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GAZİ UNIVERSITY
ENGINEERING FACULTY
2023-2024 FALL
MATH101-MATHEMATICS I MIDTERM QUESTIONS

1. Let $f(x) = \dfrac{1}{1-x}$ and $g(x) = \sqrt{x-1}$.
   How many of the following statements are true?
   I) The domain of $(f \circ f)(x)$ is $\mathbb{R} \setminus \{0\}$.
   II) The domain of $(f \circ g)(x)$ is $[1,2) \cup (2,\infty)$.
   III) $(f \circ f)(x)$ is an increasing function.
   IV) $(f \circ g)(x)$ is a one-to-one function.

   A) 1 B) 2 C) 3✓ D) 4 E) 0

2. Which of the following is true about
   $f(x) = (x+2)^2(x-1)$ at $x_0 = -2$?
   A) The equation of the tangent line is $x+y-2=0$.
   B) The equation of the normal line is $x-2y-1=0$.
   C) The equation of the tangent line is $x=0$.
   D) The equation of the tangent line is $y=-2$.
   E) The equation of the normal line is $x=-2$.✓

3. Let f be a function defined by
   $$f(x) = x^2 + sgn(x^2-4).$$
   Which one of the following is true?
   A) f is continuous on $\mathbb{R}$.
   B) f has removable discontinuity at the point -2.
   C) f has jump discontinuity at the point 0.
   D) f has removable discontinuity at the point 2.
   E) f has jump discontinuity at the point 2.✓

4. Which of the following statement is FALSE?
   A) If $g(x)$ is an odd function defined for all values of x, then $g(0)=0$.
   B) If $f(x)$ is odd, then $g(x) = f(x) - 2$ is also an odd function.✓
   C) There are two functions f and g such that $f \circ g = g \circ f$.
   D) There are two functions f and g such that their graphs are not straight lines but the graph of $f \circ g$ is a straight line.
   E) If $f(x)$ is one-to-one and $f(x)$ is never zero, then $h(x) = \dfrac{1}{f(x)}$ is also one-to-one.

5. For any $x \in \mathbb{R}$, let the function $f{:}\mathbb{R} \to \mathbb{R}$ satisfies $|f(x)| \leq 3$.
   I) $\lim_{x\to 0} e^{f(x)} \arctan x = 0$
   II) $\lim_{x\to\infty} e^{f(x)} \dfrac{x^2+4x}{x^3+x} = 0$
   III) $\lim_{x\to\infty} e^{f(x)} \sin x = 0$
   which of the above statements are true?
   A) Only I B) $I - II$✓ C) $I - III$
   D) $II - III$ E) $I - II - III$

6. Which of the following can be obtained using the Intermediate Value Theorem?

   A) The equation $e^x + x - 4 = 0$ is solvable on the interval [0,1].
   B) The equation $tanx + x + 1 = 0$ is solvable on the interval $[0, \frac{\pi}{4}]$
   C) The equation $x^2 sinx - 1 = 0$ is solvable on the interval [0,π].
   D) The equation $x^4 - 2x^2 - 1 = 0$ is solvable on the interval [0,2].✓
   E) The equation $x^3 + x^2 + 1 = 0$ is solvable on the interval [-1,1].

7. For what values of a and b is
   $$f(x) = \begin{cases} \dfrac{sin(ax)}{x} + cosbx & if \quad x < 0 \\ ax^2 + b(x-1) & if \quad x > 0 \\ 3 & if \quad x = 0 \end{cases}$$
   continuous at every x?
   A) a=2, b=-3✓
   B) a=0, b=2
   C) a=2, b=1
   D) a=-1, b=0
   E) a=-2, b=-1

8. What is the value of $f^{-1}(-3)$ for $f(x) = x.|x|+1$?

   A) 2 B) -2✓ C) 1 D) -1 E) $None$

9. Let f be a function defined by $f(x) = \ln x \sin x$. Then, what is the value of $f^{''}(\pi)$?

   A) $\frac{-2}{\pi}$✓ B) $\frac{\pi}{2}$ C) $\frac{-4}{\pi}$ D) $\frac{\ln \pi - 2}{\pi}$ E) 0

10. Let $(f \circ g)^{'}(2) = 4$, $f^{'}(0) = 1$, $g(2) = 0$ be given. Then, what is the value of $\lim\limits_{h \to 0} \frac{g(h+2)}{h}$?

    A) 0 B) 1 C) 2 D) 4✓ E) *Does not exist*

11. What is the value of $\lim_{x \to 2} \frac{\sin^2(4x-8) - \sin^2(4x-8)\cos(3x-6)}{24(x-2)^2(x^2-3x+2)^2}$?

    A) 1 B) 2 C) 3✓ D) 6 E) 12

12. What is the value of $\lim_{x \to 4} \frac{(\sqrt{x}-2).sgn(x^3-64)}{x^2-16}$?

    A) $\frac{-1}{32}$ B) 0 C) $\frac{1}{32}$ D) 1 E) *Does not exist*✓

13. Determine the largest possible domain of

$$f(x) = \left| \frac{\ln(\lfloor x-1 \rfloor - 2)}{sgn(2-x)^3} \right|$$

    Here $\lfloor x \rfloor$ denotes the greatest integer that is less than or equal to x.

14. Find the following limit without using L'Hospital's rule: $\lim_{x \to 0^+} \frac{\sqrt{x^2+4} - \sqrt{x+4}}{x - \sqrt{x}}$.

15. Let the function $f{:}\mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x\lfloor x+1 \rfloor \cos\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

    a) Is f continuous at $x = 0$?

    b) Is f differentiable at $x = 0$?

16. Let f be a function defined by $f(x) = \log_2\left(\frac{5^{-x}\cot(\ln x)}{\cos^3(x^2-1)}\right)$. Then, find the derivative of f.

*The duration is 100 minutes.
*For multiple choice questions, do not forget to mark the answers to table. Otherwise, you will not get points for those questions.
*The value of each multiple choice question is 5 points.
*The value of each classical question is 10 points.
*Use 3rd and 4th pages for answers of 13th,14th,15th&16th questions.
Good Luck!

GROUP A					
	A	B	C	D	E
1			✓		
2					✓
3					✓
4		✓			
5		✓			
6				✓	
7	✓				
8		✓			
9	✓				
10				✓	
11			✓		
12					✓