5) Let $|f(x)| \leq 3$ for any $x \in \mathbb{R}$

then $|e^{f(x)}| \leq e^3$, that is $e^{f(x)}$ is bounded.

$$\left.\begin{array}{l} \lim\limits_{x\to 0} \arctan x = 0 \\ e^{f(x)} \text{ is bounded} \end{array}\right\} \Rightarrow \lim_{x\to 0} e^{f(x)} . \arctan x = 0.$$

$$\left.\begin{array}{l} \lim\limits_{x\to\infty} \dfrac{x^2+4x}{x^3+x} = \lim\limits_{x\to\infty} \dfrac{x^2(1+4/x)}{x^3(1+1/x^2)} = 0 \\ e^{f(x)} \text{ is bounded} \end{array}\right\} \lim_{x\to 0} e^{f(x)} \cdot \frac{x^2+4x}{x^3+x} = 0$$

$$\left.\begin{array}{l} \lim\limits_{x\to\infty} \sin x \text{ does not exist} \\ e^{f(x)} \text{ is bounded} \end{array}\right\} \lim_{x\to\infty} e^{f(x)} \sin x \text{ does not exist.}$$

Answer : B) I - II.

11) $$\lim_{x\to 2} \frac{\sin^2(4x-8).\left[1-\cos(3x-6)\right]}{24 . (x-2)^2 \, (x-2)^2 . (x-1)^2} =$$

$$= \lim_{x\to 2} \frac{\sin^2(4x-8)}{24 (x-2)^2} \cdot \frac{(1-\cos(3x-6))(1+\cos(3x-6))}{(x-2)^2 . (x-1)^2 (1+\cos(3x-6))}$$

$$= \lim_{x\to 2} \frac{\sin^2(4x-8)}{24(x-2)^2} \cdot \frac{\sin^2(3x-6)}{(x-2)^2 . (x-1)^2 (1+\cos(3x-6))}$$

$$= \lim_{x\to 2} \frac{\sin(4x-8)}{\frac{4(x-2)}{4}} . \frac{\sin(4x-8)}{\frac{4(x-2)}{4}} . \frac{\sin(3x-6)}{\frac{3(x-2)}{3}} . \frac{\sin(3x-6)}{\frac{3.(x-2)}{3}} . \frac{1}{24(x-1)^2 . (1+\cos(3x-6))}$$

$$= \frac{1}{\frac{1}{4}} . \frac{1}{\frac{1}{4}} . \frac{1}{\frac{1}{3}} . \frac{1}{\frac{1}{3}} . \frac{1}{24 . 2} = 3$$

A

12) $$\lim_{x \to 4} \frac{(\sqrt{x}-2) \cdot sgn(x^3-64)}{(x+4).(\sqrt{x}+2).(\sqrt{x}-2)} = \lim_{x \to 4} \frac{sgn(x^3-64)}{(x+4).(\sqrt{x}+2)}$$

$$\left.\begin{array}{l} \lim_{x \to 4^+} \frac{sgn(x^3-64)}{(x+4).(\sqrt{x}+2)} = \frac{1}{32} \\ \lim_{x \to 4^-} \frac{sgn(x^3-64)}{(x+4).(\sqrt{x}+2)} = -\frac{1}{32} \end{array}\right\} \lim_{x \to 4} \frac{sgn(x^3-64)}{(x+4).(\sqrt{x}+2)} \text{ does not exist.}$$

15) a) $$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x \cdot 1 \cdot \cos\left(\frac{1}{x}\right) = 0$$

$\left(\cos\left(\frac{1}{x}\right) \text{ is bounded} \atop \lim_{x \to 0} x = 0 \right)$ or by squeeze thm

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} x \cdot 0 \cdot \cos\left(\frac{1}{x}\right) = 0.$$

$f(0) = 0$

Hence f is continuous at $x = 0$.

b) $$\lim_{x \to 0^+} \frac{f(x) - \overset{0}{\overset{\shortparallel}{f(0)}}}{x - \underset{0}{\underset{\shortparallel}{x_0}}} = \lim_{x \to 0^+} \frac{x \cdot 1 \cdot \cos\left(\frac{1}{x}\right)}{x} = \lim_{x \to 0^+} \cos\left(\frac{1}{x}\right)$$

does not exist

$$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - x_0} = \lim_{x \to 0^-} \frac{0}{x} = 0$$

Hence f is not differentiable at