Name Surname: B 15.11.2022
Student Number: 17:35

GAZİ UNIVERSITY
ENGINEERING FACULTY
2022-2023 FALL
MATH101-MATHEMATICS I MIDTERM QUESTIONS

1. What is the equation of the normal line to the curve $y = \sin(5x + 3\pi)$ at $x_0 = -\frac{\pi}{2}$?

A) $y = -\frac{1}{2}x + \pi$ B) $y = -\frac{1}{5}x + 1$

C) $x = -\frac{\pi}{2}$ D) $y = 1$ E) $y = -\frac{\pi}{2}$

2. What is the value of the limit $\lim_{x\to 0^+} \frac{x\sin(\pi x)}{\sqrt{x^3+x^2}-x}$?

A) π B) 2π C) 0 D) $-\pi$ E) -2π

3. Evaluate $\lim_{x\to\infty} \frac{\sqrt{x+2}}{\sqrt{9x-1}}$.

A) 1 B) 0 C)3 D) $\frac{1}{9}$ E) $\frac{1}{3}$

4. Which of the following is true about $f(x) = (2x - \pi + 1)\sin x$ at $x_0 = \frac{\pi}{2}$?

A) The equation of the tangent line is $2x - y - \pi + 1 = 0$.
B) The equation of the normal line is $x + y - \pi - 4 = 0$.
C) The equation of the tangent line is $y = 0$.
D) The equation of the normal line is $x = \frac{\pi}{2}$.
E) The equation of the tangent line is $y = 1$.

5. What is the value of the limit $\lim_{x\to 2^+} \frac{\ln\left(\lfloor x^2\rfloor - 2\right)}{(x-2)^3}$?

A) 1 B) -1 C) 0 D) $-\infty$ E) ∞

6. Evaluate $\lim_{x\to -\infty} \left(\frac{7x^2-1}{2x^3+4x+4}\right) sgn(x)$. (sgn denotes the signum function)

A) $\frac{7}{2}$ B) ∞ C) -1 D) 0 E) $-\frac{7}{2}$

7. What is the equation of the tangent line to the curve $y = 2\pi\sin x - \cos x$ at $x_0 = \frac{\pi}{2}$?

A) $2x + 2y - 5\pi = 0$ B) $2x - 2y + 3\pi = 0$

C) $x + y + 1 = 0$ D) $2x - y - 5\pi = 0$

E) $x - 2y - \pi + 1 = 0$

8.Let $f(x) = \sqrt{25 - x^2}$ be a function on the interval $[0, 5]$. Find $f^{-1}(3)$.

A) 0 B) -4 C) $\frac{1}{2}$ D) -2 E) 4

9. Let $f(x)$ be a function with domain the set of real numbers $\mathbb{R}$. Suppose that $f^2(x) - 4f(x) + 4\cos^2 x \le 0$ for every real number x. What is the value of the limit $\lim_{x\to 0} f(x)$?

A) 0 B) 1 C) 2 D) 3 E) 4

10. Let $f(x) = \frac{1-x}{1+x}$ and $g(x) = \frac{1+x}{1-x}$. Which of the following is false?

A) The range of $(f \circ g)(x)$ is $\mathbb{R}$.
B) The domain of $(f.g)(x)$ is $\mathbb{R}\backslash\{-1, 1\}$.
C) $(f \circ g)(x)$ is decreasing on the interval $(-\infty, 0)$
D) The domain of $(f \circ g)(x)$ is $\mathbb{R}\backslash\{1\}$.
E) $(g \circ g)(x)$ is increasing on the interval $(-\infty, 0)$.

11. Find the derivative of $f(x) = \frac{\cos(2x-3)}{\tan(2-x^2)}$.

A) $\frac{2\sin(2x-3)\tan(2-x^2) - 2x\cos(2x-3)\csc^2(2-x^2)}{\tan^2(x^2-2)}$

B) $\frac{-2\sin(2x-3)\cot(2-x^2) + 2x\cos(2x-3)\sec^2(2-x^2)}{\tan^2(x^2-2)}$

C) $-2\sin(2x-3)\tan(2-x^2) + 2x\cos(2x-3)\sec^2(2-x^2)$

D) $2\left(x\cos(2x-3)\csc^2(2-x^2) - \sin(2x-3)\cot(2-x^2)\right)$

E) $2\cos(2x-3)\cot(2-x^2) + 2x\sin(2x-3)\sec^2(2-x^2)$

12. For what values of b is

$$g(x) = \begin{cases} \frac{x-b}{b+1}, & if \quad x < 0 \\ x^2 + b, & if \quad x \ge 0 \end{cases}$$

continuous at every x?

A) $b = 0$ or $b = -2$ B) $b = -2$ or $b = 2$
C) $b = -1$ or $b = 1$ D) $b = 0$ or $b = 1$
E) $b = 0$ or $b = 2$

13. Let $f(0) = 1$, $\lim_{h\to 0} \frac{f(h)-1}{h} = 3$ and $g(x) = \left(3x^3 - 1\right)^2 f(x)$. Find $g'(0)$.

A) 4 B) 1 C) 2 D) 3 E) -6

14. If $f(x) = 3x + \cos(x^2 - x)$, $g(y) = \sqrt{1 - y - y^2}$ and $h(z) = \sin z$, then what is the value of $(f \circ g \circ h)'(0)$?

A) 1 B) $-\frac{1}{2}$ C) 3 D) 0 E) $-\frac{3}{2}$

15. Consider the function

$$f(x) = \begin{cases} 2x^2 - 1, & if \quad -1 < x < 0 \\ x, & if \quad 0 \le x < 1 \\ 1, & if \quad x = 1 \\ -x + 4, & if \quad 1 < x < 2 \\ 0, & if \quad 2 \le x \le 3 \end{cases}.$$

Which of the following is **false** for $f(x)$?
A) f has a removable discontinuity at $x = 0$.
B) f has a jump discontinuity at $x = 2$.
C) f is discontinuous at three points in $(-1, 3]$.
D) f is rigth-continuous at $x = 0$.
E) f is continuous at $x = 3$.

16. Which of the following is the largest possible domain of the function $f(x) = \arccos(\log_3 x)$?
A) $[-1, 1]$ B) $[-\frac{\pi}{2}, \frac{\pi}{2}]$ C) $[\frac{1}{3}, 3]$ D) $\mathbb{R}\backslash\{0\}$ E) $(0, +\infty)$

17. If $f(x) = \dfrac{(x^3 - 2)\sin(3x)}{(2 - x)^3}$, then $f'(0)$?

A) 6 B) -1 C) $\frac{1}{3}$ D) -4 E) $-\frac{3}{4}$

18. Which of the following statement is true?

A) If $\lim\limits_{x \to 0} |f(x)| = 1$, then $\lim\limits_{x \to 0} f(x) = 1$ or $\lim\limits_{x \to 0} f(x) = -1$.

B) If $f(-1) = -1$ and $f(1) = 1$, then $f(c) = 0$ for some c in $(-1, 1)$.

C) If $\lim\limits_{x \to a} f(x)$ exists but $\lim\limits_{x \to a} g(x)$ does not exist, then $\lim\limits_{x \to a} (f(x) + g(x))$ does not exist.

D) If $|f|$ is continuous at a, then so is f.

E) If $f(x) > 5$ for all x, then $\lim\limits_{x \to 3} f(x) > 5$, if the limit exists.

19. In which of the following is the expression of the function $f(x) = \arcsin h(x)$ correctly given?

A) $\ln(x + \sqrt{x - 1})$, for $x \ge 1$
B) $\ln(x - \sqrt{x^2 + 1})$, for $x \in \mathbb{R}$
C) $\ln(x + \sqrt{x^2 + 1})$, for $x \in \mathbb{R}$
D) $\ln(x - \sqrt{x + 1})$, for $x \in \mathbb{R}$
E) $\ln(x + \sqrt{x + 1})$, for $x \in \mathbb{R}$

20. Which of the following can be obtained using the Intermediate Value Theorem?

A) Function $f(x) = \dfrac{2x + 1}{x}$ has a zero in the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

B) The equation $x^3 - x - 1 = 0$ has a solution in the interval $(-2, 1)$.

C) The equation $10\cos(x + \frac{\pi}{4}).\sin x = -9$ has a solution in the interval $\left[0, \frac{\pi}{4}\right]$.

D) The equation $x^4 + 2x^2 - 3 = 0$ has a solution in the interval $[0, 2]$.
E) Function $f(x) = x^3 - 2x - 10$ takes the value $f(x) = 47$ in the interval $[2, 4]$.

The value of each question is 5 points.
The duration is 120 minutes.

GROUP B											
	A	B	C	D	E		A	B	C	D	E
1						11					
2						12					
3						13					
4						14					
5						15					
6						16					
7						17					
8						18					
9						19					
10						20					

GAZİ ÜNİVERSİTESİ MÜHENDİSLİK FAKÜLTESİ

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Group B SINAV KAĞIDI Key

1. $y = \sin(5x+3\pi)$, $x_0 = -\frac{\pi}{2} \Rightarrow y_0 = 1$

$y' = \cos(5x+3\pi) \cdot 5$

$y'\big|_{-\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) \cdot 5 = 0$; tangent line is $y=1$ and normal line is $x = -\frac{\pi}{2}$.

2. $\lim\limits_{x\to 0^+} \dfrac{x\sin(\pi x)}{\sqrt{x^3+x^2}-x} = \lim\limits_{x\to 0^+} \dfrac{x\sin(\pi x)}{x(\sqrt{x+1}-1)}$

$= \lim\limits_{x\to 0^+} \dfrac{\pi\sin(\pi x)}{\pi x} \cdot \dfrac{x}{\sqrt{x+1}-1} = \pi \cdot \lim\limits_{x\to 0^+} \dfrac{x(\sqrt{x+1}+1)}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}$

$= \pi \lim\limits_{x\to 0^+} \dfrac{\cancel{x}(\sqrt{x+1}+1)}{\cancel{x+1-1}} = 2\pi.$

3. $\lim\limits_{x\to\infty} \dfrac{\sqrt{x+2}}{\sqrt{9x-1}} = \lim\limits_{x\to\infty} \dfrac{\cancel{\sqrt{x}}\sqrt{1+\frac{2}{x}}}{\cancel{\sqrt{x}}\sqrt{9-\frac{1}{x}}} = \frac{1}{3}.$

4. $f(x) = (2x-\pi+1) \cdot \sin x$, $x_0 = \frac{\pi}{2} \Rightarrow y_0 = 1$

$f'(x) = 2 \cdot \sin x + (2x-\pi+1) \cdot \cos x$

$f'\left(\frac{\pi}{2}\right) = 2 = m_T$

The equation of the tangent line is

$y - 1 = 2\left(x - \frac{\pi}{2}\right) \Rightarrow 2x - y - \pi + 1 = 0$

5. $\lim\limits_{x\to 2^+} \dfrac{\ln(\lfloor x^2 \rfloor - 2)}{(x-2)^3} = \lim\limits_{x\to 2^+} \dfrac{\ln 2}{(x-2)^3} = +\infty$

6. $\lim\limits_{x\to -\infty} \underbrace{\left(\dfrac{7x^2-1}{2x^3+4x+4}\right)}_{\to 0} \cdot \underbrace{\operatorname{sgn}(x)}_{\to -1} = 0$

7. $y = 2\pi \sin x - \cos x$, $\quad x_0 = \frac{\pi}{2} \Rightarrow y_0 = 2\pi$

$y' = 2\pi \cos x + \sin x$

$y'\big|_{\frac{\pi}{2}} = 1 = m_T$

the equation of the tangent line is

$$y - 2\pi = 1\left(x - \frac{\pi}{2}\right) \Rightarrow 2x - 2y + 3\pi = 0$$

8. $f(x) = \sqrt{25 - x^2}$

$y = \sqrt{25 - x^2} \Rightarrow x = \sqrt{25 - y^2}$, $x \in [0, 5]$

$\Rightarrow f^{-1}(x) = \sqrt{25 - x^2}$

$\Rightarrow f^{-1}(3) = 4$

9. $f^2(x) - 4f(x) + 4\cos^2 x \leq 0$

$(f(x) - 2)^2 - 4 + 4\cos^2 x \leq 0$

$0 \leq (f(x) - 2)^2 \leq 4 - 4\cos^2 x$

$\lim\limits_{x \to 0} 0 = 0$ and $\lim\limits_{x \to 0} 4 - 4\cos^2 x = 0$. Thus, by squeeze thm. $\lim\limits_{x \to 0} (f(x) - 2)^2 = 0$.

$\therefore \lim\limits_{x \to 0} f(x) = 2.$

10. $f(x) = \frac{1-x}{1+x}$ $\quad g(x) = \frac{1+x}{1-x}$

$D(f \circ g) = \{ x \in D(g) : g(x) \in D(f) \}$

$= \{ x \in \mathbb{R} \setminus \{1\} : \frac{1+x}{1-x} \in \mathbb{R} \setminus \{-1\} \}$ $\quad \cancel{1} + \cancel{x} = -1 + \cancel{x}$

$= \mathbb{R} \setminus \{1\}$

$(f \circ g)(x) = f(g(x)) = \dfrac{1 - \frac{1+x}{1-x}}{1 + \frac{1+x}{1-x}} = -x$ and $(f \circ g)(x)$ is decreasing on $(-\infty, 0)$.

$R(f \circ g) = \mathbb{R} \setminus \{-1\}$

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$$D(f\cdot g) = D(f) \cap D(g)$$
$$= \mathbb{R} \setminus \{-1,1\}$$

$$(g \circ g)(x) = g(g(x)) = \frac{1+\frac{1+x}{1-x}}{1-\frac{1+x}{1-x}} = -\frac{1}{x}$$ is increasing on $(-\infty,0)$.

11. $$f(x) = \frac{\cos(2x-3)}{\tan(2-x^2)}$$

$$f'(x) = \frac{-\sin(2x-3)2\tan(2-x^2) + \cos(2x-3)\sec^2(2-x^2).(+2x)}{\tan^2(2-x^2)}$$
$$= -2\sin(2x-3).\cot(2-x^2) + 2x.\cos(2x-3).\csc^2(2-x^2)$$
$$= 2(x\cos(2x-3).\csc^2(2-x^2) - \sin(2x-3)\cot(2-x^2))$$

12. $$\lim_{x\to 0^-} \frac{x-b}{b+1} = \frac{-b}{b+1}$$
$$\lim_{x\to 0^+} x^2+b = b$$
$-\frac{b}{b+1} = b \Rightarrow -b = b^2+b$
$\Rightarrow b^2+2b=0$
$\Rightarrow b(b+2)=0$
$b=0$ or $b=-2$

13. $f(0)=1$, $\lim_{h\to 0} \frac{f(h)-1}{h} = 3$

$g(x) = (3x^3-1)^2.f(x) \Rightarrow g'(x) = 2(3x^3-1).9x^2.f(x) + (3x^3-1)^2.f'(x)$

$g'(0) = 1.f'(0) = 1.3 = 3$

$$f'(0) = \lim_{h\to 0} \frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{f(h)-1}{h} = 3$$

14. $f(x) = 3x + \cos(x^2 - x)$

$g(y) = \sqrt{1 - y - y^2}$

$h(z) = \sin z$

$(f \circ g \circ h)'(0) = f'(g(h(0))) \cdot g'(h(0)) \cdot h'(0)$

$h'(z) = \cos z \Rightarrow h'(0) = 1$

$h(0) = 0$, $g'(y) = \frac{1}{2}(1 - y - y^2)^{-1/2} \cdot (-1 - 2y)$

$g'(h(0)) = g'(0) = \frac{1}{2} \cdot 1 \cdot (-1) = -\frac{1}{2}$

$f'(x) = 3 - \sin(x^2 - x) \cdot (2x - 1)$

$g(h(0)) = g(0) = 1$

$f'(g(h(0))) = f'(1) = 3$

$(f \circ g \circ h)'(0) = 3 \cdot \left(-\frac{1}{2}\right) \cdot 1 = -\frac{3}{2}$

15. $\lim_{x \to 0^-} 2x^2 - 1 = 0 - 1 = -1$
$\lim_{x \to 0^+} x = 0$
] f has a jump discont. at $x = 0$.

f is right-conti at $x = 0$

$f(0) = 0$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1$
$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} -x + 4 = 3$
] f has a jump disconti at $x = 1$

$f(1) = 1$

$\lim_{x \to 2^-} -x + 4 = 2$
$\lim_{x \to 2^+} 0 = 0$
] f has a jump disconti at $x = 2$.

$\lim_{x \to 0^-} f(x) = 0 = f(3)$ and 3 is an endpointi f is conti at $x = 3$.

GAZİ ÜNİVERSİTESİ MÜHENDİSLİK FAKÜLTESİ

Adı soyadı:

Bölüm:

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B

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16. $f(x) = \arccos(\log_3 x)$

$-1 \leq \log_3 x \leq 1$ and $x > 0$

$3^{-1} \leq x \leq 3^1$ and $x > 0$

$D(f) = \left[\frac{1}{3}, 3\right]$

17. $f(x) = \dfrac{(x^3-2)\sin(3x)}{(2-x)^3}$

$$f'(x) = \frac{\left[3x^2 \cdot \sin(3x) + (x^3-2) \cdot 3\cos 3x\right](2-x)^3 - (x^3-2) \cdot \sin(3x) \cdot 3(2-x)^2 \cdot (-1)}{(2-x)^6}$$

$f'(0) = -\frac{3}{4}$

18. Let $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$

$\lim_{x\to 0} |f(x)| = 1$ but neither $\lim_{x\to 0} f(x) = 1$ nor $\lim_{x\to 0} f(x) = -1$. Similarly, $f(-1) = -1$ and $f(1) = 1$ but there is no $c \in (-1,1)$ s.t. $f(c) = 0$.

For $f(x) = \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases}$, $|f|$ is cont. at $x=0$. But f is not cont. at $x=0$.

If $f(x) > 5$ for all x, then $\lim_{x\to 3} f(x) \geq 5$, if the limit exists.

19. $f(x) = \text{arcsih}(x)$

Let $y = \sinh^{-1}(x)$. Then $x = \sinh y = \frac{e^y - e^{-y}}{2} = \frac{e^{2y} - 1}{2e^y}$.

Therefore, $(e^y)^2 - 2xe^y - 1 = 0$. This is a quadratic equation in e^y, and it can be solved by the quadratic formula:

$$e^y = \frac{2x \pm \sqrt{4x^2+4}}{2} = x \pm \sqrt{x^2+1}.$$

Note that $x^2+1 > x$. Since e^y cannot be negative, we need to use the positive sign:

$$e^y = x + \sqrt{x^2+1}. \text{ Hence, } y = \ln(x + \sqrt{x^2+1})$$

for $x \in \mathbb{R}$.

20. $f(x) = x^4 + 2x^2 - 3$

f is cont. on $[0,2]$,

$\left.\begin{array}{l} f(0) = -3 \\ f(2) = 21 \end{array}\right]$ $-3 < 0 < 21$. By IVT there exists a number $c \in (0,2)$ s.t. $f(c) = 0$.

Hence, the equation $x^4 - 2x^2 - 3 = 0$ has a solution in the interval $[0,2]$.