EEE472 POWER SYSTEM ANALYSIS II

BUS ADMITANCE MATRIX

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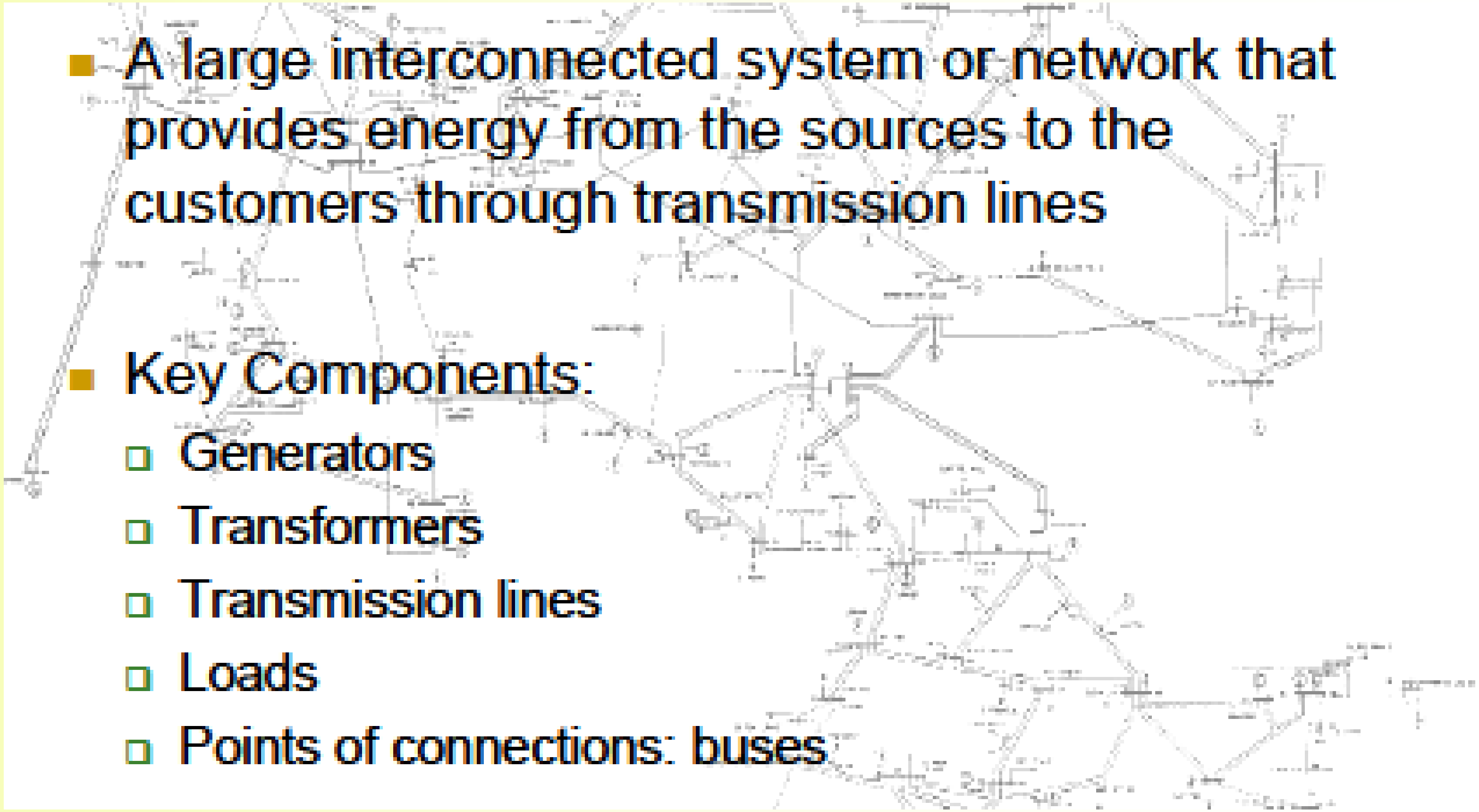
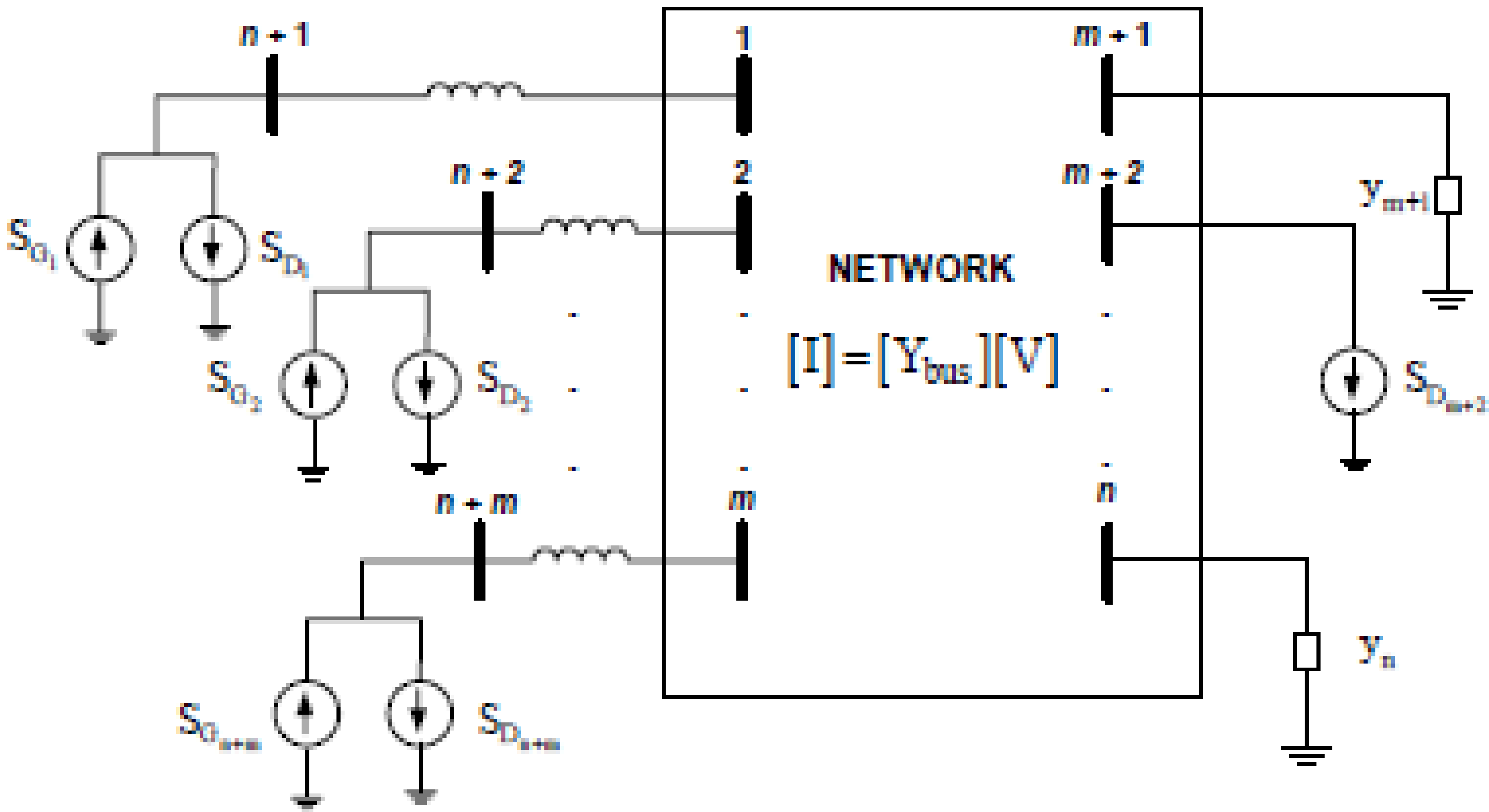
What is a Power System?

- A large interconnected system or network that provides energy from the sources to the customers through transmission lines

- Key Components:
  - Generators
  - Transformers
  - Transmission lines
  - Loads
  - Points of connections: buses



Electric Power Network

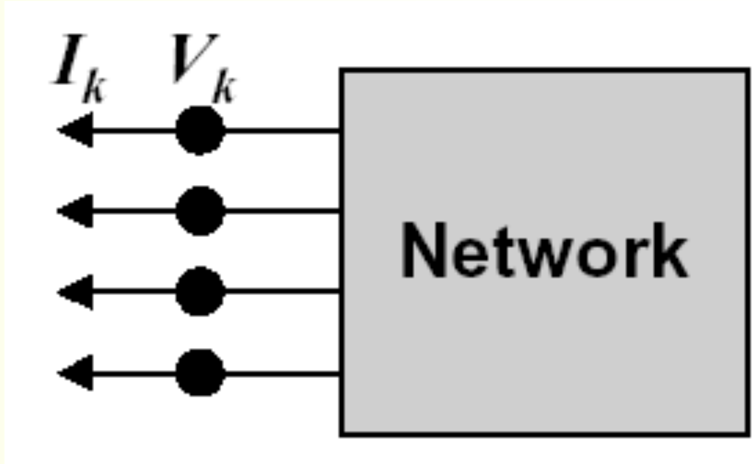


Bus Admittance Matrix

- The injected currents at the nodes of the interconnected network are related to the voltages at the nodes via an admittance representation

$$\mathbf{I} = \mathbf{Y}_{\mathbf{bus}}\mathbf{V}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$



where I = vector of injected node currents

$\mathrm{Y}_{\mathrm{bus}}$ = bus admittance matrix

V = vector of node voltages

Bus Admittance Matrix

- Used to form the network model of an interconnected power system
  - Nodes represent substation bus bars
  - Branches represent transmission lines and transformers
  - Injected currents are the flows from generator and loads
- Notes on bus admittance:
  - Large
  - Has a large number of zero entries

Bus Admittance Matrix or Y_{bus}

- First step in solving the power flow is to create what is known as the bus admittance matrix, often called the $\mathbf{Y}_{bus}$.
- The $\mathbf{Y}_{bus}$ gives the relationships between all the bus current injections, $\mathbf{I}$, and all the bus voltages, $\mathbf{V}$, $\mathbf{I} = \mathbf{Y}_{bus}\mathbf{V}$
- The $\mathbf{Y}_{bus}$ is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances.

Bus Admittance Matrix

- Let bus j be an arbitrary bus

$$\sum_{i=1}^{n} I_{ji} = I_j$$

where I_j = injected current at bus j

I_{ij} = current leaving bus j to bus i

- Branch current I_{ji}

$$I_{ji} = y_{ji}(V_j - V_i) \qquad y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + j\,x_{ij}}$$

where y_{ij} = line admittance from bus j to bus i

V_j, V_i = bus voltages

Bus Admittance Matrix

- Injected current I_j

$$I_j = \sum_{i=1}^{n} y_{ji}(V_j - V_i)$$

- n equations

$$I_1 = \left(\sum_{i=1}^{n} y_{1i}\right)V_1 + (-y_{12})V_2 + \cdots + (-y_{1n})V_n$$

$$I_2 = (-y_{21})V_1 + \left(\sum_{i=1}^{n} y_{2i}\right)V_2 + \cdots + (-y_{2n})V_n$$

$$\vdots$$

$$I_n = (-y_{n1})V_1 + (-y_{n2})V_2 + \cdots + \left(\sum_{i=1}^{n} y_{ni}\right)V_n$$

Ybus Building Algorithm

- "Inspection" method
- Diagonal entries Y_{jj} : summing the **primitive admittance** of lines and ties to the reference at bus j $(Y_{jj} = \sum_{i}^{n} y_{ji})$
- Off diagonal entries Y_{ij} : the negatives of the admittances of lines between buses i and j $(Y_{ji} = Y_{ij} = -y_{ij})$
- If there is no line between i and j, this term is zero

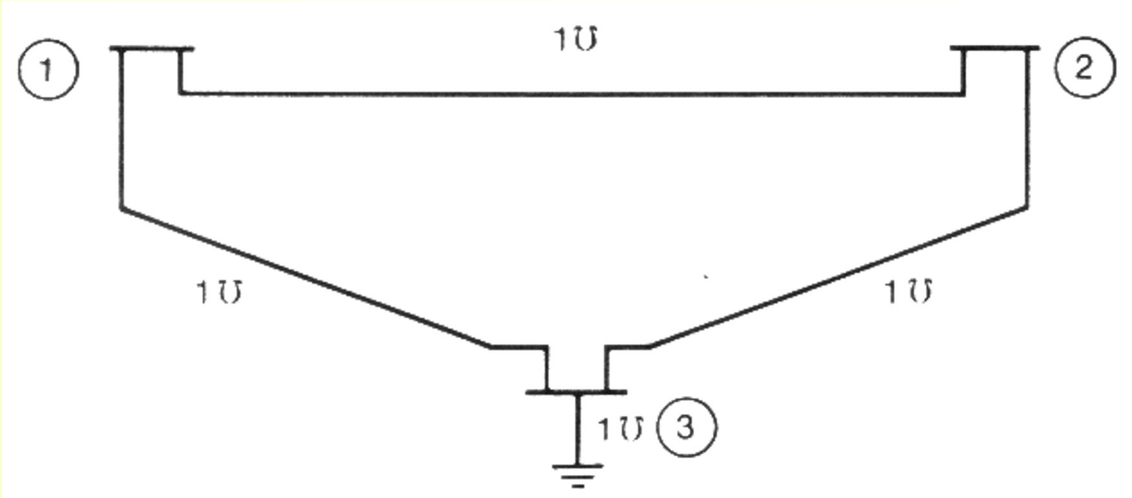
Properties of Ybus Matrix

- The matrix is complex and symmetric
- The matrix is sparse since each bus is connected to only a few nearby buses
- The percent sparsity increases with the matrix dimension
- If there is a nonzero admittance tie to the reference bus, Ybus is nonsingular
- If there are no ties to the reference bus, Ybus is singular

Y_{bus} -Summary

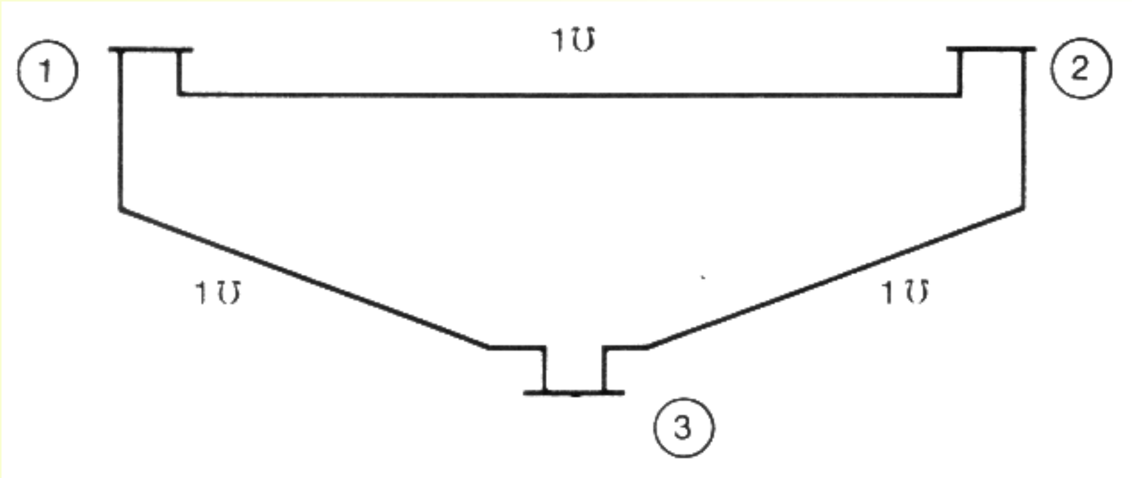
- The diagonal terms, Y_{jj}, are the "self admittance" terms, equal to the sum of the admittances of all devices incident to bus *k*.
- The off-diagonal terms, Y_{ij}, are equal to the negative of the admittance joining the two buses.
- With large systems Y_{bus} is a sparse matrix (that is, most entries are zero):
  - sparsity is key to efficient numerical calculation.
- Shunt terms, such as in the equivalent π line model, only affect the diagonal terms.

Example 1



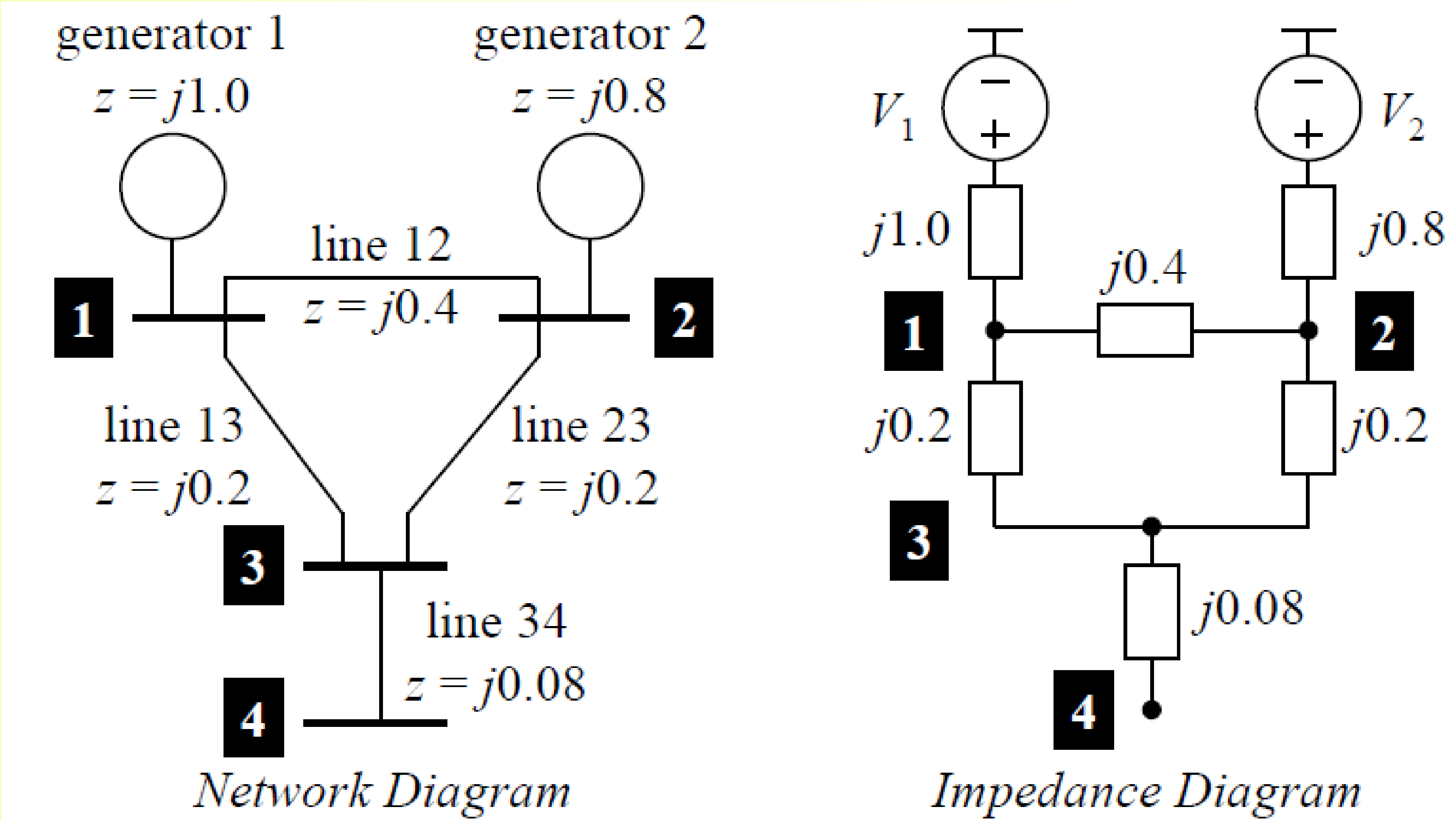
$$\begin{array}{c} \\ ① \\ ② \\ ③ \end{array}\begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{bmatrix} 1+1 & -1 & -1 \\ -1 & 1+1 & -1 \\ -1 & -1 & 1+1+1 \end{bmatrix} \end{array} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

Example 2



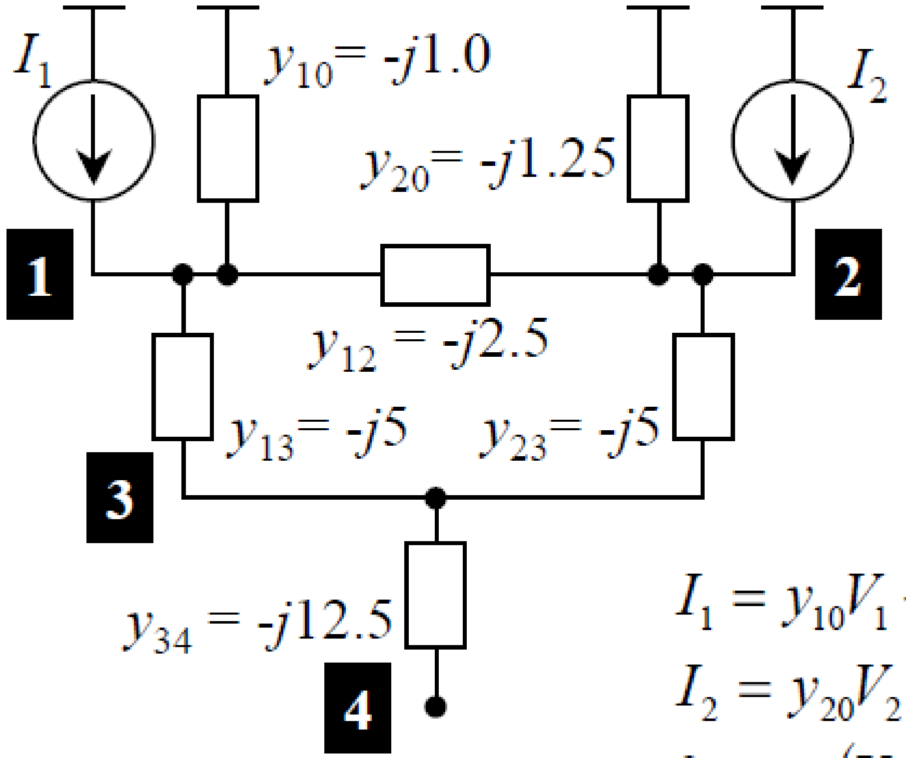
$$
\begin{array}{cc}
 & \begin{array}{ccc} ① & ② & ③ \end{array} \\
\begin{array}{c} ① \\ ② \\ ③ \end{array} &
\begin{bmatrix} 1+1 & -1 & -1 \\ -1 & 1+1 & -1 \\ -1 & -1 & 1+1 \end{bmatrix}
\end{array}
= \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}
$$

Example 3



generator 1
z = j1.0
generator 2
z = j0.8
line 12
z = j0.4
1
2
line 13
z = j0.2
line 23
z = j0.2
3
line 34
z = j0.08
4
Network Diagram
V1
V2
j1.0
j0.8
j0.4
1
2
j0.2
j0.2
3
j0.08
4
Impedance Diagram

Example 3



Admittance Diagram

KCL Equations

$$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$$
$$I_2 = y_{20}V_2 + y_{21}(V_2 - V_1) + y_{23}(V_2 - V_3)$$
$$0 = y_{31}(V_3 - V_1) + y_{32}(V_3 - V_2) + y_{34}(V_3 - V_4)$$
$$0 = y_{43}(V_4 - V_3)$$

Example 3

Rearranging the KCL Equations

$$I_1 = (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3$$

$$I_2 = -y_{21}V_1 + (y_{20} + y_{21} + y_{23})V_2 - y_{23}V_3$$

$$0 = -y_{31}V_1 - y_{32}V_2 + (y_{31} + y_{32} + y_{34})V_3 - y_{34}V_4$$

$$0 = -y_{43}V_3 + y_{43}V_4$$

Matrix Formation of the Equations

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (y_{10} + y_{12} + y_{13}) & -y_{12} & -y_{13} & 0 \\ -y_{21} & (y_{20} + y_{21} + y_{23}) & -y_{23} & 0 \\ -y_{31} & -y_{32} & (y_{31} + y_{32} + y_{34}) & -y_{34} \\ 0 & 0 & -y_{43} & y_{43} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Example 3

Completed Matrix Equation

$$Y_{11} = (y_{10} + y_{12} + y_{13}) = -j8.50$$

$$Y_{12} = Y_{21} = -y_{12} = j2.50$$

$$Y_{13} = Y_{31} = -y_{13} = j5.00$$

$$Y_{22} = (y_{20} + y_{21} + y_{23}) = -j8.75$$

$$Y_{23} = Y_{32} = -y_{23} = j5.00$$

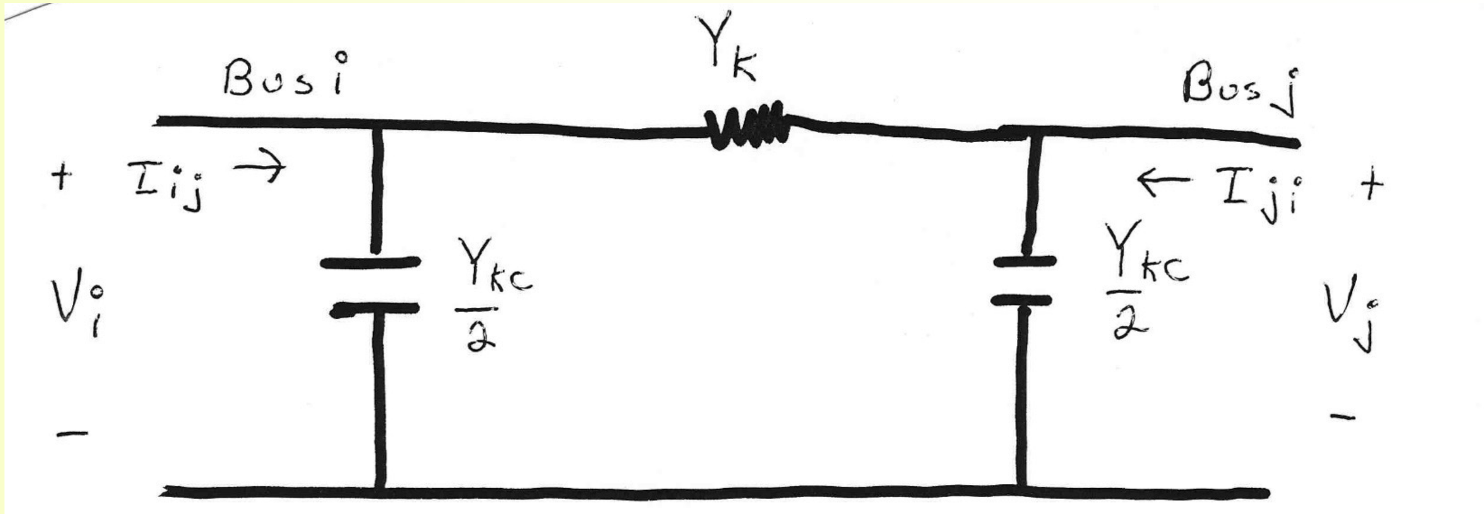
$$Y_{33} = (y_{31} + y_{32} + y_{34}) = -j22.50$$

$$Y_{34} = Y_{43} = -y_{34} = j12.50$$

$$Y_{44} = y_{34} = -j12.50$$

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j8.50 & j2.50 & j5.00 & 0 \\ j2.50 & -j8.75 & j5.00 & 0 \\ j5.00 & j5.00 & -j22.50 & j12.50 \\ 0 & 0 & j12.50 & -j12.50 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Modeling Shunts in Y_{bus}

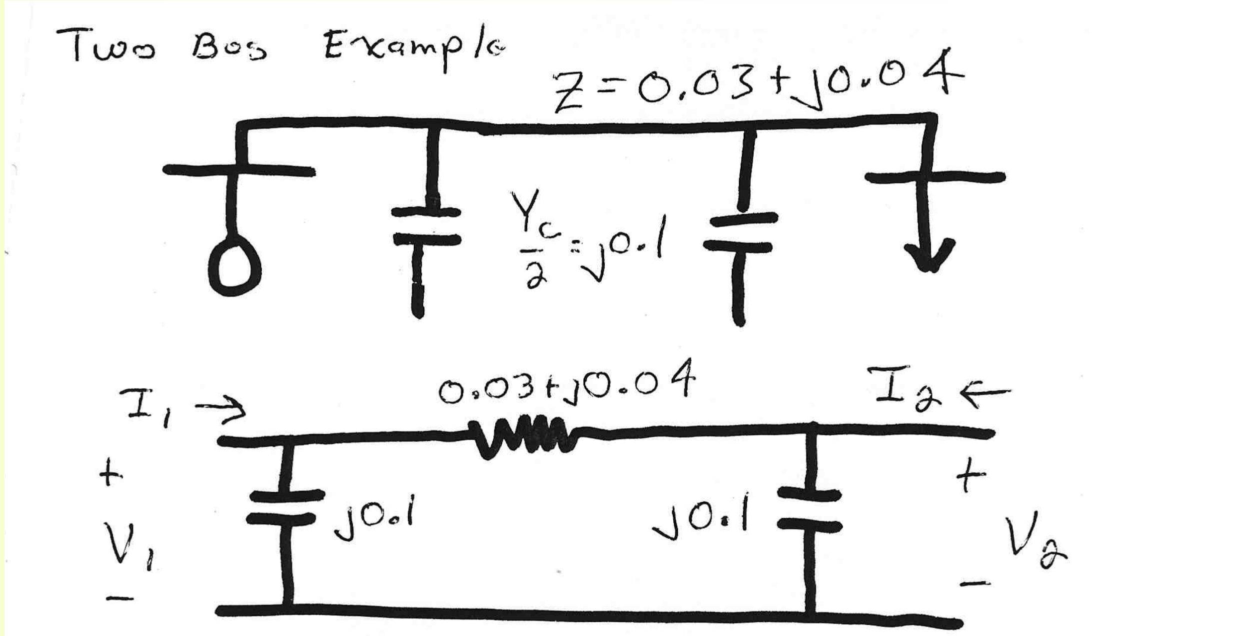


$$\text{Since} \quad I_{ij} = (V_i - V_j)Y_k + V_i \frac{Y_{kc}}{2}$$

$$Y_{ii} = Y_{ii}^{\text{from other lines}} + Y_k + \frac{Y_{kc}}{2}$$

$$\text{Note} \quad Y_k = \frac{1}{Z_k} = \frac{1}{R_k + jX_k}\frac{R_k - jX_k}{R_k - jX_k} = \frac{R_k - jX_k}{R_k^2 + X_k^2}$$

Example 4



$$I_1 = \frac{(V_1 - V_2)}{Z} + V_1 \frac{Y_c}{2}, \text{ where } \frac{1}{Z} = \frac{1}{0.03 + j0.04} = 12 - j16.$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Using the $\mathbf{Y}_{\text{bus}}$

If the voltages are known then we can solve for the current injections:

$$\mathbf{Y}_{\text{bus}}\mathbf{V} = \mathbf{I}$$

If the current injections are known then we can solve for the voltages:

$$\mathbf{Y}_{\text{bus}}^{-1}\mathbf{I} = \mathbf{V} = \mathbf{Z}_{\text{bus}}\mathbf{I}$$

where $\mathbf{Z}_{\text{bus}} = \mathbf{Y}_{\text{bus}}^{-1}$ is the bus impedance matrix.

Example 5:Solving for Bus Currents

For example, in previous case assume:

$$\mathbf{V} = \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix}.$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix} = \begin{bmatrix} 5.60 - j0.70 \\ -5.58 + j0.88 \end{bmatrix}$$

Therefore the power injected at bus 1 is:

$$S_1 = V_1 I_1^* = 1.0 \times (5.60 + j0.70) = 5.60 + j0.70$$

$$S_2 = V_2 I_2^* = (0.8 - j0.2) \times (-5.58 - j0.88) = -4.64 + j0.41$$

Example 6: Solving for Bus Voltages

As another example, in previous case assume

$$\mathbf{I} = \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix}.$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix}^{-1} \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix} = \begin{bmatrix} 0.0738 - j0.902 \\ -0.0738 - j1.098 \end{bmatrix}$$

Therefore the power injected is

$$S_1 = V_1 I_1^* = (0.0738 - j0.902) \times 5 = 0.37 - j4.51$$

$$S_2 = V_2 I_2^* = (-0.0738 - j1.098) \times (-4.8) = 0.35 + j5.27$$

Modifications in Bus Admittance Matrix

Line Outages

- Line outages
  - Let Y_{bus} be given for a system in which a line is to be outaged
  - The line outage is equivalent to adding a new line of admittance $-y_{out}$ in parallel with the line to be outaged
  - The combination of y_{out} and $-y_{out}$ is a zero admittance (open circuit)

- Procedure
  - Diagonal entries: Add $-y_{out}$ to the ii and jj entries
  - Off diagonal entries: Add y_{out} to the ij and ji entries

Deletion of a Bus-Kron Reduction

- A system of n buses in which m buses to be deleted (m<n)
- Partition I_{bus} and V_{bus}

$$I_{bus} = \begin{array}{l} \left.\begin{bmatrix} I_a \\ \cdots \end{bmatrix}\right\} n-m \\ \left.\begin{bmatrix} I_b \end{bmatrix}\right\} \quad m \end{array} \qquad V_{bus} = \begin{array}{l} \left.\begin{bmatrix} V_a \\ \cdots \end{bmatrix}\right\} n-m \\ \left.\begin{bmatrix} V_b \end{bmatrix}\right\} \quad m \end{array}$$

- Deleted buses = no injected currents
  - Let $I_b = 0$

Deletion of a Bus-Kron Reduction

- Need to find an equivalent admittance matrix with *m* buses deleted
  - Note: the equivalent admittance matrix is (n-m)x(n-m)

$$I_{bus} = Y_{bus} V_{bus}$$

$$\begin{bmatrix} I_a \\ \hline I_b \end{bmatrix} = \left[\begin{array}{c|c} Y_{aa} & Y_{ab} \\ \hline Y_{ab}^T & Y_{bb} \end{array}\right] \begin{bmatrix} V_a \\ \hline V_b \end{bmatrix} \begin{array}{l} \}\, n-m \\ \}\, m \end{array}$$

Deletion of a Bus-Kron Reduction

- Set $I_b = 0$ and eliminate V_b

$$I_b = 0 = Y_{ab}^T V_a + Y_{bb} V_b$$

$$V_b = -Y_{bb}^{-1} Y_{ab}^t V_a \quad \Rightarrow \quad \textbf{Ybb is assumably invertible}$$

- Substitute Vb in n-m rows

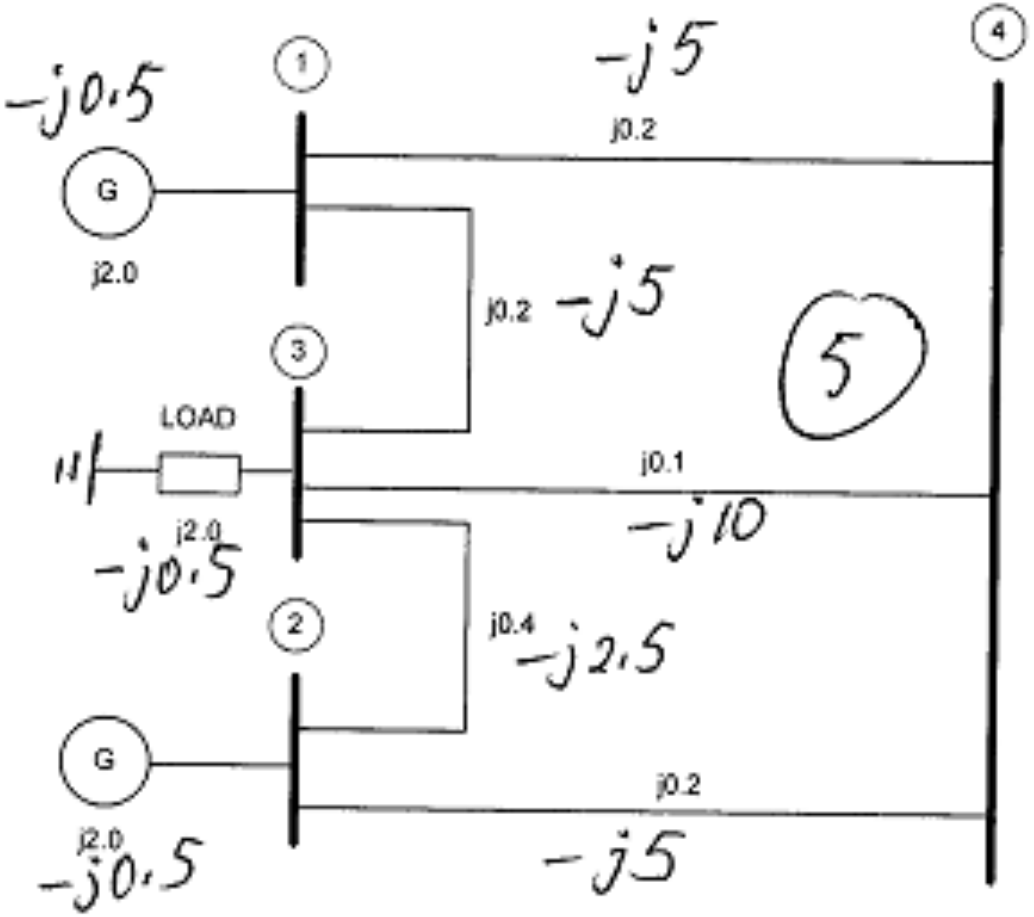
$$I_a = Y_{aa} V_a + Y_{ab} V_b$$

$$= Y_{aa} V_a - Y_{ab} Y_{bb}^{-1} Y_{ab}^t V_a$$

$$= \underbrace{\left[Y_{aa} - Y_{ab} Y_{bb}^{-1} Y_{ab}^t \right]}_{Y_{bus}^{eq}} V_a$$

Example 1

For the two-bus system given below.



a) Find Y_{bus} matrix **(15).**

b) Eliminate buses 3 and 4 using Kron Reduction metod **(15).**

c) Find the reactances of generators and the line of the reduced 2-bus system **(5).**

Example 1: Solution

c) Find the reactances of generators and the

a)

$$
Y_{Bus} = \begin{array}{c} ① \\ ② \\ ③ \\ ④ \end{array}
\left[\begin{array}{cc|cc}
-j10.5 & 0 & j5 & j5 \\
0 & -j8 & j2.5 & j5 \\
\hline
j5 & j2.5 & -j18 & j10 \\
j5 & j5 & j10 & -j20
\end{array}\right]
$$

10

b)

$$Y_{aa} = \begin{bmatrix} -j10.5 & 0 \\ 0 & -j8 \end{bmatrix}$$

$$Y_{ab} = \begin{bmatrix} j5 & j5 \\ j2.5 & j5 \end{bmatrix}$$

$$Y_{ab}^{T} = \begin{bmatrix} j5 & j2.5 \\ j5 & j5 \end{bmatrix}$$

5

$$Y_{bb} = \begin{bmatrix} -j18 & j10 \\ j10 & -j20 \end{bmatrix}$$

$$Y_{red} = Y_{aa} - Y_{ab} Y_{bb}^{-1} Y_{ab}^{T}$$

$$Y_{bb}^{-1} = \frac{1}{(-260)} \begin{bmatrix} -j20 & -j10 \\ -j10 & -j18 \end{bmatrix}$$

5

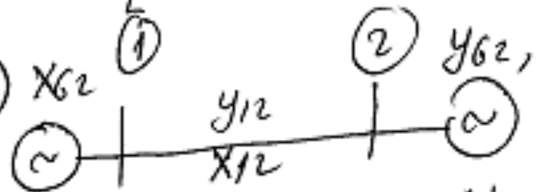
$$Y_{bb}^{-1} = \begin{bmatrix} j0.0769 & j0.0385 \\ j0.0385 & j0.0692 \end{bmatrix}$$

$$Y_{ab} Y_{bb}^{-1} Y_{ab}^{T} = \begin{bmatrix} -j5.575 & -j4.135 \\ -j4.135 & -j3.173 \end{bmatrix}$$

$$Y_{red} = \begin{bmatrix} -j4.925 & j4.135 \\ j4.135 & -j4.827 \end{bmatrix}$$

5

c) X_{G2} $\quad$ y_{G2}, X_{G2}



$Y_{12} = j4.135 \longrightarrow y_{12} = -j4.135$

5

$X_{12} = j0.242$ pu

$y_{G1} = -j4.925 + j4.135 = -j0.79$ pu

$y_{G2} = -j4.827 + j4.135 = -j0.692$ pu

$= j1.266$ pu

$X_{G1} = \dfrac{1}{-j0.79} = j1.266$ pu $\qquad X_{G2} = \dfrac{1}{-j0.692} = j1.445$ pu

Example 2

Name-Last name: SAYASUN

Department:

Student ID #:

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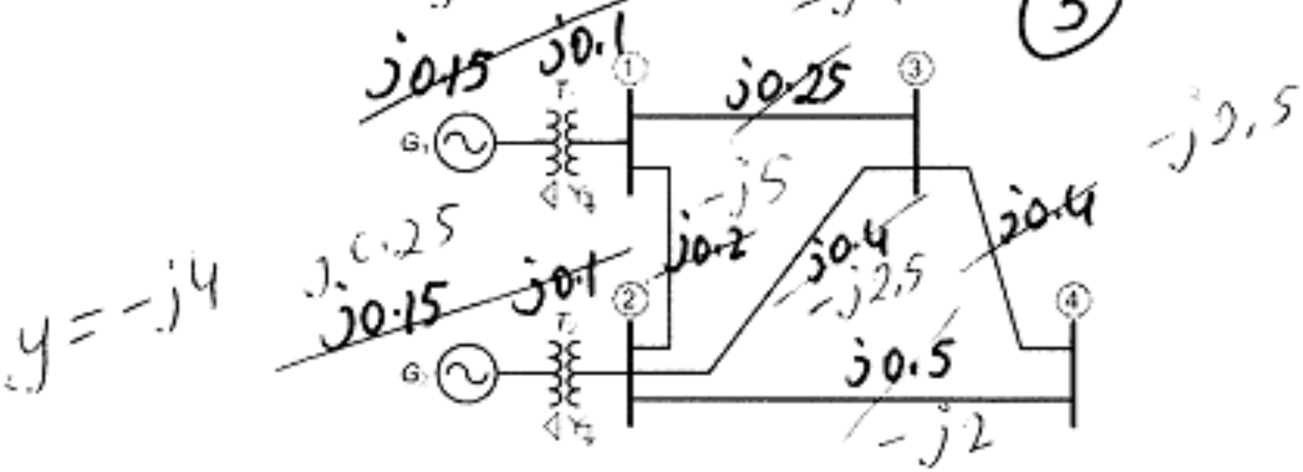


G1 and G2: $x = j0.15$ pu, T_1 and T_2: $x = j0.1$ pu, Lines: $x_{12} = j0.2$ pu, $x_{13} = j0.25$ pu, $x_{23} = x_{34} = j0.4$ pu and $x_{24} = j0.5$ pu

$$I_a = Y_{aa} V_a + Y_{ab} V_b$$
$$0 = Y_{ab}^T V_a + Y_{bb} V_b$$

2. For the power system given above

a) Find $\mathbf{Y_{bus}}$ matrix **(15 pts).**

b) Eliminate the bus# 3 and 4 using **Kron Reduction method** **(15 pts).**

c) Find the reactances of generators and the line of the 2-bus reduced system **(5 pts)**

Example 2: Solution

a)

$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & Y_{bus} = \begin{bmatrix} -j13 & j5 & j4 & 0 \\ j5 & -j13.5 & j2.5 & j2 \\ j4 & j2.5 & -j9 & j2.5 \\ 0 & j2 & j2.5 & -j4.5 \end{bmatrix} \end{array}$$

Y_{ab}, Y_{ab}^T

10

b) $Y_{bus}^{red} = Y_{aa} - Y_{ab} Y_{bb}^{-1} Y_{ab}^{T}$

$$Y_{bb} = \begin{bmatrix} -j9 & j2.5 \\ j2.5 & -j4.5 \end{bmatrix}$$

$$Y_{bb}^{-1} = \frac{1}{(-34.25)} \begin{bmatrix} -j4.5 & -j2.5 \\ -j2.5 & -j9 \end{bmatrix}$$

$$Y_{bb}^{-1} = \begin{bmatrix} j0.13 & j0.073 \\ j0.073 & j0.26 \end{bmatrix}$$

5

$$Y_{ab} Y_{bb}^{-1} Y_{ab}^{T} = \begin{bmatrix} j4 & 0 \\ j2.5 & j2 \end{bmatrix} \begin{bmatrix} j0.13 & j0.073 \\ j0.073 & j0.26 \end{bmatrix} \begin{bmatrix} j4 & j2.5 \\ 0 & j2 \end{bmatrix}$$

$$Y_{ab} Y_{bb}^{-1} Y_{ab}^{T} = \begin{bmatrix} -j2.08 & -j1.884 \\ -j1.884 & -j2.583 \end{bmatrix}$$

5

$$Y_{bus}^{red} = Y_{aa} - Y_{ab} Y_{bb}^{-1} Y_{ab}^{T}$$

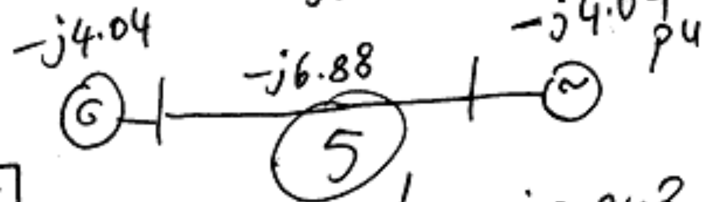
$$Y_{bus}^{red} = \begin{bmatrix} -j13 & j5 \\ j5 & -j13.5 \end{bmatrix} - \begin{bmatrix} -j2.08 & -j1.884 \\ -j1.884 & -j2.583 \end{bmatrix}$$

$$Y_{bus}^{red} = \begin{bmatrix} -j10.92 & j6.88 \\ j6.88 & -j10.92 \end{bmatrix}$$

5

c) $y_{12} = y_{21} = -j6.88$ pu

Thus, $X_{12} = \dfrac{1}{-j6.88} = j0.145$ pu



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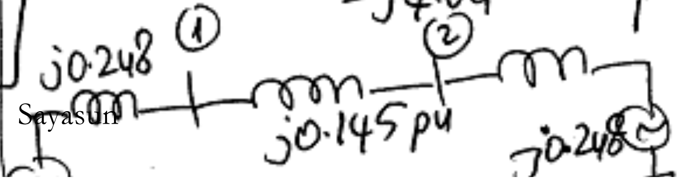
$X_{G1} = X_{G2} = \dfrac{1}{-j4.04} = j0.248$ pu

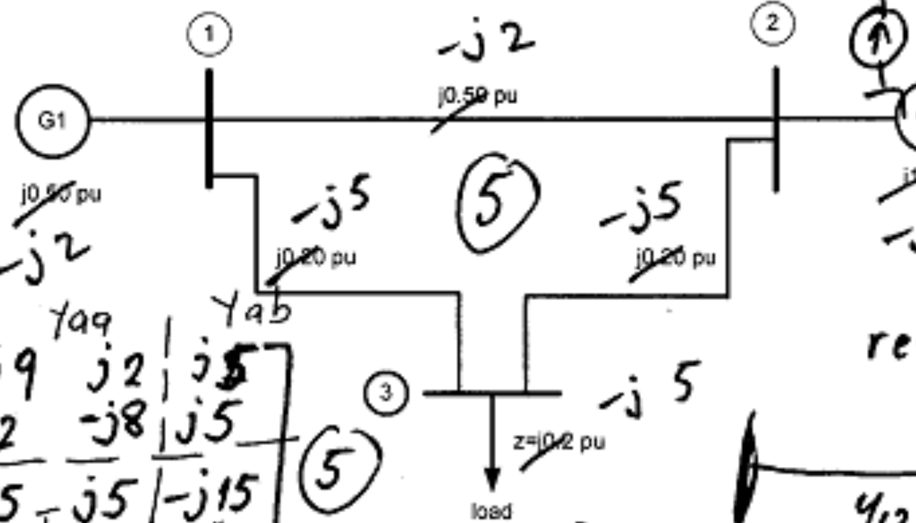


Example 3

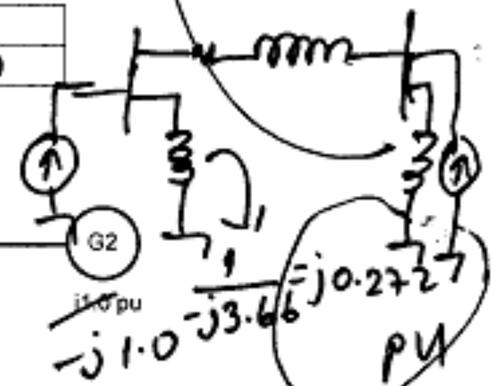
Student ID #:

POWER SYSTEM ANALYSIS
2017-2018 FALL TERM MIDTERM EXAM, Nov. 05, 2017
100 MINUTES, PLEASE SHOW ALL YOUR EFFORTS.
GOOD LUCK!!!
PROF. DR. SAFFET AYASUN

Q #	1	2	3	Total
Points	/35	/30	/35	/100



$$\frac{1}{-j3.67} = j0.272 \text{ pu}$$

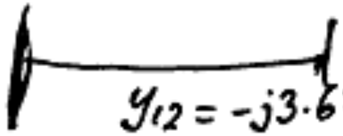


$$\frac{1}{-j3.66} = j0.2727 \text{ pu}$$

reduced system

$$a)\ Y_{Bus} = \begin{array}{ccc} Y_{aa} & & Y_{ab} \\ \left[\begin{array}{cc|c} -j9 & j2 & j5 \\ j2 & -j8 & j5 \\ \hline j5 & j5 & -j15 \end{array}\right] \\ Y_{ab} = Y_{ba} & & Y_{bb} \end{array} \quad 5$$

c)



$$y_{12} = -j3.67$$

$$y_{10} = -j7.33 + j3.67 = -j3.66$$

$$y_{12} = -j3.67$$

$$y_{20} = -j6.33 + j3.67 = -j2.67$$

1. For the power system given above
a) Find Y_{bus} matrix **(10 pts).**
b) Eliminate the bus#3 using **Kron Reduction method** (10 pts).
c) Find bus voltages V_1 and V_2 if current injections are $I_1 = 1.0$ pu and $I_2 = 1.2\angle -10^0$ pu **(10 pts)**
d) Find the reactances of generators and the line of the 2-bus reduced system **(5 pts)**

Example 3: Solution

b)

$$Y_{red} = Y_{aa} - Y_{ab} Y_{bb}^{-1} Y_{ab}^{T}$$

$$= \begin{bmatrix} -j9 & j2 \\ j2 & -j8 \end{bmatrix} - \begin{bmatrix} j5 \\ j5 \end{bmatrix} \left(\frac{1}{-j15}\right) \begin{bmatrix} j5 & j5 \end{bmatrix} \quad 5$$

$$= \begin{bmatrix} -j9 & j2 \\ j2 & -j8 \end{bmatrix} + \begin{bmatrix} j1.67 & j1.67 \\ j1.67 & j1.67 \end{bmatrix}$$

$$= \begin{bmatrix} -j7.33 & j3.67 \\ j3.67 & -j6.33 \end{bmatrix} \quad 5$$

c) $I = Y V$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Y^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

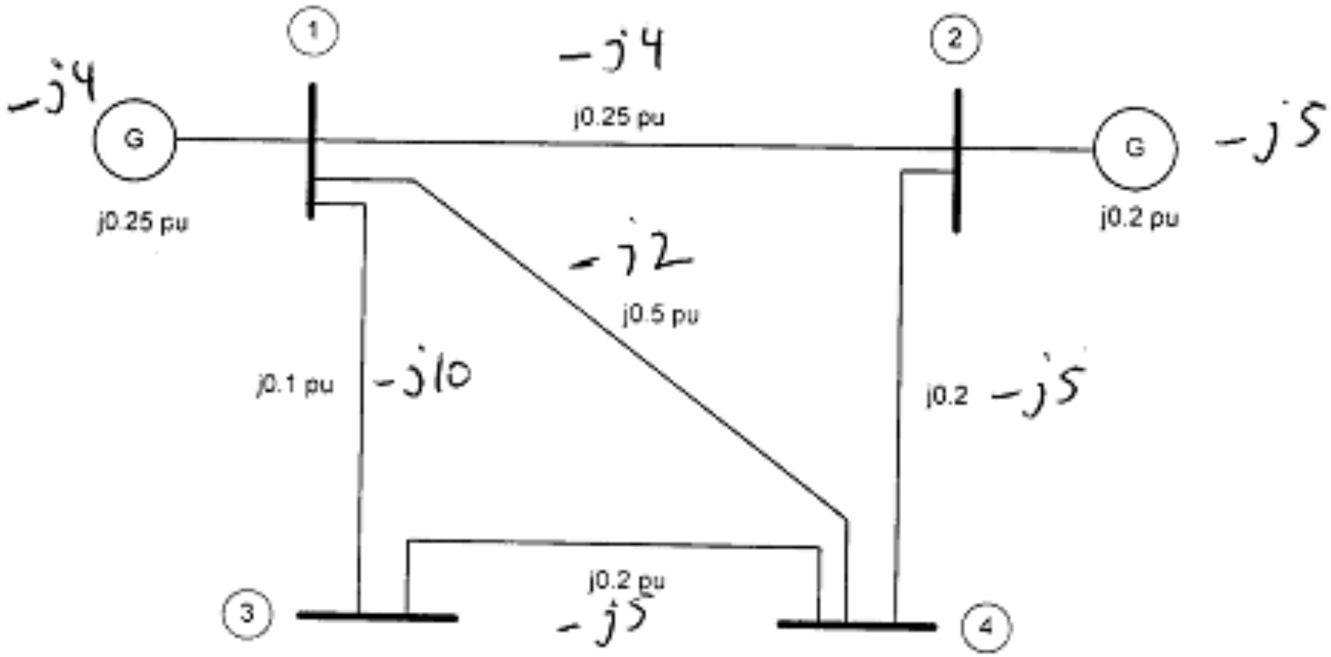
$$= \begin{bmatrix} -j7.33 & j3.67 \\ j3.67 & -j6.33 \end{bmatrix}^{-1} \begin{bmatrix} 1.0 \\ 1.2\angle -10^\circ \end{bmatrix}$$

$$= \begin{bmatrix} j0.192 & j0.111 \\ j0.111 & j0.223 \end{bmatrix} \begin{bmatrix} 1.0 \\ 1.2\angle -10^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0.324\angle 85.93^\circ \\ 0.378\angle 82.95^\circ \end{bmatrix} pu$$

Example 4

Department:
Student ID #:



2. For the power system given above

a) Find Y_{bus} matrix **(10 pts).**

b) Eliminate buses 3 and 4 using **Kron Reduction method** **(15 pts).**

Example 4: Solution

a)

$$Y_{11} = -j4 - j4 - j2 - j10 = -j20\ pu$$

$$Y_{22} = -j5 - j5 - j4 = -j14\ pu$$

$$Y_{33} = -j10 - j5 = -j15\ pu$$

$$Y_{44} = -j5 - j2 - j5 = -j12$$

5

$$Y_{Bus} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} -j20 & j4 & j10 & j2 \\ j4 & -j14 & 0 & j5 \\ j10 & 0 & -j15 & j5 \\ j2 & j5 & j5 & -j12 \end{bmatrix} \end{array}$$

b)

$$Y_{aa} = \begin{bmatrix} -j20 & j4 \\ j4 & -j14 \end{bmatrix}$$

5

$$Y_{ab} = \begin{bmatrix} j10 & j2 \\ 0 & j5 \end{bmatrix}$$

$$Y_{ab}^{T} = \begin{bmatrix} j10 & 0 \\ j2 & j5 \end{bmatrix}, \quad Y_{bb} = \begin{bmatrix} -j15 & j5 \\ j5 & -j12 \end{bmatrix}$$

5

$$Y_{red} = Y_{aa} - Y_{ab} Y_{bb}^{-1} Y_{ab}^{T}$$

$$Y_{bb}^{-1} = \frac{1}{-155} \begin{bmatrix} -j12 & -j5 \\ -j5 & -j15 \end{bmatrix} = \begin{bmatrix} j0.077 & j0.032 \\ j0.032 & j0.097 \end{bmatrix}$$

$$Y_{ab} Y_{bb}^{-1} Y_{ab}^{T} = \begin{bmatrix} -j9.363 & -j2.57 \\ -j2.57 & -j2.425 \end{bmatrix}$$

5

$$Y_{red} = \begin{bmatrix} -j20 & j4 \\ j4 & -j14 \end{bmatrix} - \begin{bmatrix} -j9.363 & -j2.57 \\ -j2.57 & -j2.425 \end{bmatrix}$$

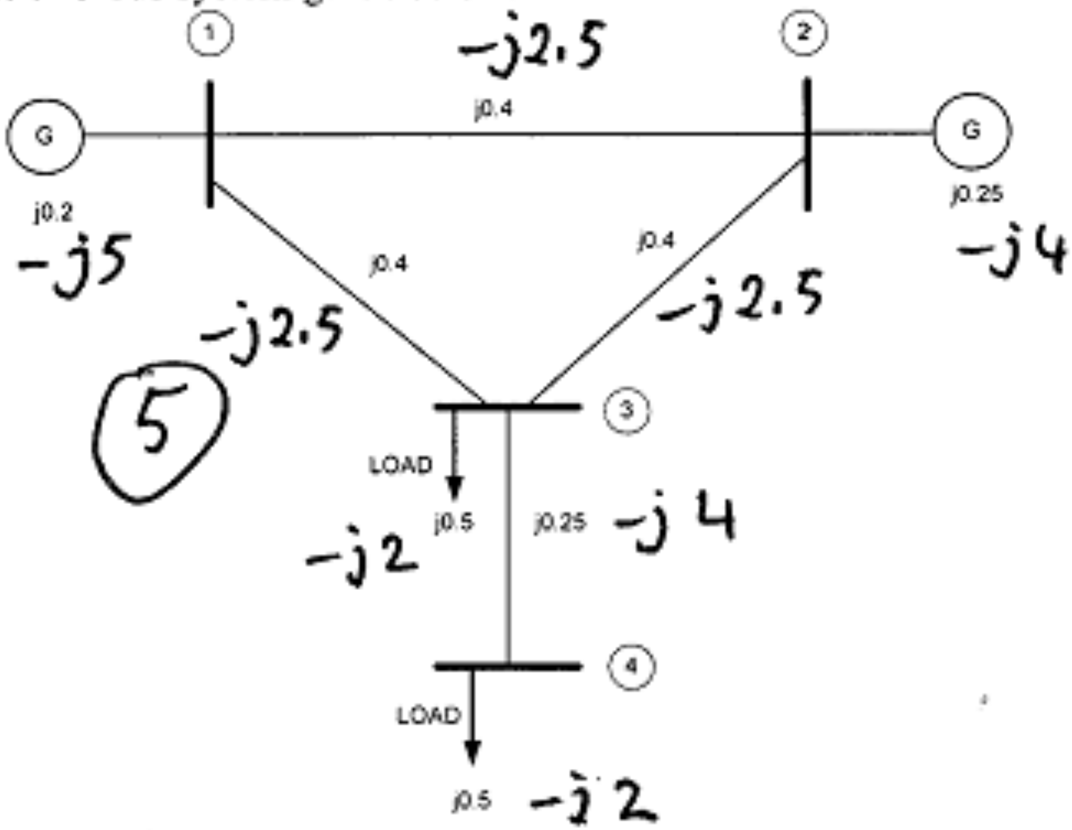
$$Y_{red} = \begin{bmatrix} -j10.632 & j6.57 \\ j6.57 & -j11.575 \end{bmatrix}$$

5

Example 5

Name/Last Name: 3
Student ID #:
Signature:

3. For the two-bus system given below.



a) Find Y_{bus} matrix (15).

b) Eliminate buses 3 and 4 using Kron Reduction metod (15).

Example 5:Solution

b) Eliminate buses 3 and 4 using Kron Reduction metod (15).

a)

$$Y_{Bus} = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cc|cc} -j10 & j2.5 & j2.5 & 0 \\ j2.5 & -j9 & j2.5 & 0 \\ \hline j2.5 & j2.5 & -j11 & j4 \\ 0 & 0 & j4 & -j6 \end{array}\right] \end{array}$$

(10)

b)

$$Y_{bb} = \begin{bmatrix} -j11 & j4 \\ j4 & -j6 \end{bmatrix}$$

$$Y_{bb}^{-1} = -\frac{1}{50}\begin{bmatrix} -j6 & -j4 \\ -j4 & -j11 \end{bmatrix}$$

$$Y_{bb}^{-1} = \begin{bmatrix} j0.12 & j0.08 \\ j0.08 & j0.22 \end{bmatrix}$$

(5)

$$Y^{red} = Y_{aa} - Y_{ab} Y_{bb}^{-1} Y_{ab}^{T}$$

$$Y_{bb}^{-1} Y_{ab}^{T} = \begin{bmatrix} j0.12 & j0.08 \\ j0.08 & j0.22 \end{bmatrix}\begin{bmatrix} j2.5 & j2.5 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.3 & -0.3 \\ -0.2 & -0.2 \end{bmatrix}$$

(-) sign !!

$$Y_{ab} Y_{bb}^{-1} Y_{ab}^{T} = \begin{bmatrix} j2.5 & 0 \\ j2.5 & 0 \end{bmatrix}\begin{bmatrix} -0.3 & -0.3 \\ -0.2 & -0.2 \end{bmatrix}$$

(5)

$$= \begin{bmatrix} -j0.75 & -j0.75 \\ -j0.75 & -j0.75 \end{bmatrix}$$

$$Y^{red} = \begin{bmatrix} -j10 & j2.5 \\ j2.5 & -j9 \end{bmatrix} - \begin{bmatrix} -j0.75 & -j0.75 \\ -j0.75 & -j0.75 \end{bmatrix}$$

$$= \begin{bmatrix} -j9.25 & j3.25 \\ j3.25 & -j8.25 \end{bmatrix}$$

(5)