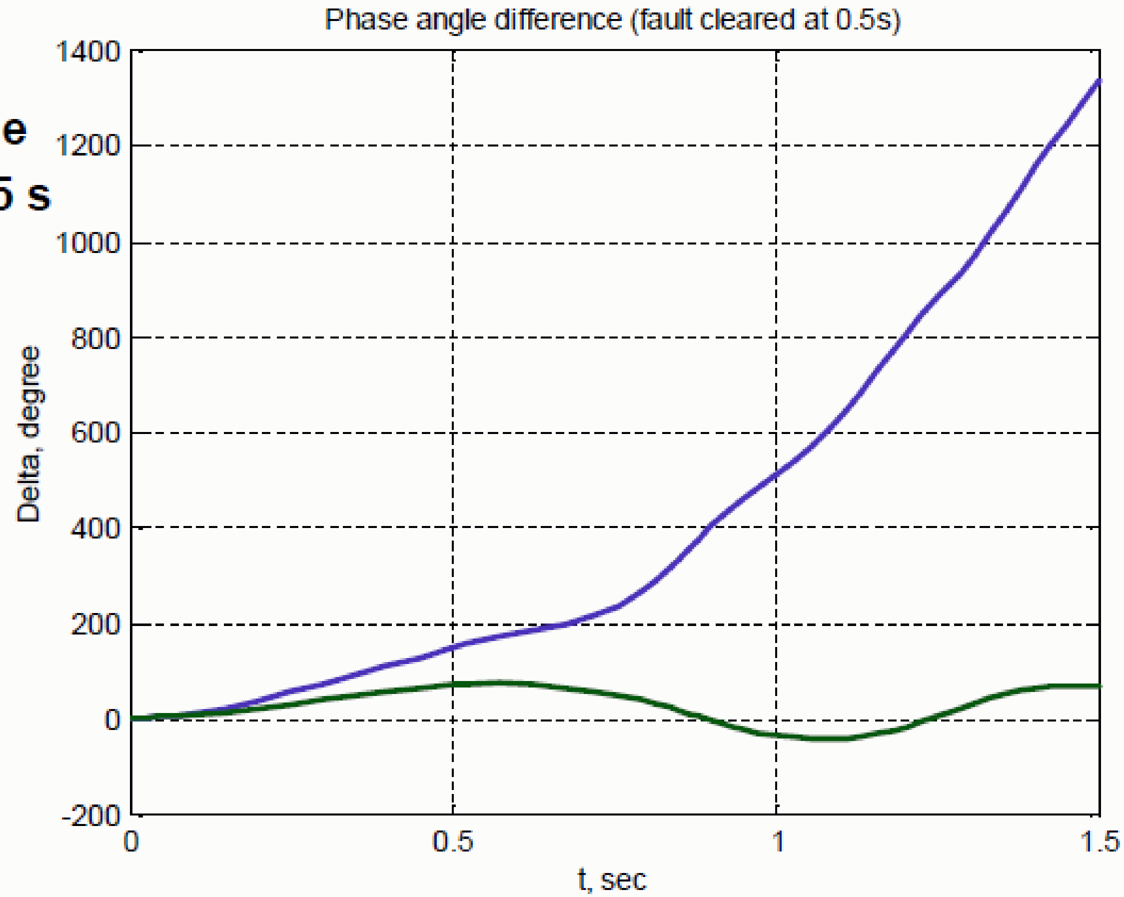
Power angle / time
fault clearing in 0.4 s



Power Systems I

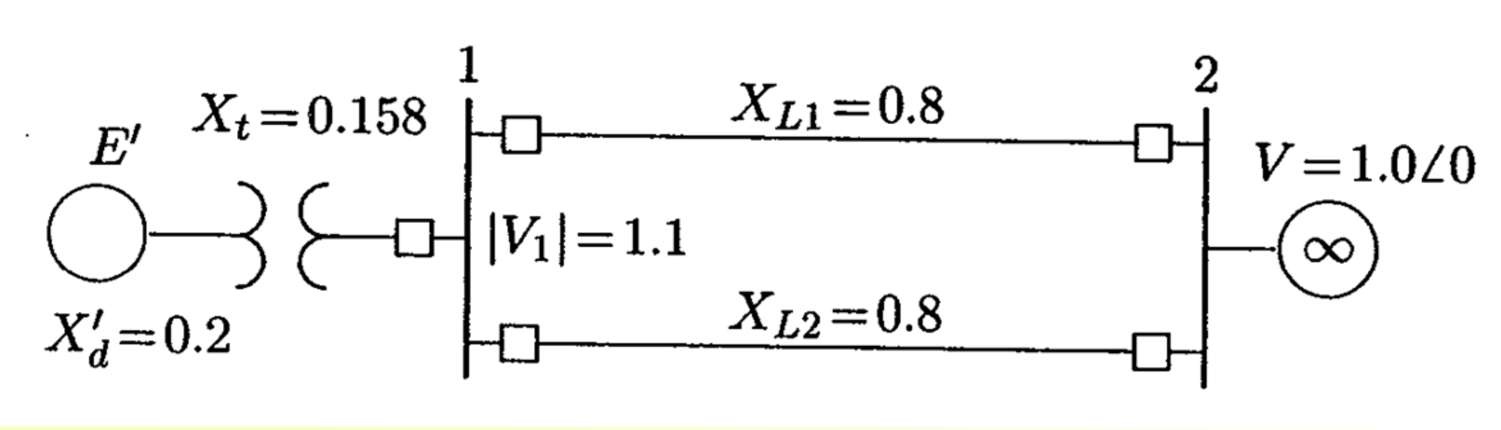
EXAMPLE

Power angle / time
fault clearing in 0.5 s



Power Systems I

Homework 1 (20 points)

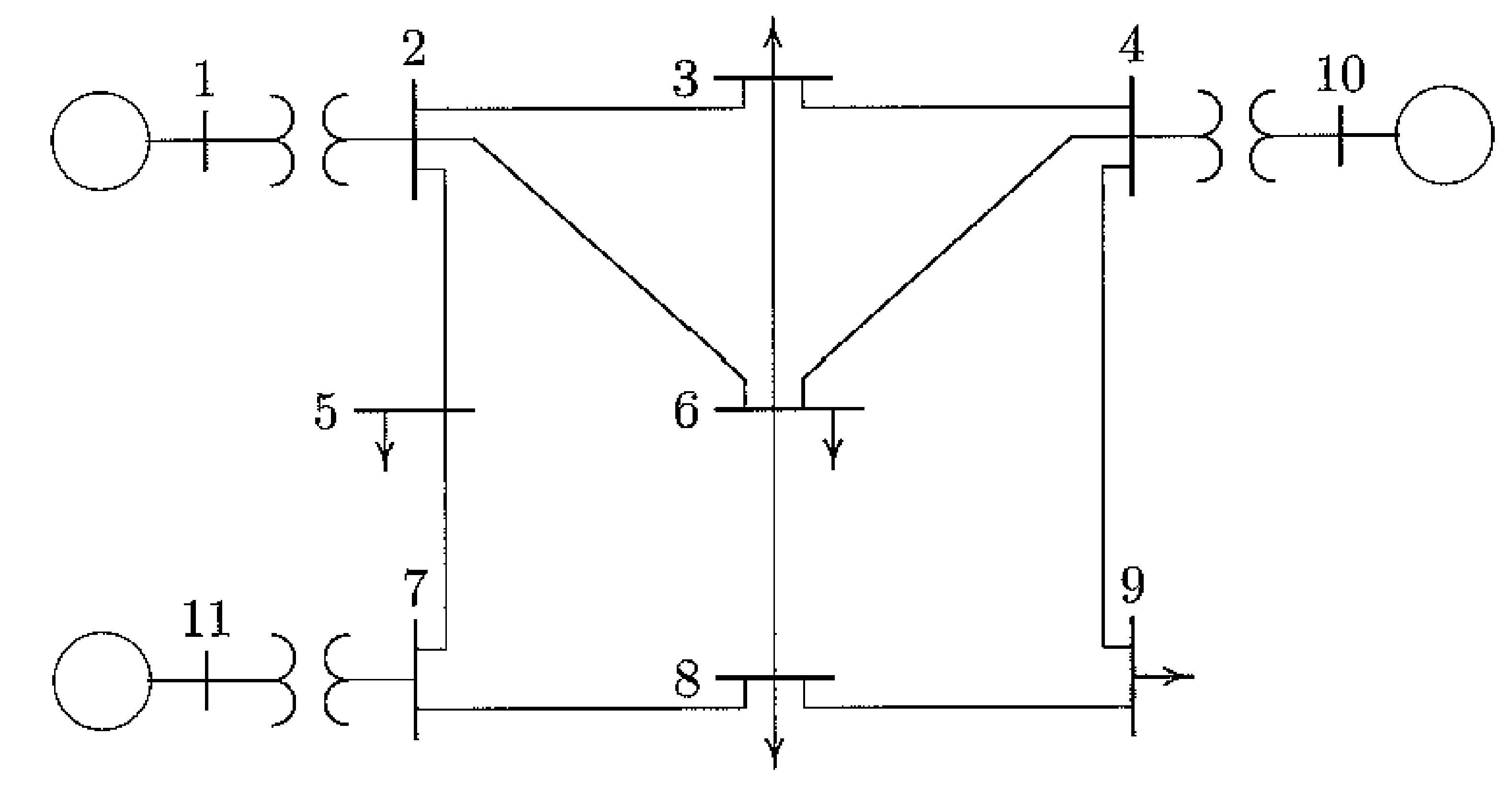


. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit. A three-phase fault at the middle of one line is cleared by isolating the faulted circuit simultaneously at both ends.

(a) The fault is cleared in 0.2 second. Obtain the numerical solution of the swing equation for 1.5 seconds. Select one of the functions **swingmeu**, **swingrk2**, or **swingrk4**.

(b) Repeat the simulation and obtain the swing plots when fault is cleared in 0.4 second, and for the critical clearing time.

Homework 2: One-line diagram



Homework 2: Load and generator data

LOAD DATA					
Bus No.	Load MW	Load Mvar	Bus No.	Load MW	Load Mvar
1	0.0	0.0	7	0.0	0.0
2	0.0	0.0	8	110.0	90.0
3	150.0	120.0	9	80.0	50.0
4	0.0	0.0	10	0.0	0.0
5	120.0	60.0	11	0.0	0.0
6	140.0	90.0			

GENERATION SCHEDULE				
Bus No.	Voltage Mag.	Generation, MW	Mvar Limits Min.	Mvar Limits Max.
1	1.040			
10	1.035	200.0	0.0	180.0
11	1.030	160.0	0.0	120.0

Homework 2: Line and generator data

LINE AND TRANSFORMER DATA				
Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2}B$, PU
1	2	0.000	0.006	0.000
2	3	0.008	0.030	0.004
2	5	0.004	0.015	0.002
2	6	0.012	0.045	0.005
3	4	0.010	0.040	0.005
3	6	0.004	0.040	0.005
4	6	0.015	0.060	0.008
4	9	0.018	0.070	0.009
4	10	0.000	0.008	0.000
5	7	0.005	0.043	0.003
6	8	0.006	0.048	0.000
7	8	0.006	0.035	0.004
7	11	0.000	0.010	0.000
8	9	0.005	0.048	0.000

MACHINE DATA			
Gen.	R_a	X'_d	H
1	0	0.20	12
10	0	0.15	10
11	0	0.25	9

Homework 2 (20 points)

A three-phase fault occurs on line 4–9, near bus 4, and is cleared by the simultaneous opening of breakers at both ends of the line. Using the **trstab** program, perform a transient stability analysis. Determine the stability for

(a) When the fault is cleared in 0.4 second

(b) When the fault is cleared in 0.8 second

(c) Repeat the simulation to determine the critical clearing time.